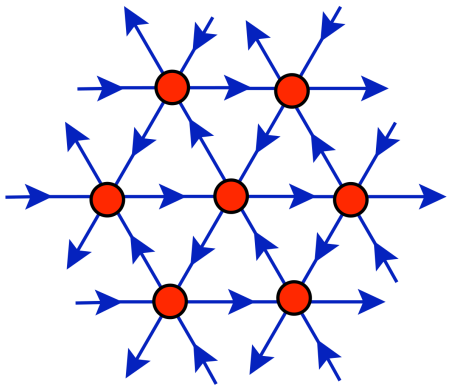
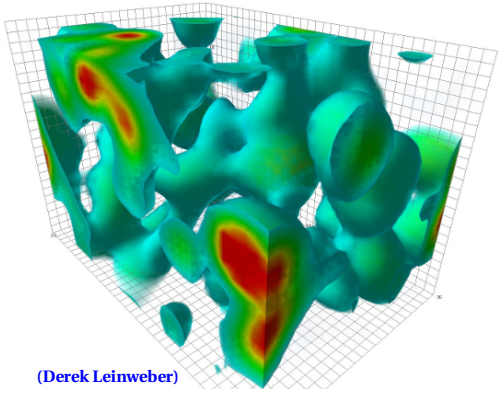
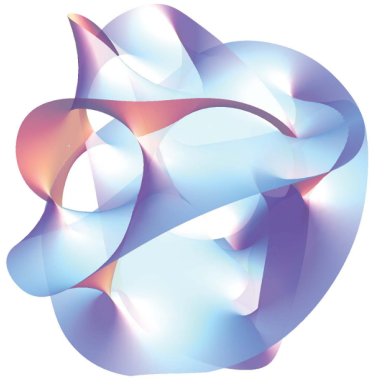
Progress and prospects of lattice supersymmetry







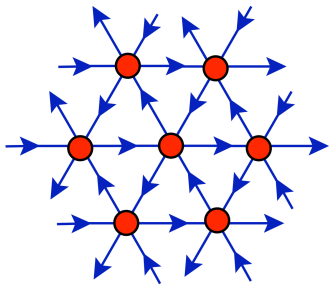
David Schaich (University of Bern)

Lattice 2018, July 24

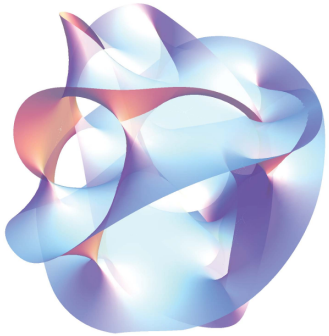
schaich@itp.unibe.ch

www.davidschaich.net

Overview



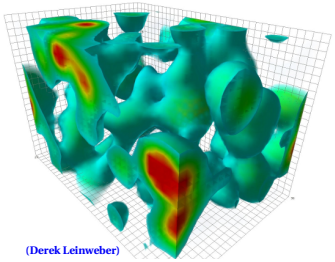
Significant progress currently being made
in lattice studies of supersymmetric QFTs



Motivation and background

Progress

- Lower-dimensional systems
- Minimally ($\mathcal{N} = 1$) supersymmetric Yang–Mills
- Maximally ($\mathcal{N} = 4$) supersymmetric Yang–Mills



Prospects and challenges

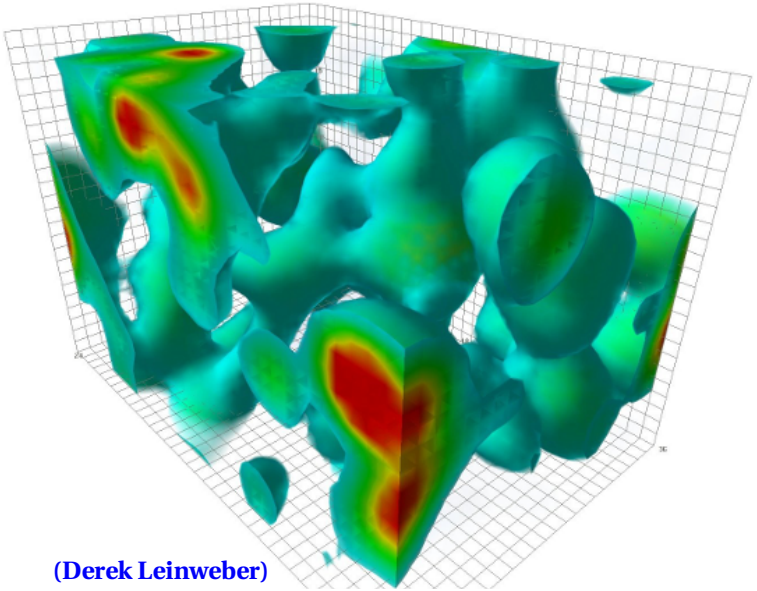
Motivations

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

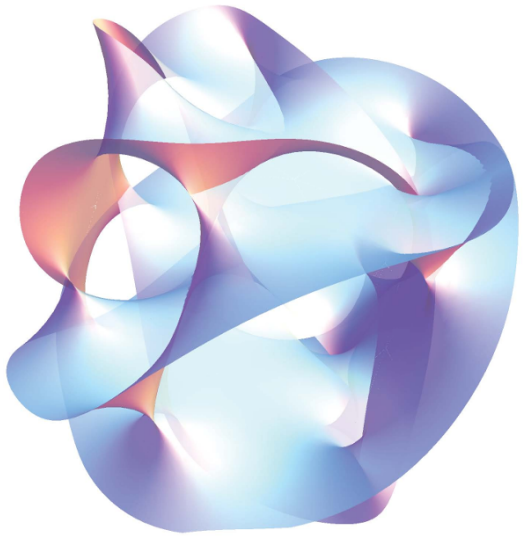
Many directions to be pursued

BSM

QFT



Holography



Obstruction

SUSY is a space-time symmetry

Extend 4d Poincaré symmetry

by $4\mathcal{N}$ spinor generators Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ with $\mathrm{I} = 1, \cdots, \mathcal{N}$

Super-Poincaré algebra includes $\left\{ Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{J}}_{\dot{\alpha}} \right\} = 2\delta^{\mathrm{IJ}} \sigma^{\mu}_{\alpha\dot{\alpha}} P_{\mu}$

$\longrightarrow$ infinitesimal translations that don't exist in discrete space-time

Constructions can become complicated

Supersymmetry $\longleftrightarrow$ Leibniz rule $\quad \partial\left[\phi\eta\right] = \left[\partial\phi\right]\eta + \phi\partial\eta$

Non-ultralocal product operator $\longrightarrow$ lattice Leibniz rule
but not gauge invariance (D'Adda–Kawamoto–Saito, 1706.02615)

Cyclic Leibniz rule $\longrightarrow$ partial lattice supersymmetry
but only (0+1)d QM so far (Kato–Sakamoto–So, 1609.08793)

Significant recent progress from three simplifications

Reduce dimensions | **Avoid scalars** | **Maximize symmetries**

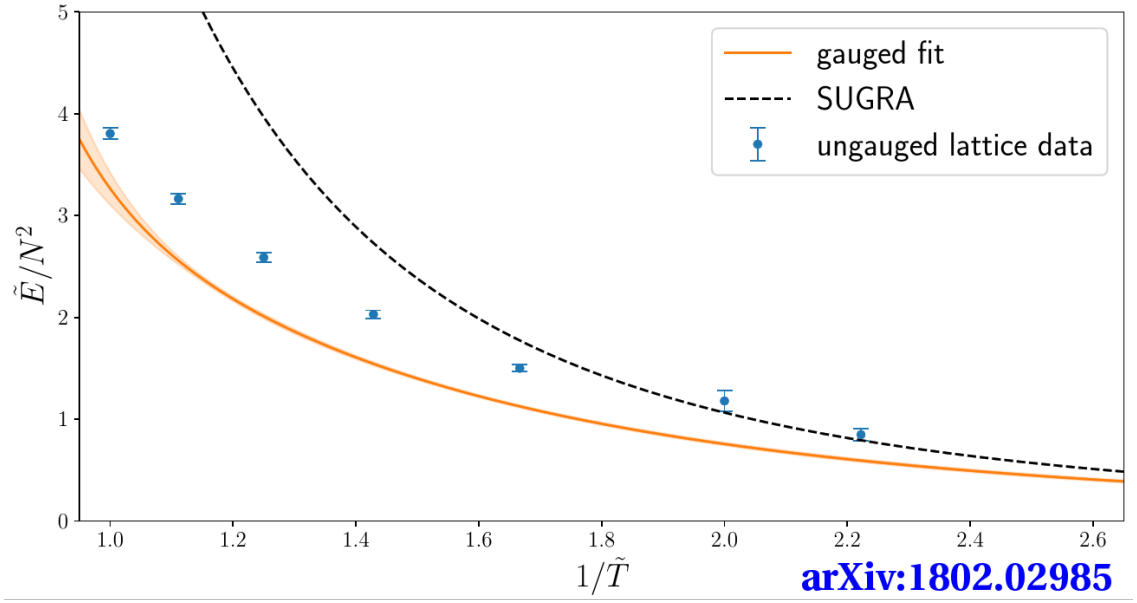
(I) Reduce dimensions

Ultimate simplification — compactify all spatial dimensions

4d SU(N) super-Yang–Mills $\longrightarrow$ quantum mechanics of $N \times N$ matrices

Many studies over past decade

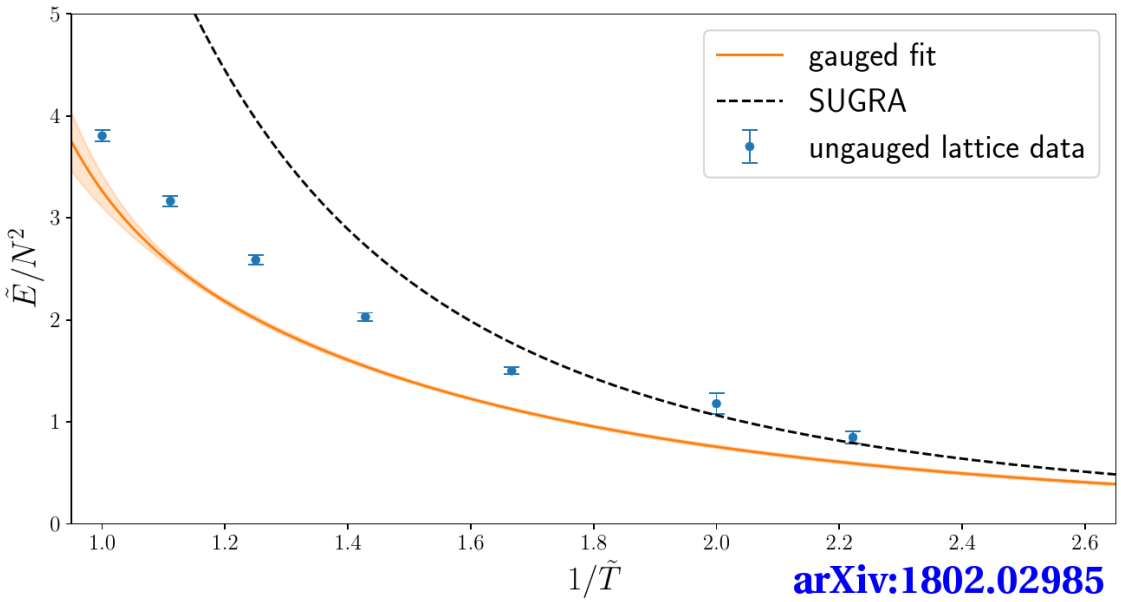
- Can use non-lattice cutoff (Nishimura et al., 1308.3525, 1311.5607)
- Can 'ungauge' SU(N) (Maldacena–Milekhin, 1802.00428
  Berkowitz–Hanada–Rinaldi–Vranas, 1802.02985)



(I) Reduce dimensions

Ultimate simplification — compactify all spatial dimensions

4d SU(N) super-Yang–Mills $\longrightarrow$ quantum mechanics of $N \times N$ matrices

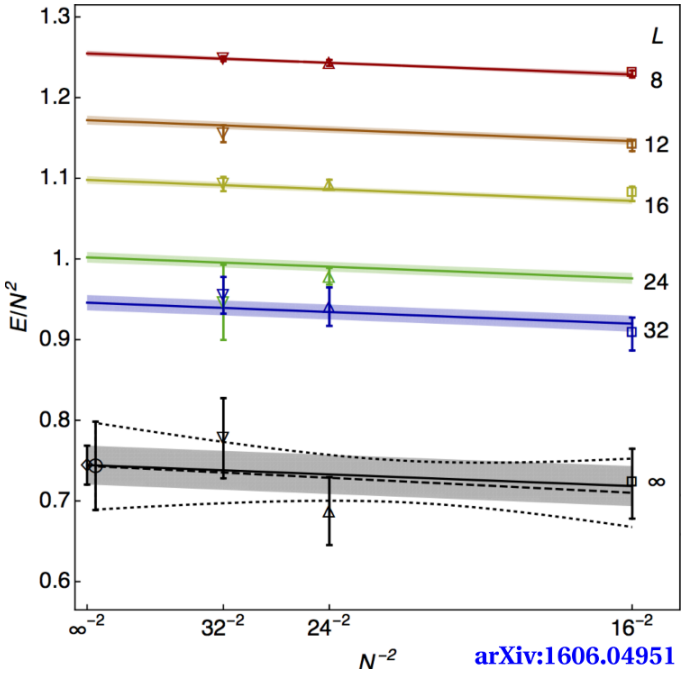


Holographic duality conjecture

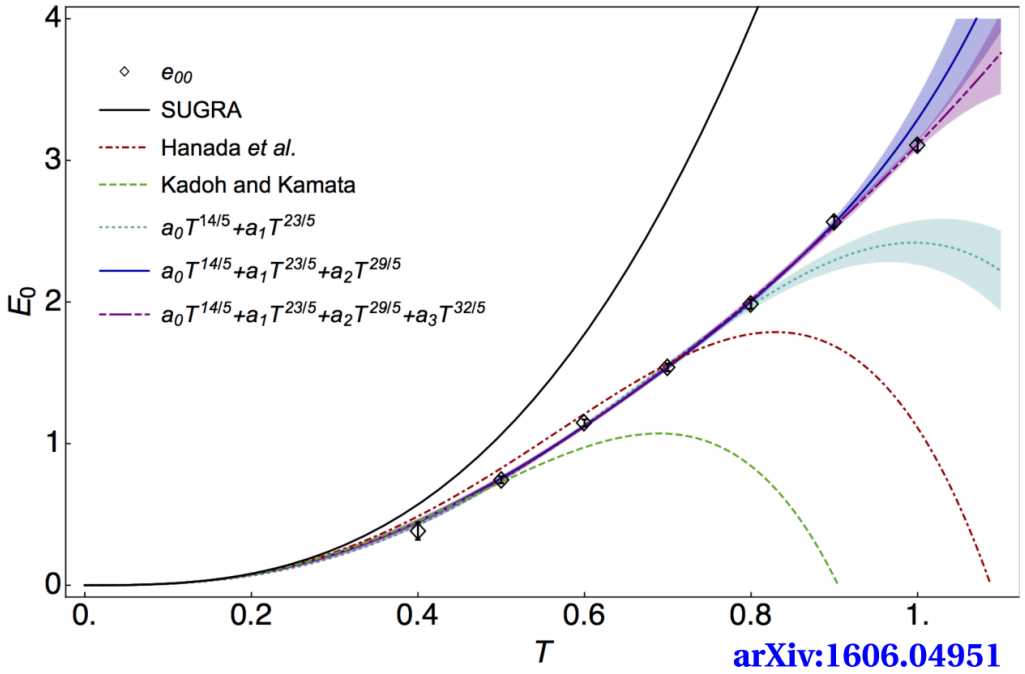
Thermodynamics of maximal SYM $\longleftrightarrow$ black holes in string theory

Testing holography with lattice super-Yang–Mills QM

State-of-the-art large-N continuum extrapolations
by Monte Carlo String/M-Theory Collaboration [1606.04951]



$\rightsquigarrow$



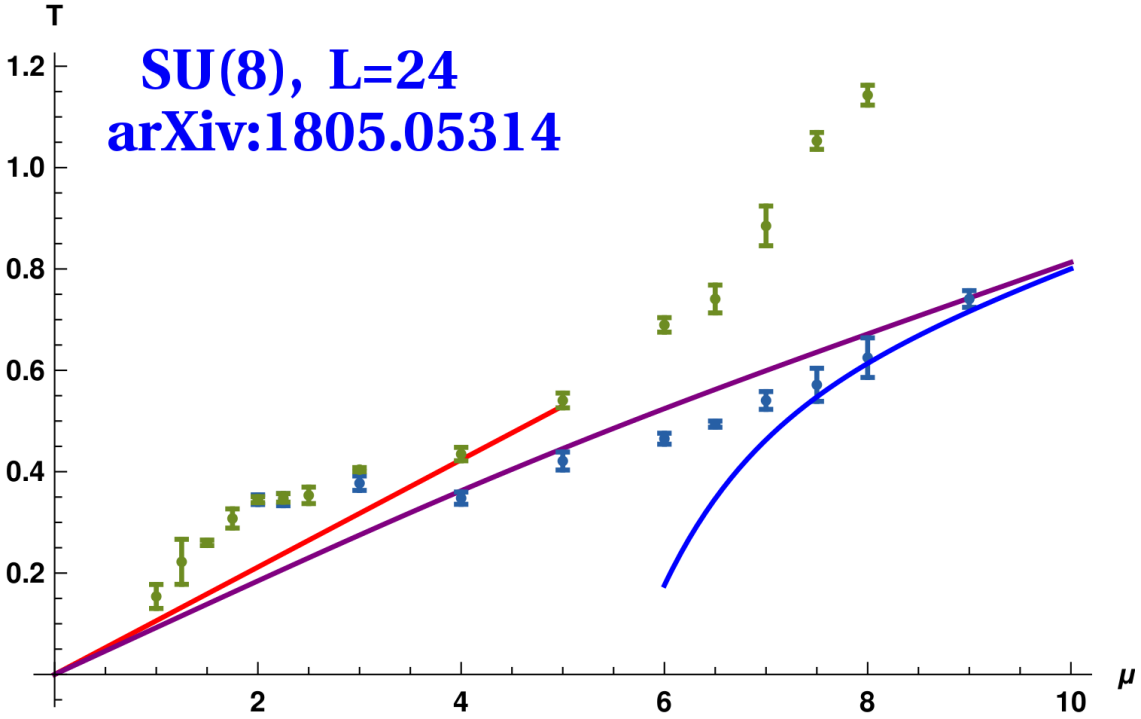
$16 \leq N \leq 32$ — MMMM code parallelizes individual $N \times N$ matrices

Recent progress: Supersymmetric mass deformation

Berenstein–Maldacena–Nastase, hep-th/0202021

Deform SYM QM while preserving maximal supersymmetry

$\longrightarrow$ more interesting features to study



Asano–Filev–Kováčik–O'Connor, 1805.05314

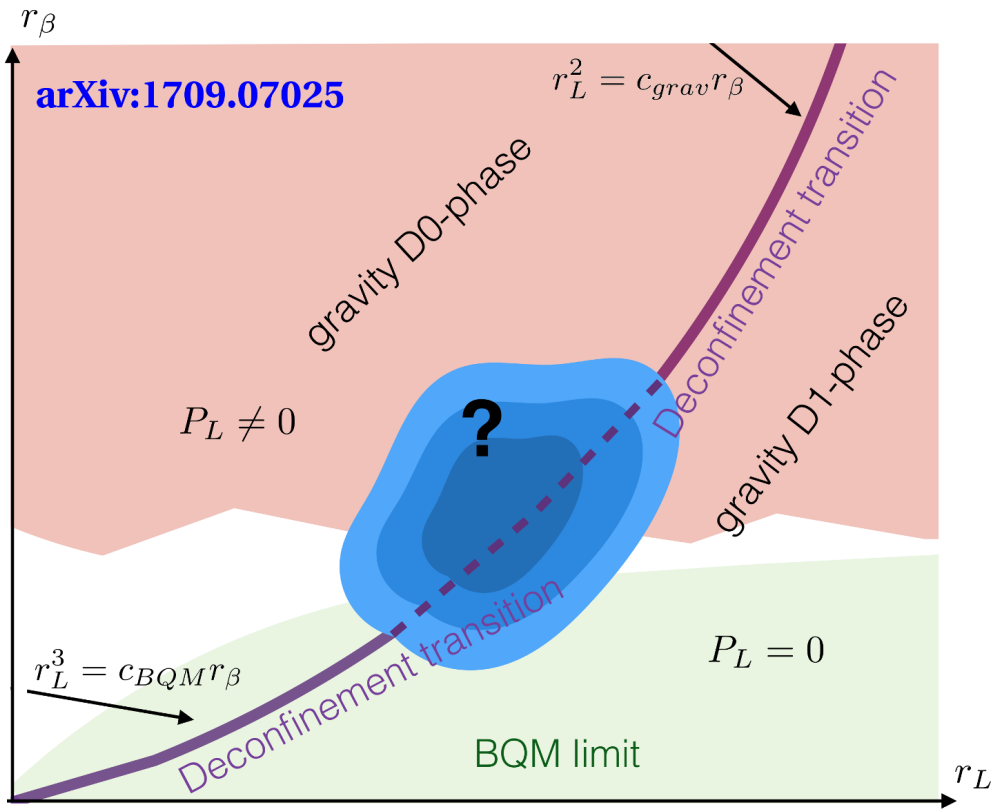
Lattice phase diagram, temperature vs. mass μ

For small μ, close to leading-order holography

Flavor and confinement transitions split for $\mu \gtrsim 3$

Recent progress: Two-dimensional super-Yang–Mills

Maximal SYM on $r_\beta \times r_L$ torus $\longrightarrow$ temperature $t = 1/r_\beta$



For decreasing r_L at low t
and large N

homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

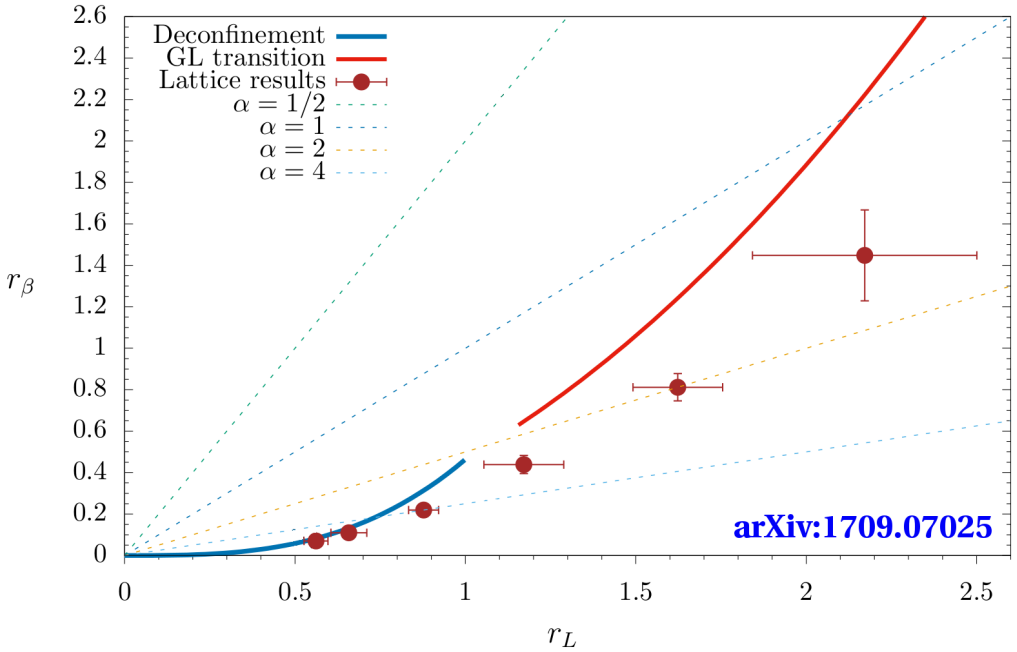
$\updownarrow$

"spatial deconfinement"
signalled by Wilson line

Recent progress: Two-dimensional super-Yang–Mills

Maximal SYM on $r_\beta \times r_L$ torus $\longrightarrow$ temperature $t = 1/r_\beta$

Catterall–Jha–DS–Wiseman, 1709.07025



Consistent with holography,
considering $6 \leq N \leq 16$,
multiple lattice sizes

Controlled extrapolations
are work in progress

[JHA—TUE]

(Independent work on 3d SYM: [GIEDT—THU])

(II) Avoid scalars: 4d $\mathcal{N} = 1$ super-Yang–Mills

SU(N) gauge theory with single massless adjoint Majorana fermion

Domain wall / overlap fermions

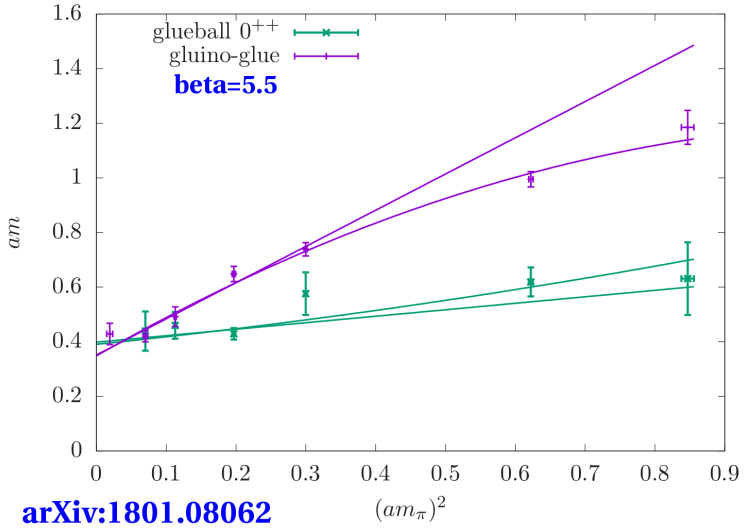
$\longrightarrow$ automatic ('accidental') supersymmetry in continuum limit

Current work uses Wilson fermions

Fine-tune gluino mass $\longrightarrow$ supersymmetry in chiral continuum limit

Supermultiplets in $\mathcal{N} = 1$ SYM spectrum

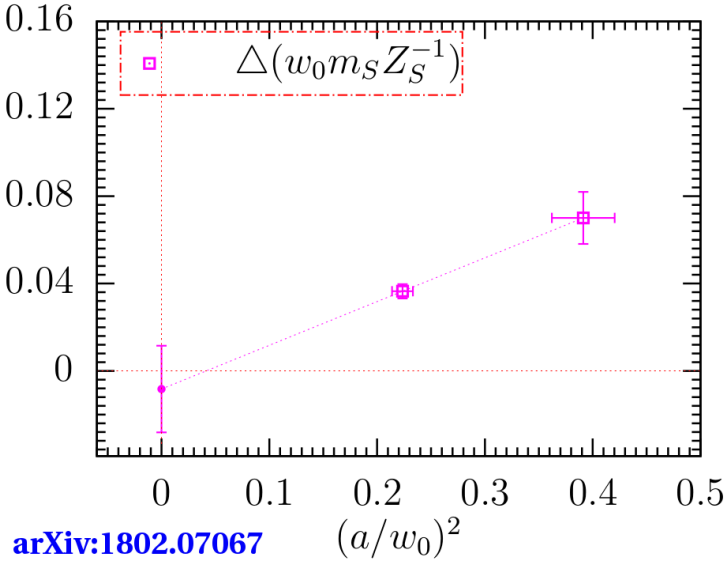
Desy–Münster–Regensburg–Jena, 1801.08062 & 1802.07067



Glueball degenerate
with fermionic partner in chiral limit

susy-breaking effects at $\beta = 5.5$
smaller than uncertainties

[Gerber—Fri], 'baryons': [Ali—Fri]



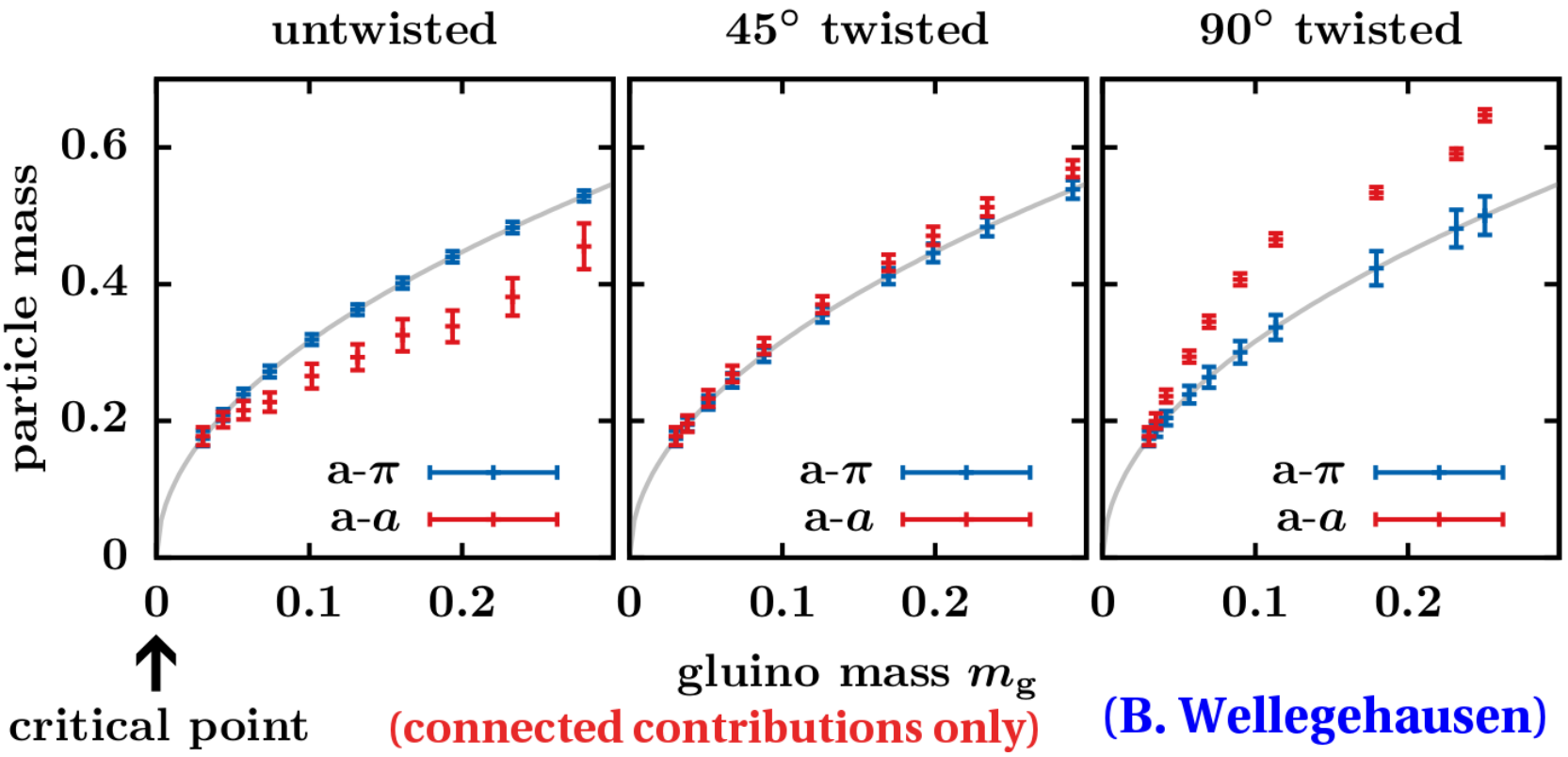
Supersymmetry breaking
from Ward identities

Consistent with $\mathcal{O}(a^2)$ lattice artifacts
expected for clover action

Other recent $\mathcal{N} = 1$ SYM progress

Analog of twisted-mass Wilson action for SYM

$\longrightarrow$ improve approach to continuum limit [STEINHAUSER—FRI]



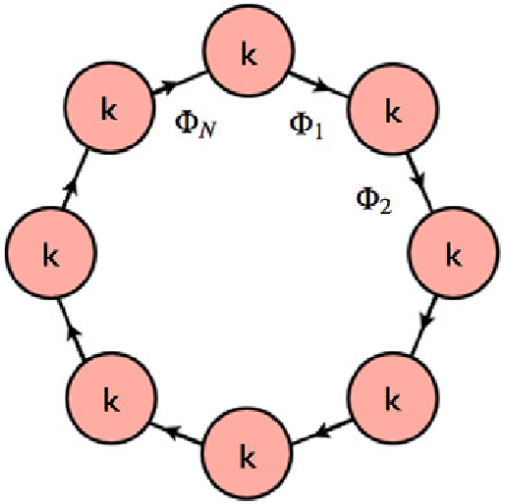
Phase structure: [LOPEZ—FRI] SYM gradient flow: [KADOH—FRI]

(III) Maximally supersymmetric Yang–Mills

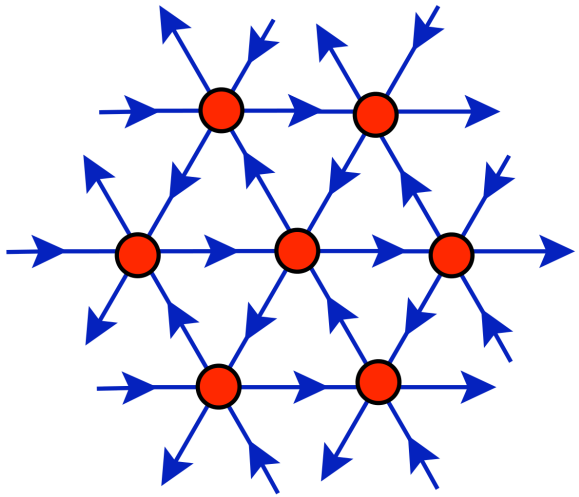
Conformal SU(N) gauge theory with $\mathcal{N} = 4$ fermions and 6 scalars

Reformulate theory to preserve closed sub-algebra $\{\mathcal{Q}, \mathcal{Q}\} = 0$

$\Longrightarrow$ recover other 15 $\mathcal{Q}$'s in continuum limit



Review:
Catterall, Kaplan & Ünsal
arXiv:0903.4881

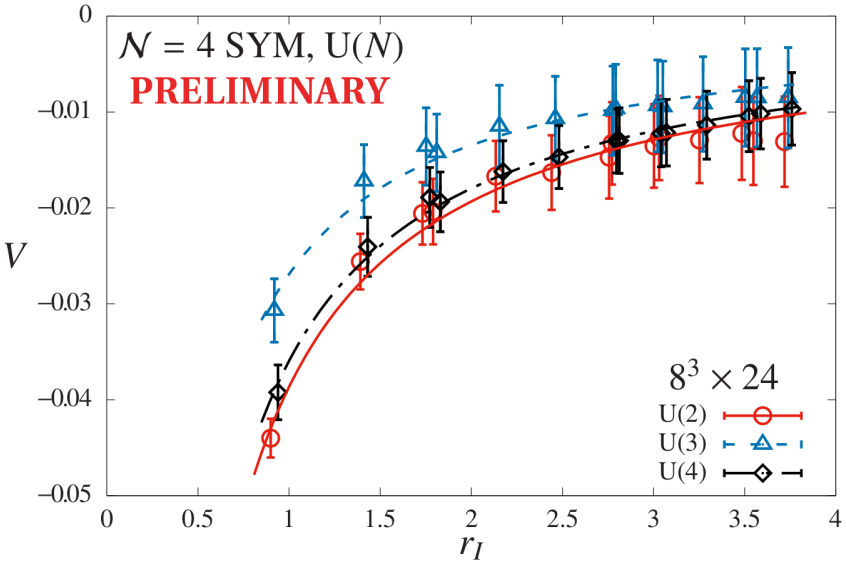


Public code: `github.com/daschaich/susy`

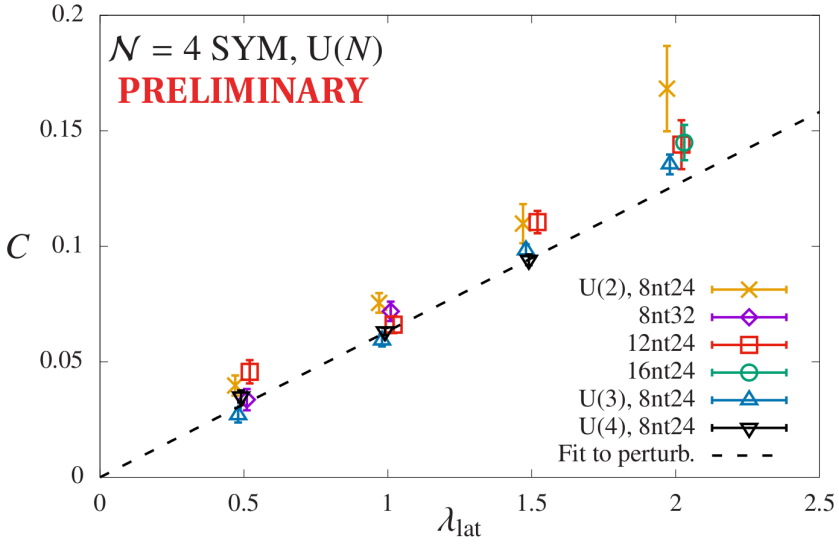
$\mathcal{N} = 4$ SYM static potential is Coulombic

Fitting $V(r) = A - C/r + \sigma r$

$\longrightarrow$ vanishing string tension σ for all 't Hooft couplings $\lambda = g^2 N$



$\rightsquigarrow$



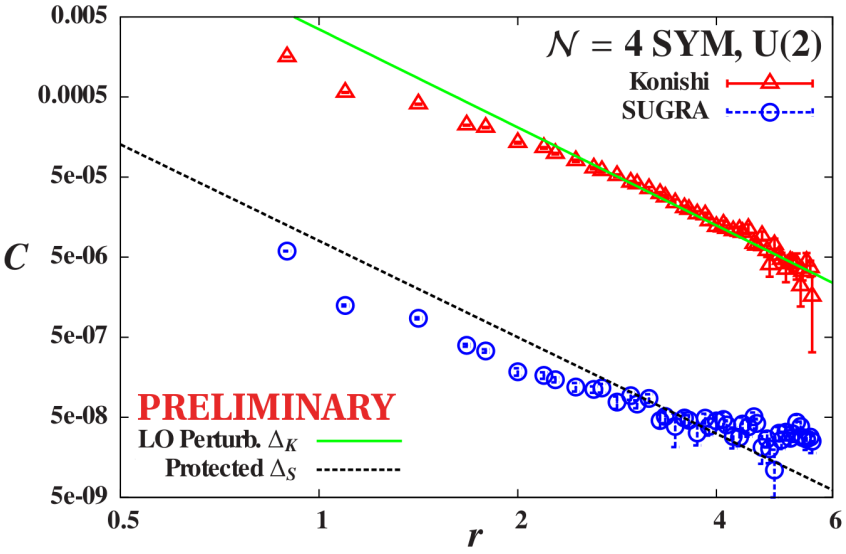
Results for Coulomb coefficient $C(\lambda)$

consistent with continuum perturbation theory: $C(\lambda) = \lambda/(4\pi) + \mathcal{O}(\lambda^2)$

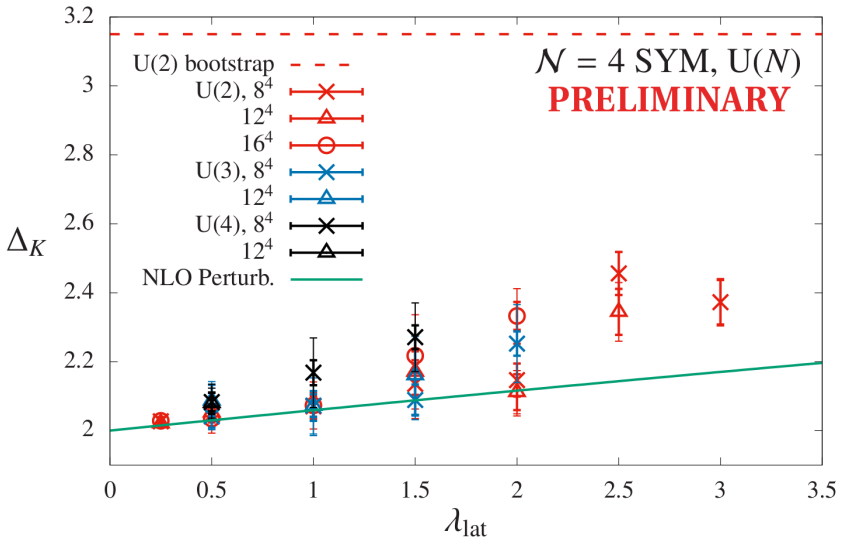
Conformal operator scaling dimensions

Conformality $\longrightarrow$ spectrum of scaling dimensions $\Delta(\lambda)$
govern power-law decays of correlation functions

$$\mathcal{O}_K(x) = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}(x)\Phi^{\mathrm{I}}(x)\right] \qquad C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K}$$



$\rightsquigarrow$



Observe power-law decay for simple Konishi correlator,
$\Delta_K(\lambda)$ from Monte Carlo RG consistent with perturbation theory

Future challenge: supersymmetric QCD

Add ‘quarks’ and squarks $\longrightarrow$ study electric–magnetic dualities, dynamical supersymmetry breaking and more

Fine-tuning back with a vengeance — $\mathcal{O}(10)$ parameters

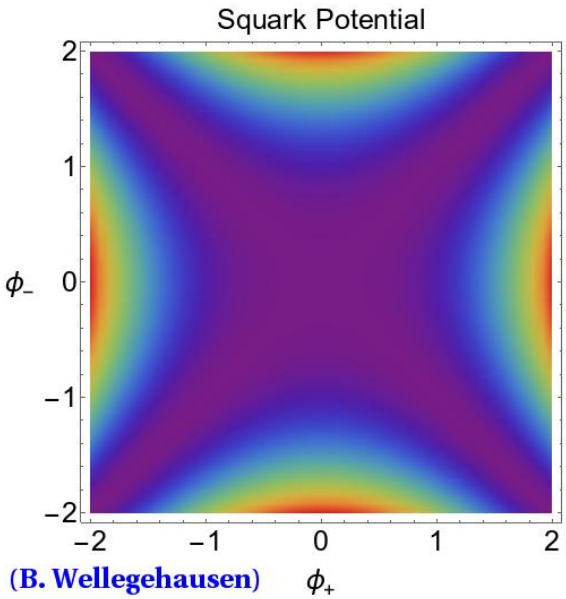
Ginsparg–Wilson fermions $\longrightarrow$ offline reweighting (Giedt [0903.2443])

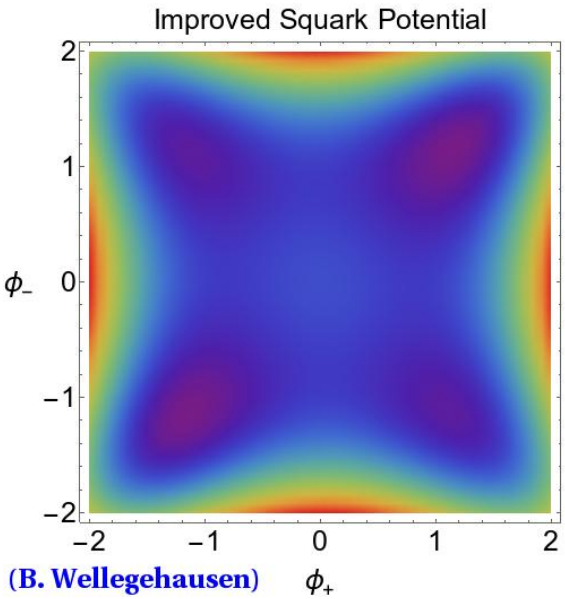
Fine-tuning Wilson-fermion superQCD

Current work uses Wilson fermions

Lattice perturbation theory guides fine-tuning [WELLEGEHAUSEN—FRI]

(also Costa–Panagopoulos [1706.05222])





Alternately include only fundamental + adjoint fermions

save scalars for future work [BERGNER—FRI]

Quiver construction for superQCD in $d < 4$

Preserve twisted supersymmetry sub-algebra on the lattice

Proposed by Matsuura [0805.4491] and Sugino [0807.2683],

first numerical study by Catterall & Veernala [1505.00467]

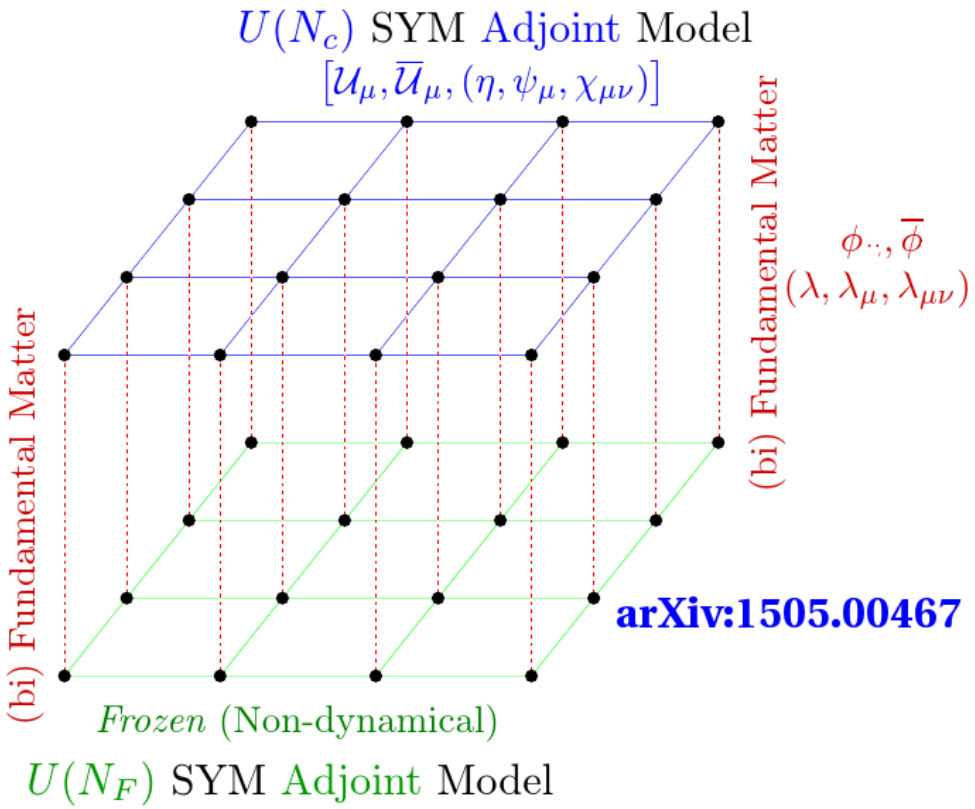
2-slice lattice SYM
with U(N) $\times$ U(F) gauge group

Adj. fields on each slice

Bi-fundamental in between

Decouple U(F) slice

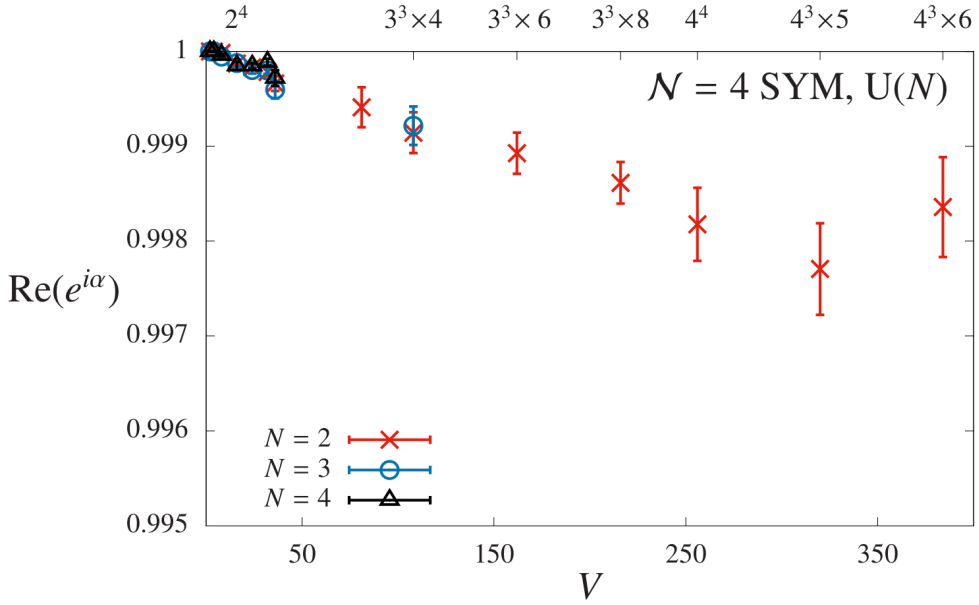
$\longrightarrow$ U(N) SQCD in $d-1$ dims.
with F fund. hypermultiplets

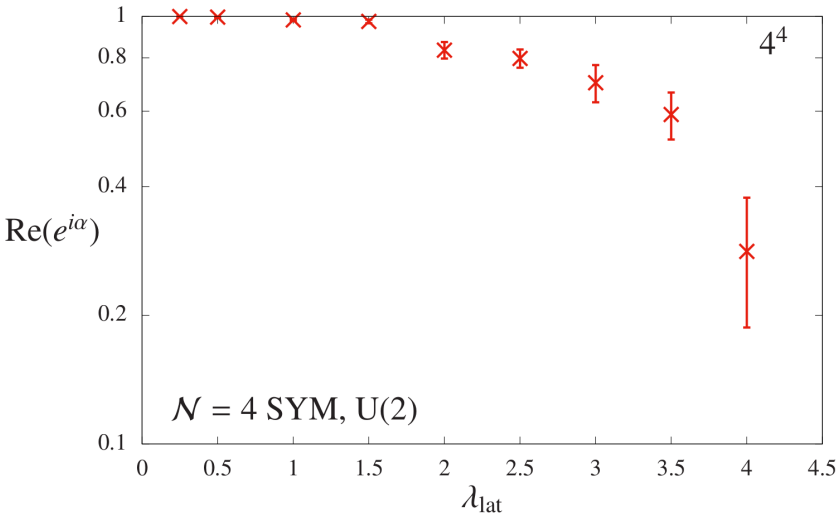


$\mathcal{N} = 4$ SYM sign problem

Fix $\lambda_{\text{lat}} = g^2 N = 0.5$

Pfaffian nearly real positive for all accessible volumes





Fix 4^4 volume

Fluctuations increase with coupling

Signal-to-noise becomes obstruction for $\lambda_{\text{lat}} \gtrsim 4$

Still need better understanding of $\mathcal{N} = 4$ SYM pfaffian phase

Recap: An exciting time for lattice supersymmetry

Significant progress currently being made
in lattice studies of supersymmetric QFTs

- Advancing to less-simplified lower-dimensional systems
- No scalars $\longrightarrow$ fine-tuned $\mathcal{N} = 1$ SYM with Wilson fermions
- Exact lattice supersymmetry $\longrightarrow$ $\mathcal{N} = 4$ super-Yang–Mills
- Many challenges being tackled: superQCD, sign problems, ...

Thank you!

Input: E. Rinaldi, J. Nishimura, H. So,
M. Costa, G. Bergner, B. Wellegehausen

schaich@itp.unibe.ch

www.davidschaich.net

Recap: An exciting time for lattice supersymmetry

Significant progress currently being made
in lattice studies of supersymmetric QFTs

- Advancing to less-simplified lower-dimensional systems
- No scalars $\longrightarrow$ fine-tuned $\mathcal{N} = 1$ SYM with Wilson fermions
- Exact lattice supersymmetry $\longrightarrow$ $\mathcal{N} = 4$ super-Yang–Mills
- Many challenges being tackled: superQCD, sign problems, ...

Thank you!

Input: E. Rinaldi, J. Nishimura, H. So,
M. Costa, G. Bergner, B. Wellegehausen

schaich@itp.unibe.ch www.davidschaich.net

Backup: Breakdown of Leibniz rule on the lattice

$\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$ is problematic

$\Longrightarrow$ try finite difference $\partial\phi(x) \longrightarrow \Delta\phi(x) = \frac{1}{a}\left[\phi(x+a) - \phi(x)\right]$

Crucial difference between ∂ and Δ

$$\Delta\left[\phi\eta\right] = a^{-1}\left[\phi(x+a)\eta(x+a) - \phi(x)\eta(x)\right]$$
$$= \left[\Delta\phi\right]\eta + \phi\Delta\eta + a\left[\Delta\phi\right]\Delta\eta$$

Only recover Leibniz rule $\partial\left[\phi\eta\right] = \left[\partial\phi\right]\eta + \phi\partial\eta$ when $a \to 0$

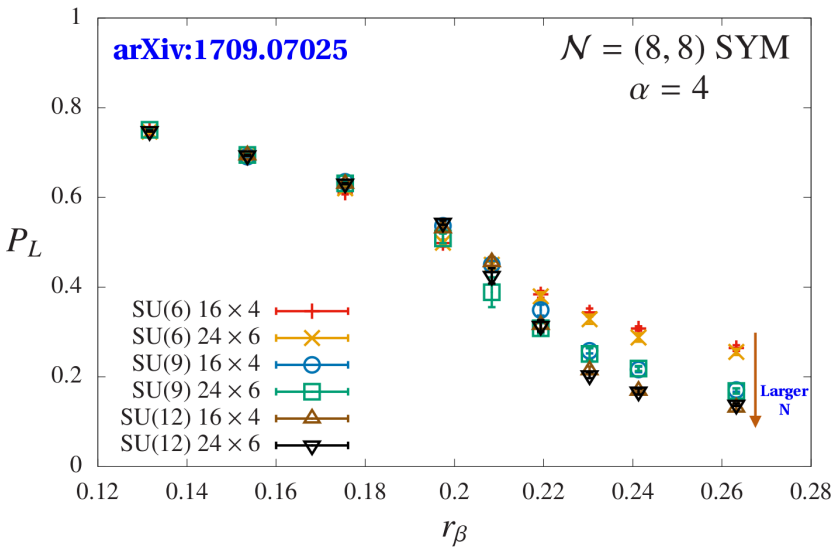
$\Longrightarrow$ "discrete supersymmetry" breaks down on the lattice

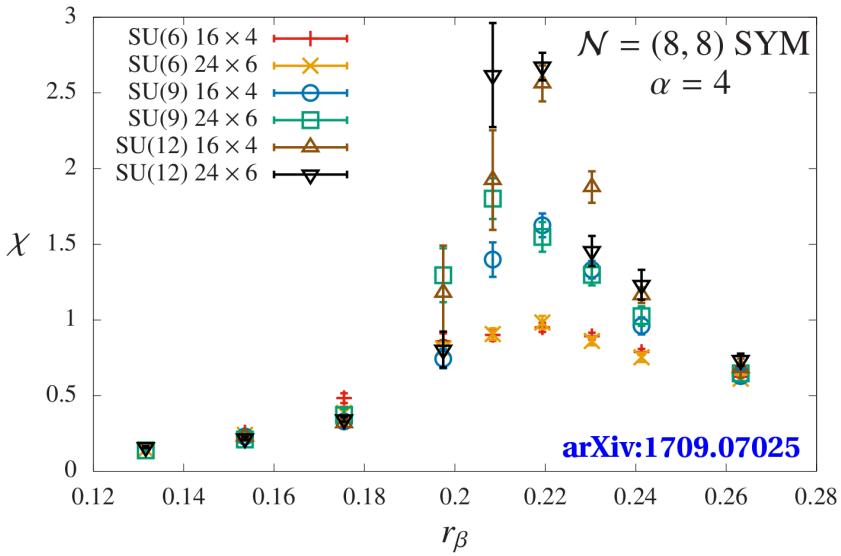
Backup: $\mathcal{N} = (8,8)$ SYM transition

Example spatial deconfinement transition

Clear transition in Wilson line and peak in corresponding susceptibility

Comparing $6 \leq N \leq 12$ for both 16×4 and 24×6 lattices with fixed $\alpha = r_L / r_\beta = 4$

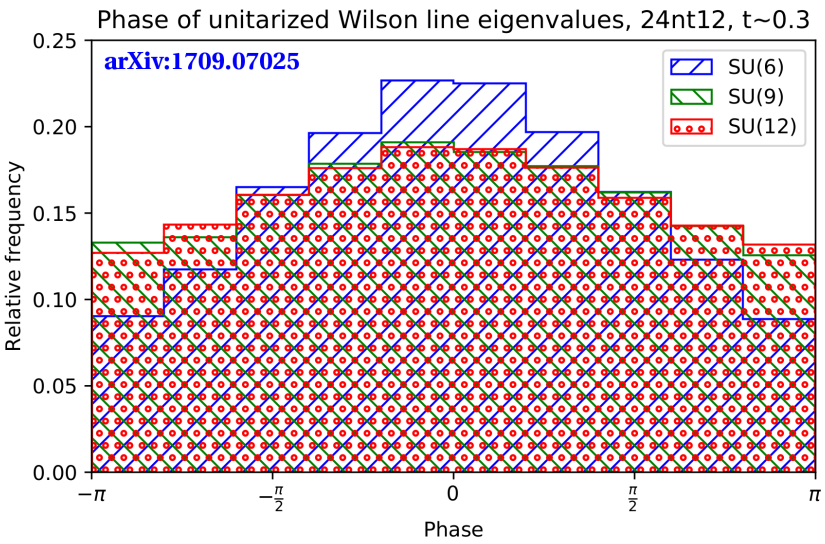


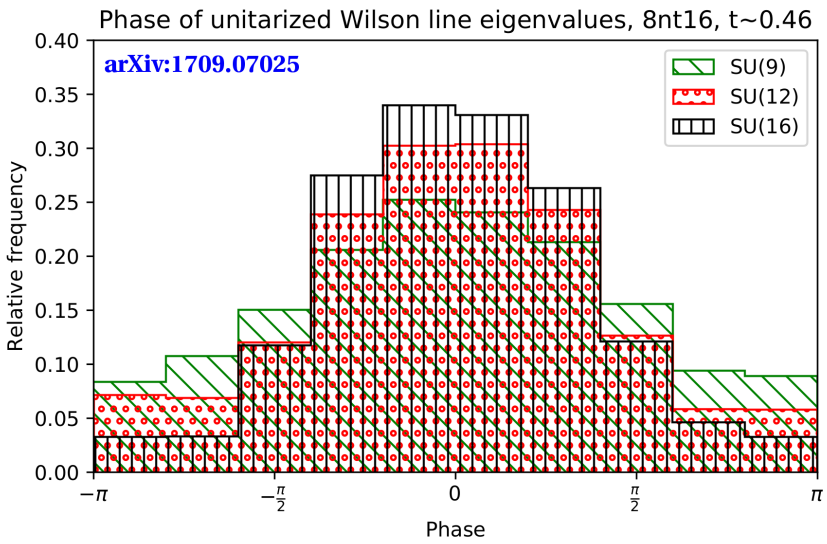


Next step: Controlled large-N and continuum extrapolations

Backup: $\mathcal{N} = (8, 8)$ SYM Wilson line eigenvalues

Check 'spatial deconfinement' through histograms
of Wilson line eigenvalue phases





Left: $r_L = 2r_\beta$ distributions more extended as N increases
$\longrightarrow$ dual gravity describes homogeneous black string (D1 phase)

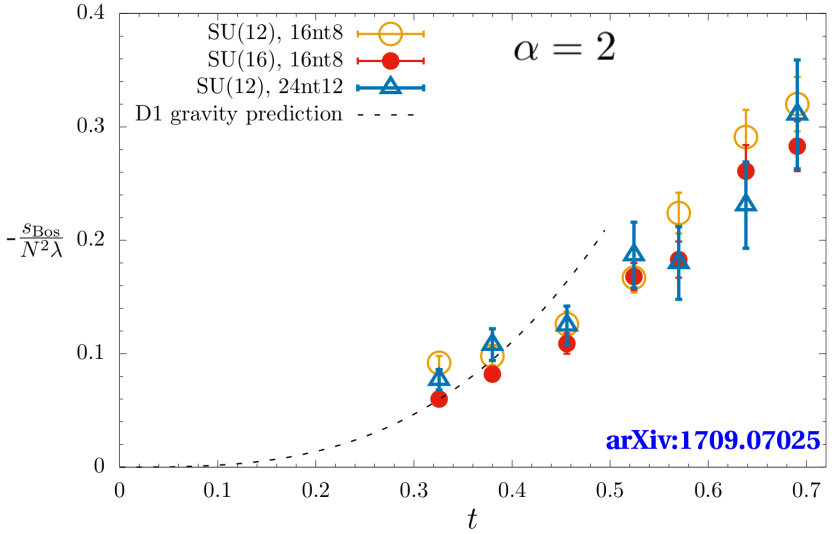
Right: $r_L = r_\beta/2$ distributions more compact as N increases
$\longrightarrow$ dual gravity describes localized black hole (D0 phase)

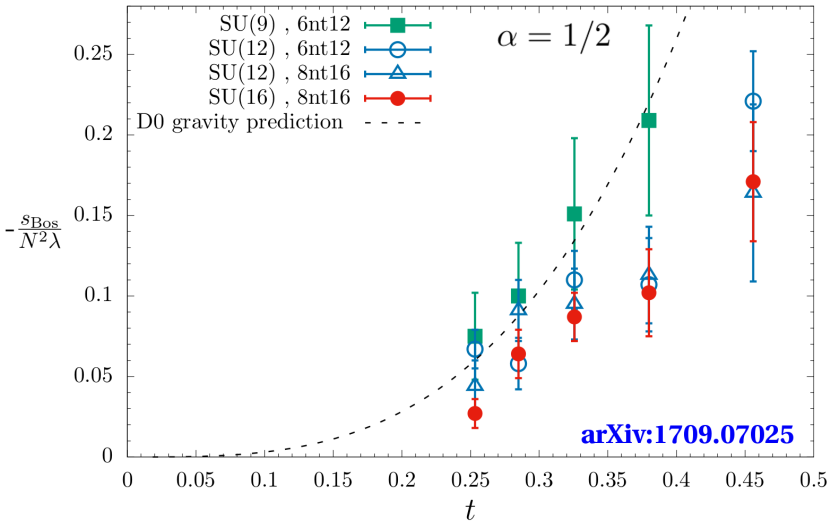
Backup: Dual black hole thermodynamics

Holography predicts bosonic action for corresponding dual black holes

$\propto t^3$ for large-r_L D1 phase $\propto t^{3.2}$ for small-r_L D0 phase

Lattice results consistent with holography for sufficiently low $t \lesssim 0.4$





Need larger $N > 16$ to avoid instabilities at lower temperatures

Backup: Improved $\mathcal{N} = 4$ SYM lattice action

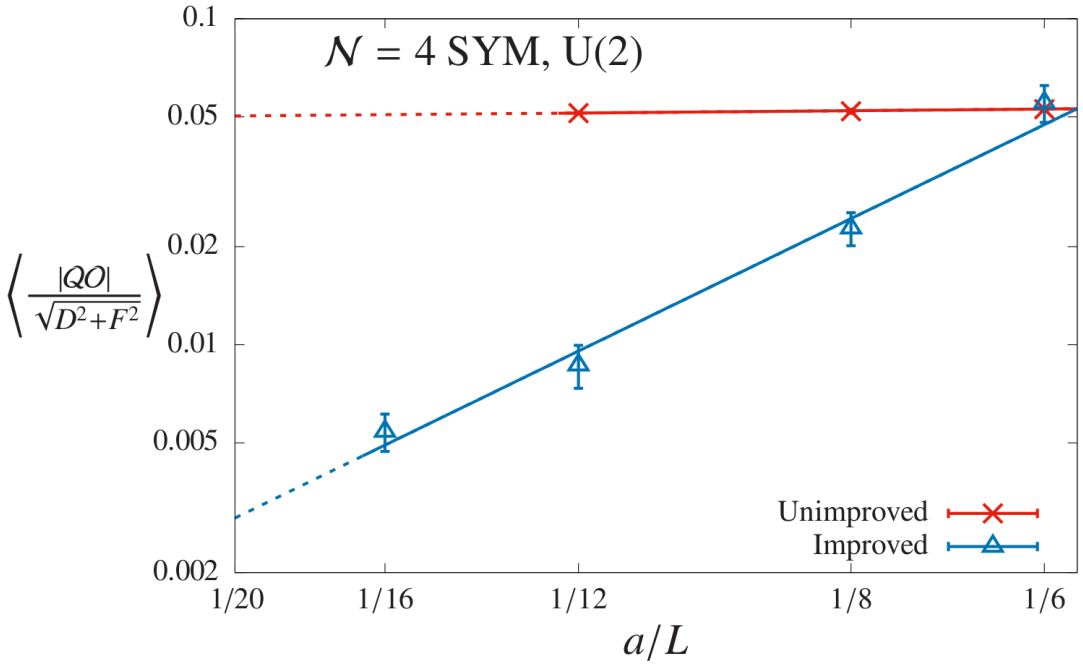
Ward identity violations

$\langle \mathcal{Q}\mathcal{O} \rangle \neq 0$

measure susy breaking

Here considering

$\mathcal{Q}\left[\eta \mathcal{U}_a \overline{\mathcal{U}}_a\right] = d\mathcal{U}_a \overline{\mathcal{U}}_a - \eta \psi_a \overline{\mathcal{U}}_a$



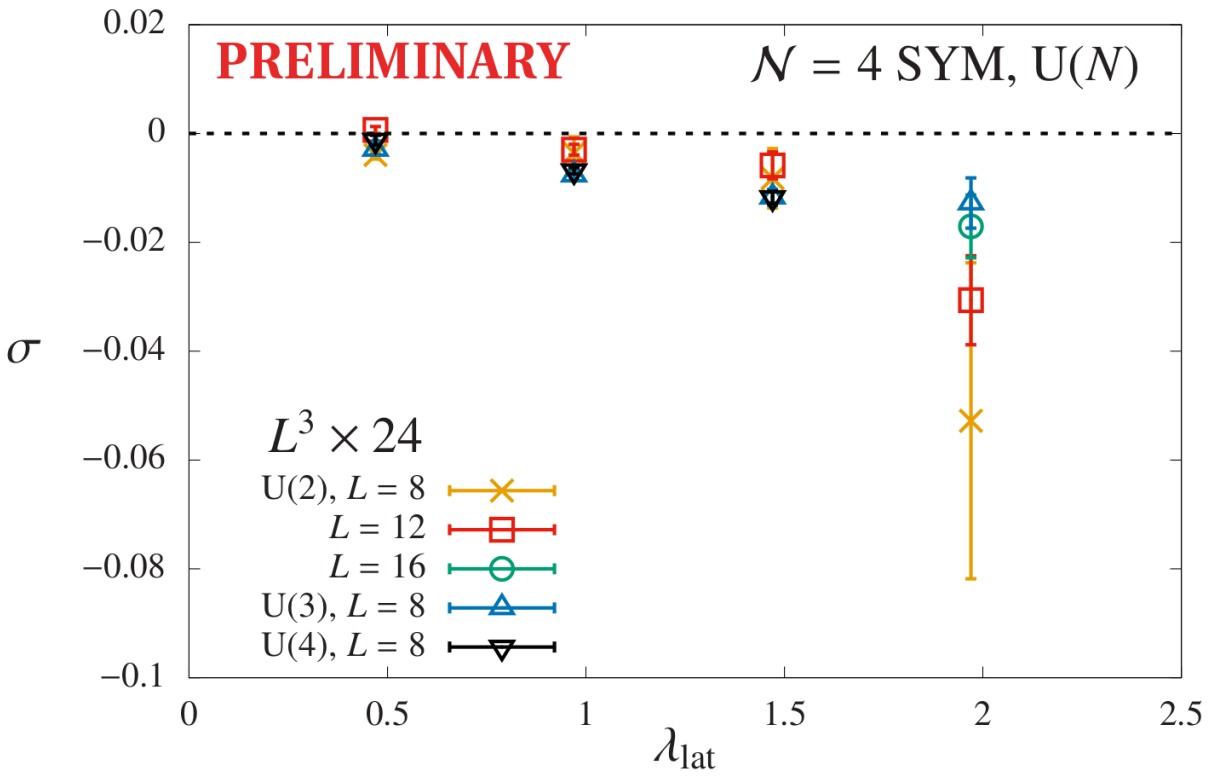
Improved action [arXiv:1505.03135] $\longrightarrow$ $\langle \mathcal{Q}\mathcal{O} \rangle \propto (a/L)^2$

Much better than action used before

Effective '$O(a)$ improvement', since $\mathcal{Q}$ forbids all dim-5 operators

Backup: $\mathcal{N} = 4$ SYM string tension vanishes

String tension σ from fits to confining form $V(r) = A - C/r + \sigma r$



Slightly negative values flatten $V(r)$ for $r \lesssim L/2$

$\sigma \to 0$ as accessible range of r increases on larger volumes

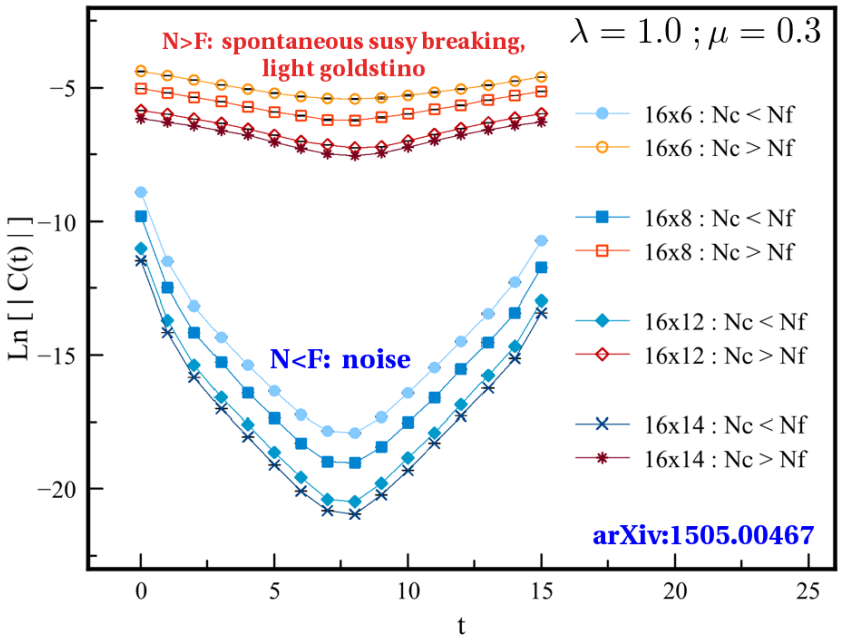
Backup: Dynamical susy breaking in 2d quiver SQCD

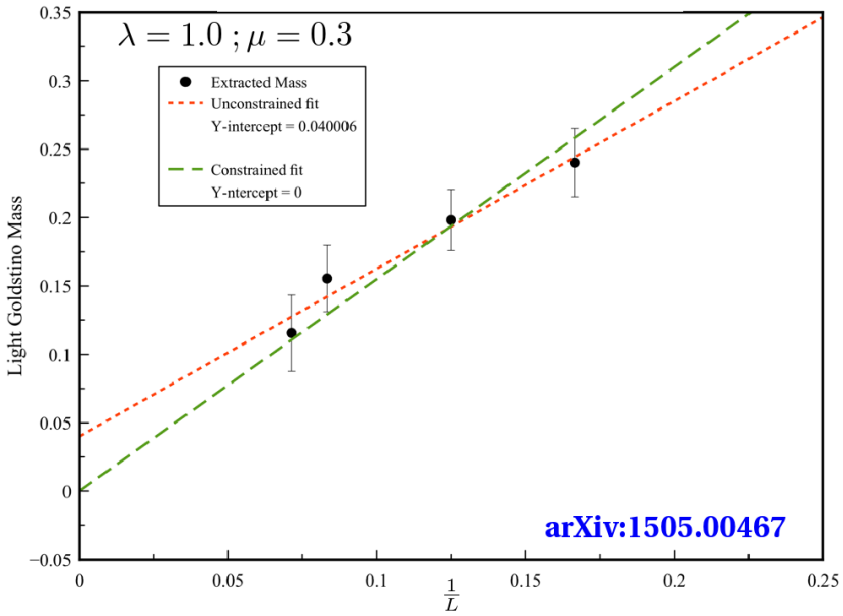
Auxiliary field e.o.m. $\longrightarrow$ Fayet–Iliopoulos D-term potential

$$d = \overline{\mathcal{D}}_a \mathcal{U}_a + \sum_{i=1}^{F} \phi_i \overline{\phi}_i + r \mathbb{I}_N \quad \longrightarrow \quad S_D \propto \sum_{i=1}^{F} \left(\mathrm{Tr}\left[\phi_i \overline{\phi}_i + r \mathbb{I}_N \right] \right)^2$$

Zero out N diagonal elements via F scalar vevs

or else susy breaking, $\langle \mathcal{Q}\eta \rangle = \langle d \rangle \neq 0 \longleftrightarrow \langle 0 | H | 0 \rangle > 0$





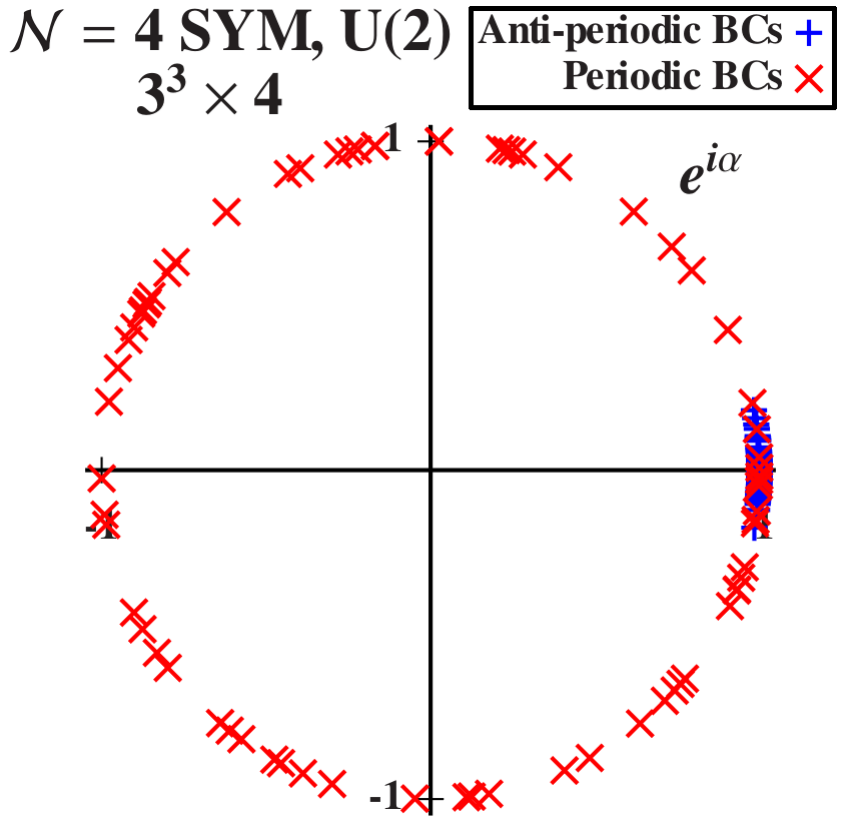
Backup: $\mathcal{N} = 4$ SYM sign problem puzzles

Periodic temporal boundary conditions for the fermions

$\longrightarrow$ obvious sign problem, $\langle e^{i\alpha} \rangle_{pq} \approx 0$

Anti-periodic BCs $\longrightarrow$ $e^{i\alpha} \approx 1$, phase reweighting negligible

Why such sensitivity to the BCs?

Other $\langle \mathcal{O} \rangle_{pq}$ are nearly identical for these two ensembles

Why doesn't sign problem affect other observables?