MATH327: StatMech and Thermo, Spring 2026

Extra practice — Relativistic ideal gas

Consider a classical gas of N indistinguishable particles in a volume $V = L^3$, with temperature $T = 1/\beta$. The canonical partition function for a single particle is

$$Z_1 = \left(\frac{L}{2\pi\hbar}\right)^3 \int e^{-E/T}\, d^3p.$$

Each particle moves non-relativistically with energy

$$E_n = p_n c,$$

where $p_n = \sqrt{(p_x)_n^2 + (p_y)_n^2 + (p_z)_n^2}$ is the magnitude of the nth particle's momentum $\vec{p}_n = (p_x, p_y, p_z)_n$ and c is the speed of light (which can be set to $c = 1$ by using natural units).

(a) Calculate the Helmholtz free energy of the gas.

(b) What are the internal energy and heat capacity of the gas?

(c) What is the entropy S of the gas?

(d) Compute the pressure of the gas in terms of N, T and V, comparing the resulting equation of state to the ideal gas law we derived for a non-relativistic gas.