

MATH327: StatMech and Thermo, Spring 2026

Extra practice — Moving spins

Consider N distinguishable spins that can move in a two-dimensional plane at fixed temperature $T = 1/\beta$. There is a background magnetic field of strength $H > 0$ perpendicular to the plane. Each spin has mass m and energy

$$E(\vec{p}, s) = \frac{1}{2m}p^2 - Hs$$

where $p^2 = p_x^2 + p_y^2 \geq 0$ while s can take the two values $s = 1$ (aligned with the background field) and $s = -1$ (anti-aligned against the background field). The single-particle partition function is

$$Z_1(\beta) = \sum_s \int \exp[-\beta E(\vec{p}, s)] d^2p.$$

- Compute the Helmholtz free energy for this system.
- Calculate the internal energy $\langle E \rangle$ and the entropy S .
- Describe the lowest-energy state of the system. What is the corresponding ground-state energy E_0 ? Show that $\langle E \rangle \rightarrow E_0$ in the zero-temperature limit.