

MATH327: StatMech and Thermo, Spring 2026

Extra practice — Central limit theorem

Consider a game of roulette that is slightly more realistic than the one introduced in our first tutorial. We now place bets on a particular red or black pocket, getting back ten times our bet if we win. To encourage players to keep playing, the casino will refund the bet if the ball ends up in a different pocket with the same colour as the one we bet on. We only lose our money if the ball ends up in a pocket with a different colour.

As in the tutorial, compute the gains and probabilities for winning $W = 0, 1, \dots, N$ times out of N total spins. Compare these exact results to predictions from the central limit theorem — either numerically integrating over appropriate intervals, or applying the approximation discussed in the tutorial and taking the probability distribution to be roughly constant within each interval.

Would you bet on the same pocket for every spin, or change your bet? Assuming you play for many spins $N \gg 1$, which game would you rather play — this one or the one from the tutorial?