

MATH327: StatMech and Thermo, Spring 2026

Extra practice — Walk with random step lengths

Consider a random walk that starts at $x = 0$ and consists of N steps. Each step has a length x_i randomly selected from the probability distribution

$$p(x) = \begin{cases} Ce^{-x} & \text{for } x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where C is a constant. The final position of the walker is

$$X = \sum_{i=1}^N x_i.$$

- Compute the constant C .
- What is the probability that any given step has length $x_i \leq 1$?
- What are the mean μ and variance σ^2 of the distribution $p(x)$?
- Assuming $N \gg 1$, how many steps are needed for the expectation value $\langle X \rangle$ of the final position of the walker to be at least one hundred (i.e., $\langle X \rangle \geq 100$)? After this number of steps, what is the interval $\langle X \rangle - \Delta \leq X \leq \langle X \rangle + \Delta$ within which the walker can be found with 50% probability?