MATH327: StatMech and Thermo, Spring 2026

Extra practice — Negative temperature

Consider a system of N distinguishable particles in which the energy of each particle can take only two distinct values, $\varepsilon > 0$ and 10ε. Denote by n_0 the number of particles that have energy ε, and by $n_1 = N - n_0$ the number of particles that have energy 10ε. Assume the system is in thermodynamic equilibrium with both $n_0 \gg 1$ and $n_1 \gg 1$.

Suppose the system is isolated and governed by the micro-canonical ensemble with conserved internal energy E.

(a) Approximating $\log(n!) \approx n \log n - n$, what is the entropy of the system in terms of N, E and ε?

(b) What is the temperature T of the system in terms of N, E and ε? Show that the temperature is negative when $\frac{N\varepsilon}{E} < C$ and determine the constant C.

(c) Explain what happens when a negative-temperature system is brought into thermal contact with a positive-temperature system.