MATH327: StatMech and Thermo, Spring 2026

Extra practice — Gaussian integrals

As mentioned in the module's logistical information, I anticipate that you have previously learned about gaussian integrals. However, several students struggled with these in years past, so a review may be helpful.

First show

$$\mathcal{G} \equiv \int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}.$$

There are several ways this can be done. My favourite is to evaluate $\mathcal{G}^2$ using polar coordinates.

Next use a change of variables to show

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2}\, dx = \sqrt{\frac{\pi}{a}},$$

assuming $a > 0$. What happens if $a \leq 0$?

Finally consider the gaussian probability distribution

$$p(x) = C \exp\left[-\frac{(x - N\mu)^2}{2N\sigma^2}\right],$$

with finite N, μ and $\sigma^2 > 0$. Based on the properties of probabilities, determine the positive coefficient C and compare your result against Eq. 10 in the lecture notes.