# MATH327: StatMech and Thermo, Spring 2026

# Extra practice — 2d fermions

Consider a gas of fermions with mass $m$ confined to a two-dimensional surface of area $A$, with temperature $T = 1/\beta$ and chemical potential $\mu$. Its grand-canonical potential is

$$\Phi(T, \mu) = -\frac{mAT}{2\pi\hbar^2} \int_0^\infty \log\left[1 + e^{-(E-\mu)/T}\right] dE\,.$$

(a) Show that the particle density is

$$\frac{\langle N \rangle}{A} = \alpha \int_0^\infty F(E)\, dE, \qquad\qquad F(E) = \frac{1}{e^{(E-\mu)/T} + 1},$$

where the constant $\alpha = \frac{m}{2\pi\hbar^2}$ and $F(E)$ is the usual Fermi function.

(b) Evaluate the particle density in the low-temperature limit where $F(E)$ becomes a step function. Use the result to find the Fermi energy $E_F = \mu$.

(c) Starting from the internal energy density

$$\frac{\langle E \rangle}{A} = \frac{m}{2\pi\hbar^2} \int_0^\infty E\; F(E)\, dE,$$

show $\langle E \rangle = \gamma \langle N \rangle^2$ in the low-temperature limit, and find the constant $\gamma$.

(d) The pressure is now the force per unit length on the boundary of the area $A$. In the low-temperature limit it is given by

$$P = -\left.\frac{\partial \langle E \rangle}{\partial A}\right|_N .$$

Calculate $P$ to show $PA = \kappa \langle N \rangle E_F$ in the low-temperature limit, and determine the constant $\kappa$.