Wed 22 Apr 86 03 39

Plan

Dark matter and CMB

Einstein solid heat capacity → improvements

$$S = \log M(N, K) \qquad E = K\hbar\omega$$

Putting K indist'able balls into N dist'able boxes

Count sequences of K balls separated by $N-1$ box walls

$$\bullet\ \bullet\ |\ \bullet\ |\ |\ \bullet \;\rightarrow\; (2, 1, 0, 1) \qquad K=4 \text{ in } N=4$$

Choose K symbols to be balls out of $K+N-1$ total symbols $\Big\}$ $M = \binom{K+N-1}{K}$

$$\beta = \frac{1}{T} = \frac{\partial S}{\partial E} \simeq \frac{1}{\hbar\omega}\frac{\partial}{\partial K}\log\left(\frac{(K+N)!}{K!\,N!}\right) \qquad N-1 \simeq N \text{ for } N \gg 1$$

$E = K\hbar\omega$

$$\frac{\partial}{\partial K}\Big[(K+N)\log(K+N) - K\log K - N\log N\Big]$$

$$\log(K+N) \cancel{+1} - \log K \cancel{-1}$$

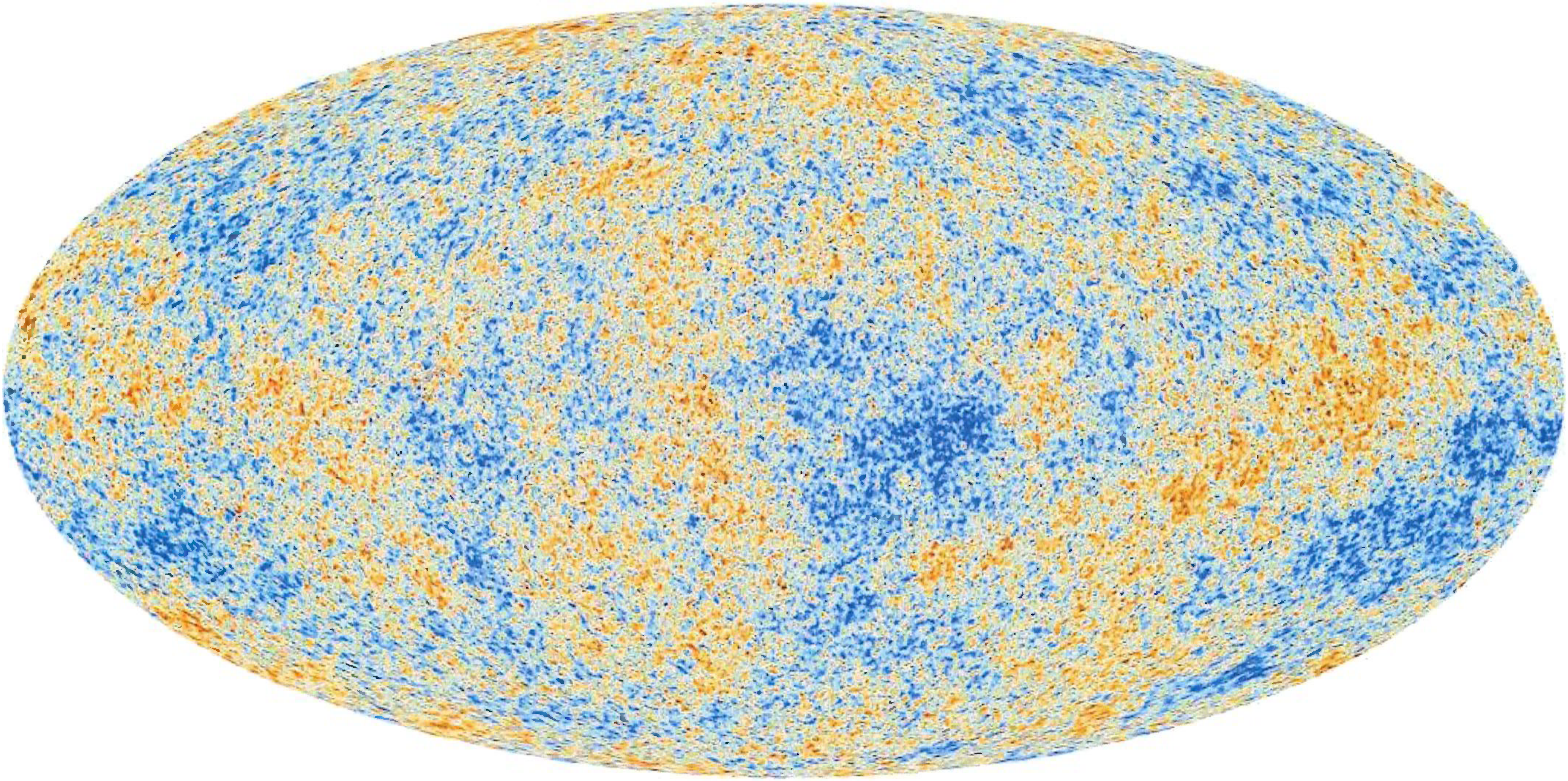
$$\rightarrow \beta\hbar\omega = \log\left(1+\frac{N}{K}\right) = \log\left(1+\frac{N\hbar\omega}{E}\right) > 0$$

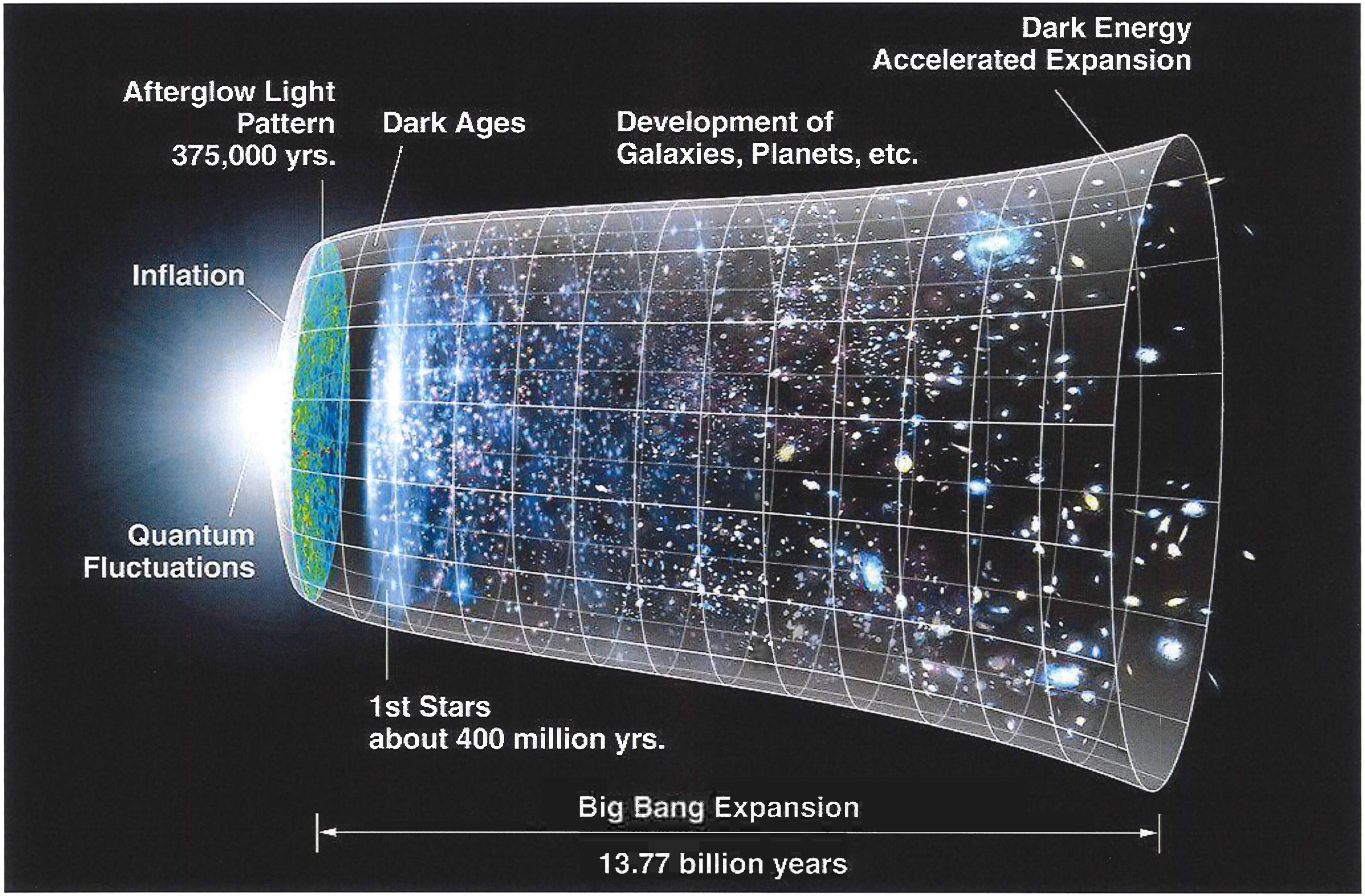
Invert: $\dfrac{N\hbar\omega}{E} = e^{\beta\hbar\omega} - 1 \rightarrow E = \dfrac{N\hbar\omega}{e^{\beta\hbar\omega}-1}$

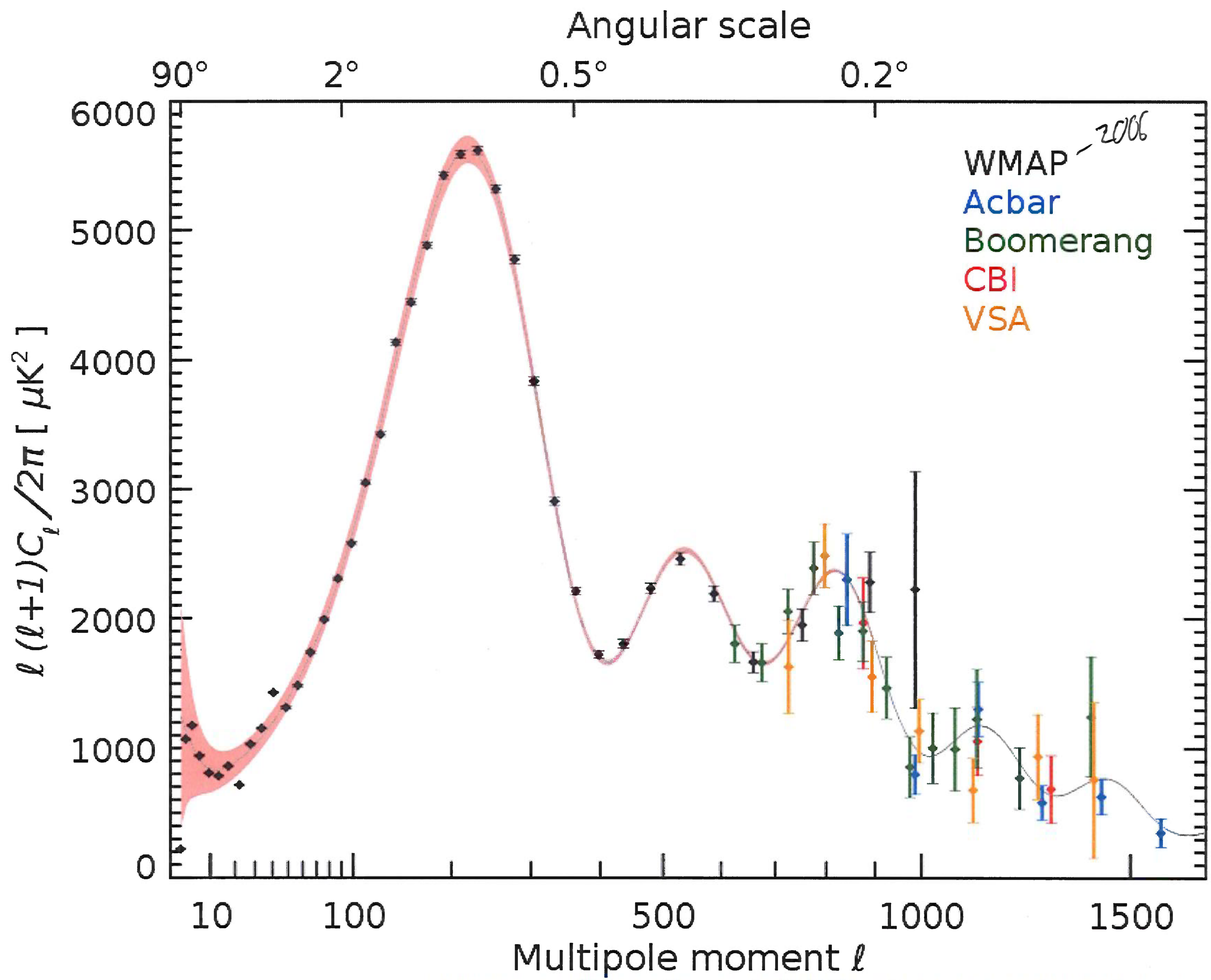
$$C_V = -\beta^2\frac{\partial E}{\partial \beta} = +\beta^2\frac{N\hbar\omega\,(e^{\beta\hbar\omega})(\hbar\omega)}{(e^{\beta\hbar\omega}-1)^2} = \frac{N x^2 e^x}{(e^x-1)^2}$$

$x = \beta\hbar\omega$

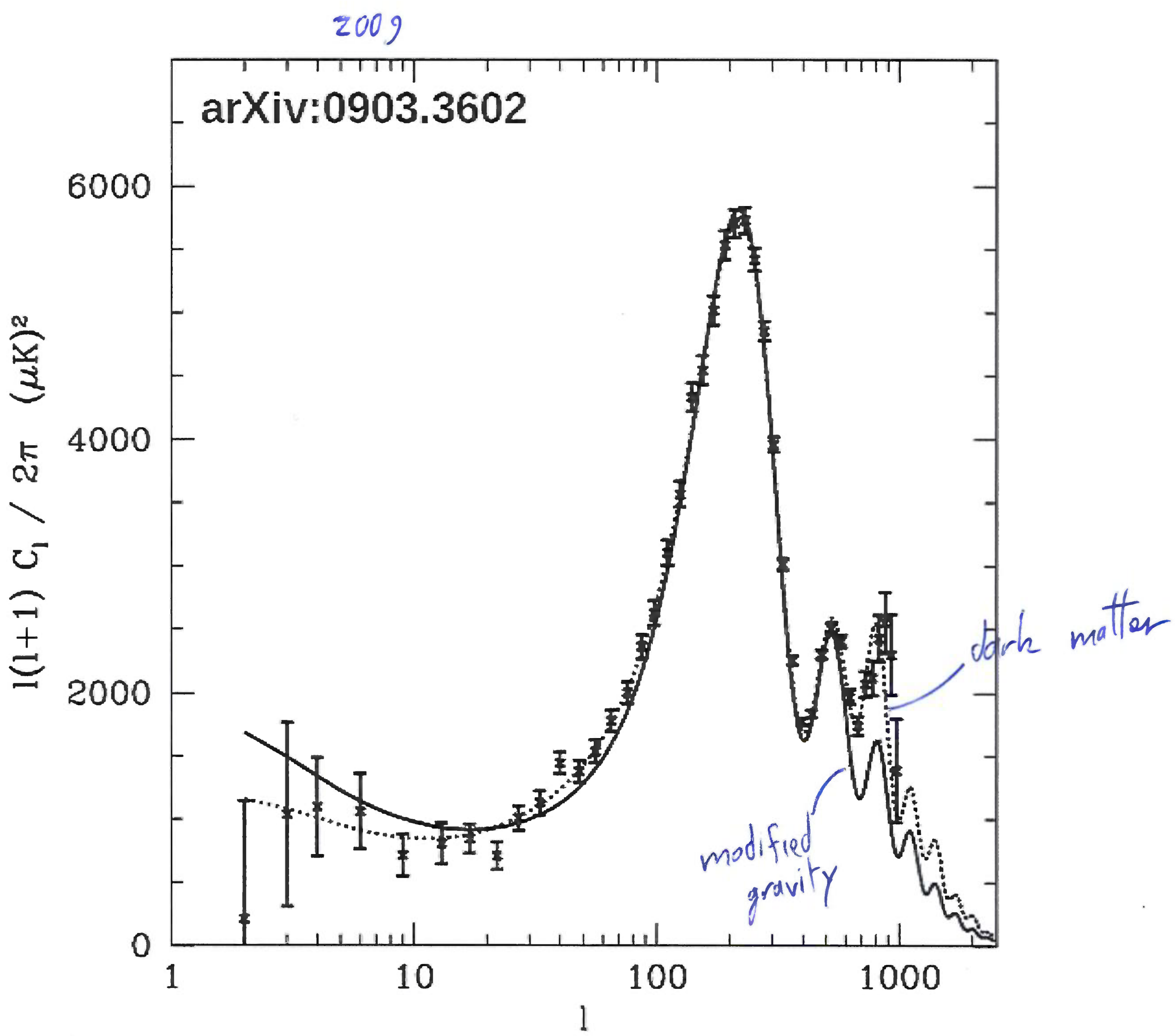
Planck, 2013




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years



commons.wikimedia.org/wiki/File:PowerSpectrumExt.svg


2009
arXiv:0903.3602
l(l+1) C_l / 2π (μK)²
6000
4000
2000
0
1
10
100
1000
l
dark matter
modified gravity

Problem:

$T \to 0 \quad \beta \to \infty \quad x \to \infty \qquad \frac{C_V}{N} \approx \frac{x^2 e^x}{e^{2x}} = \frac{x^2}{e^x} \to 0$ ✓

Einstein solid over-simplification

Oscillations correlated across multiple atoms

waves of coherent motion

↳ phonons inspired by photons

3, c_s — 2, c

massless bosons

$E = \hbar\omega, \quad \mu = 0$

1) Adapt photon gas to phonons

Restrict to minimum wavelength from atom separation

→ maximum frequency $\omega_{max} = T_D/\hbar$

Focus on functional form of c_V

constant at $T \gg T_D$

$\propto T^3$ for $T \ll T_D$

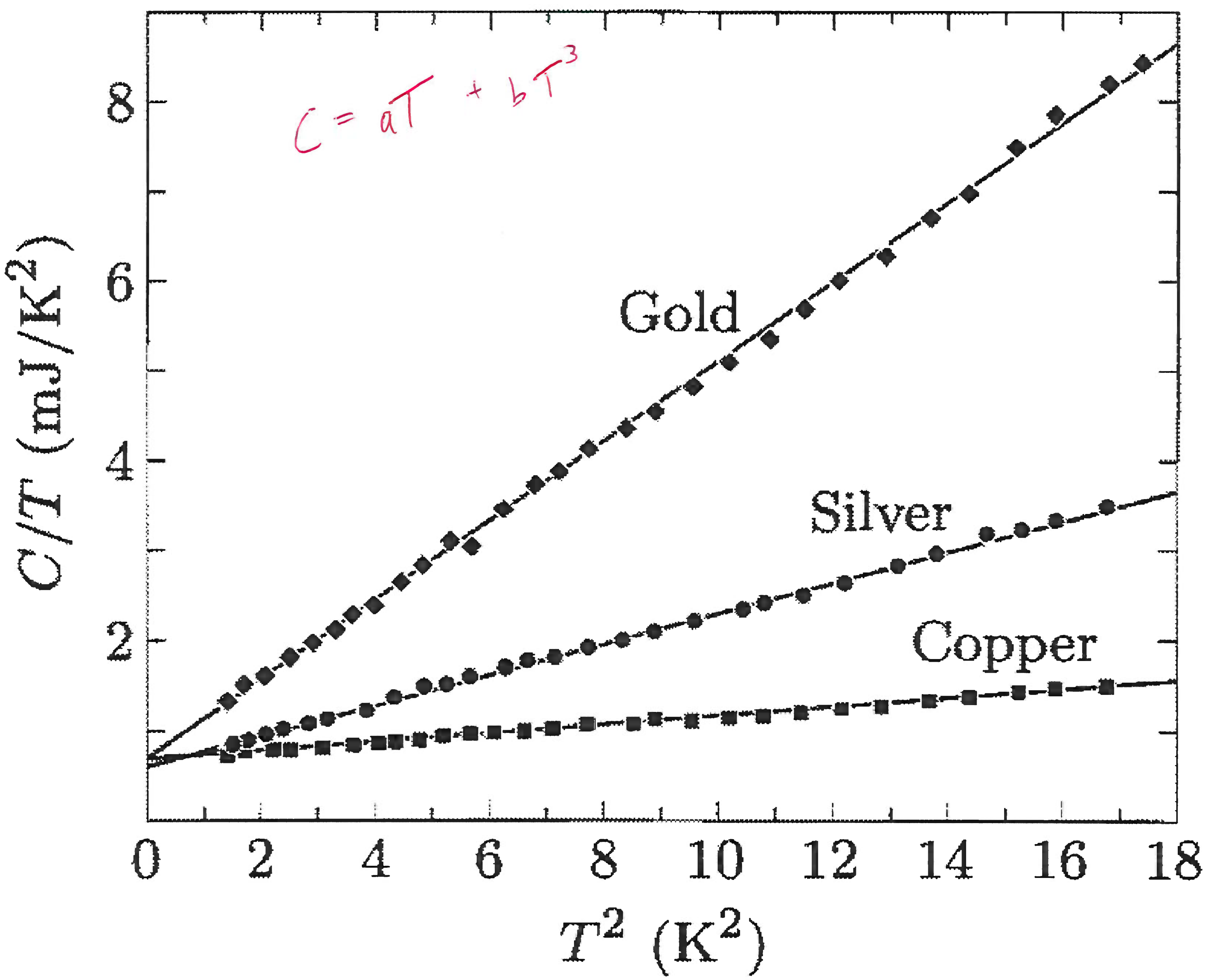
2) $c_V \propto T$ for very low T

Not enough energy for phonons — atoms frozen in place

Electron gas has non-zero $\langle E \rangle_F$ and c_V for $T \to 0$

Integrate $\langle E \rangle_F \propto \int_0^\infty F(E)\, E^{3/2}\, dE$

$\to c_V \propto T$


C/T (mJ/K²)
T² (K²)
Gold
Silver
Copper
C = aT + bT³
0
2
4
6
8
10
12
14
16
18