Wed 15 Apr 19 76 74

Plan

Brief general Feedback HW

Second HW due 24 April

Heat capacities and Einstein solid

Heat capacity

Already seen ideal gas gives $c_V = \frac{3}{2} N$ ✗

For dist'able spins $E = -NH \tanh(\beta H)$ $H > 0$

$$c_V = \frac{\partial E}{\partial T} = -\beta^2 \frac{\partial E}{\partial \beta} = N\beta^2 H \left(\frac{H\cosh(\beta H)}{\cosh(\beta H)} - \frac{H\sinh^2(\beta H)}{\cosh^2(\beta H)} \right)$$

$$= \frac{N\beta^2 H^2}{\cosh^2(\beta H)} > 0 \quad ✓$$

Low-T $\beta \to \infty$ $\cosh^2(\beta H) \approx \frac{1}{4} e^{2\beta H}$ $c_V \to 0$ ✓

High-T $\beta \to 0$ $\cosh^2(\beta H) \to 1$ $c_V \to 0$ ✗

Einstein solid

Atoms in solid held in place by interacting with neighbours
→ oscillators

Suppose oscillators non-interacting with energies $\varepsilon_n = 0, \hbar\omega, 2\hbar\omega, \ldots$

Consider assigning K units of energy to N oscillators

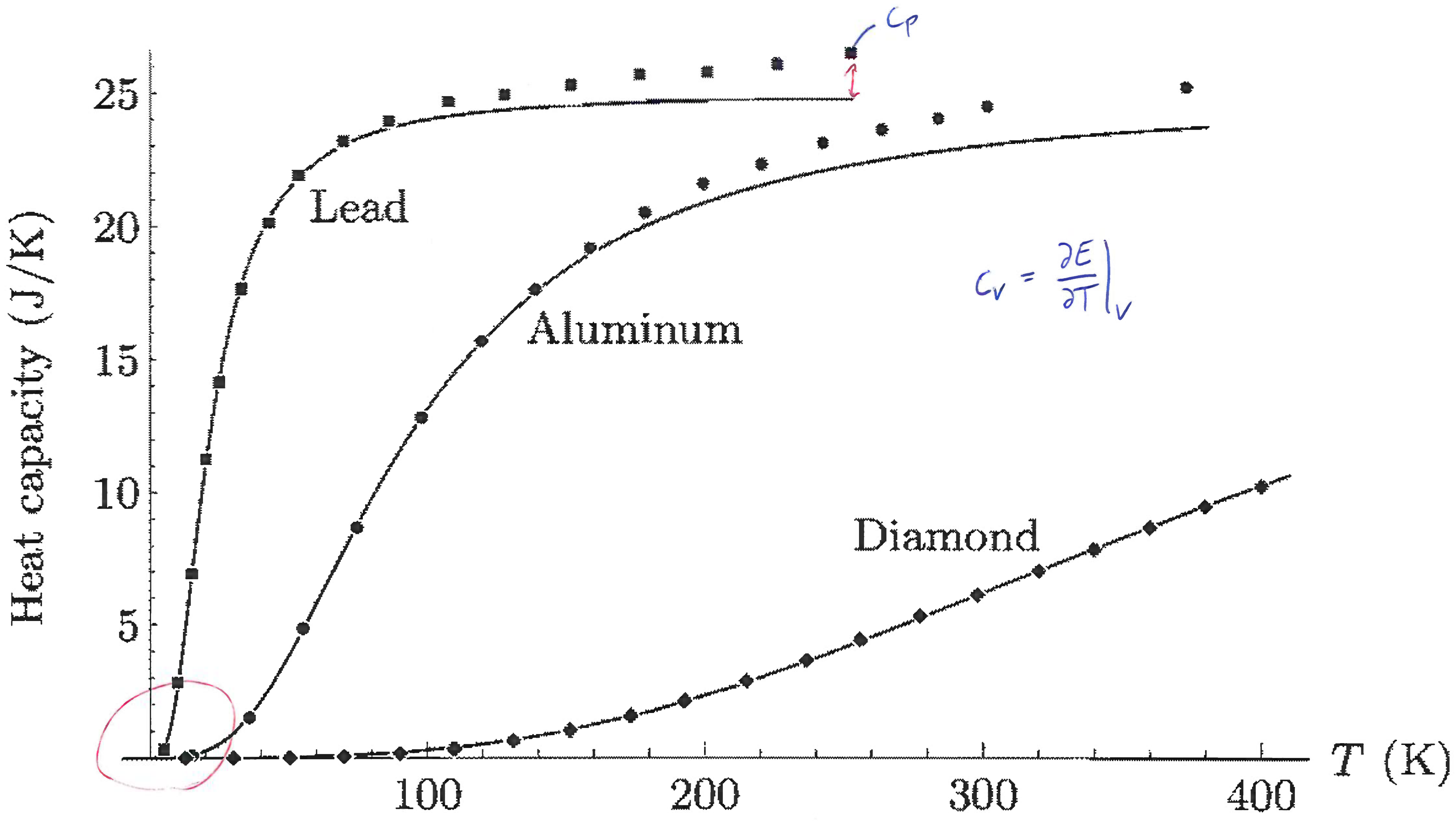
$$E = K\hbar\omega = \sum_n k_n \hbar\omega = \sum_n \varepsilon_n \qquad K = \sum_n k_n$$

Find $M \to S = \log M \to \frac{1}{T} = \frac{\partial S}{\partial E}$

N^K corrected for over-counting → mess

Invert $T(E)$ to find canonical $E(T)$ $\rightarrow$ $C_V = \frac{\partial E}{\partial T}$

Warm-up: $N=3$ with $k = 0, 1, 2, 3, \ldots$



Heat capacity (J/K)
25
20
15
10
5
100
200
300
400
T (K)
Lead
Aluminum
Diamond
c_p
$c_v = \frac{\partial E}{\partial T}\Big|_v$