Wed 4 Mar

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Plan

Entropy bounds

Stirling's formula

Mixing entropy

Bounds:

$$N! = N(N-1)(N-2)\cdots 1 < N\cdot N\cdot N\cdots N = N^N$$

$$\rightarrow \log(N!) < N\log N \checkmark$$

$$e^N = \sum_{k=0}^{\infty} \frac{N^k}{k!} > \frac{N^N}{N!} \qquad \rightarrow \log(N!) > N\log N - N \checkmark$$

$$1 - \frac{1}{\log N} < \frac{\log(N!)}{N\log N} < 1$$

$\frac{1}{\log N} \to 0$

$\log(N!) \sim N\log N$ asymptotically $N \gg 1$

$$\frac{d}{da}\left(\int_0^\infty e^{-ax}\,dx\right) = \frac{d}{da} a^{-1}$$

$$\int_0^\infty -x e^{-ax}\,dx = -a^{-2}$$

$$\frac{d^N}{da^N} \rightarrow \int_0^\infty (+x)^N e^{-\cancel{a}x}\,dx = \cancel{(-1)^N} N!\, a^{-(N+1)}$$

$a \to 1$

$$\int_0^\infty x^N e^{-x}\,dx = N! \qquad \square$$

Maximize: $\frac{d}{dx} x^N e^{-x} = N \cancel{x^{N-1}} \cancel{e^{-x}} - \cancel{x^N} \cancel{e^{-x}} = 0$

$$x = N$$

Change var: $y = x - N$ $\quad \left|\frac{y}{N}\right| \ll 1$

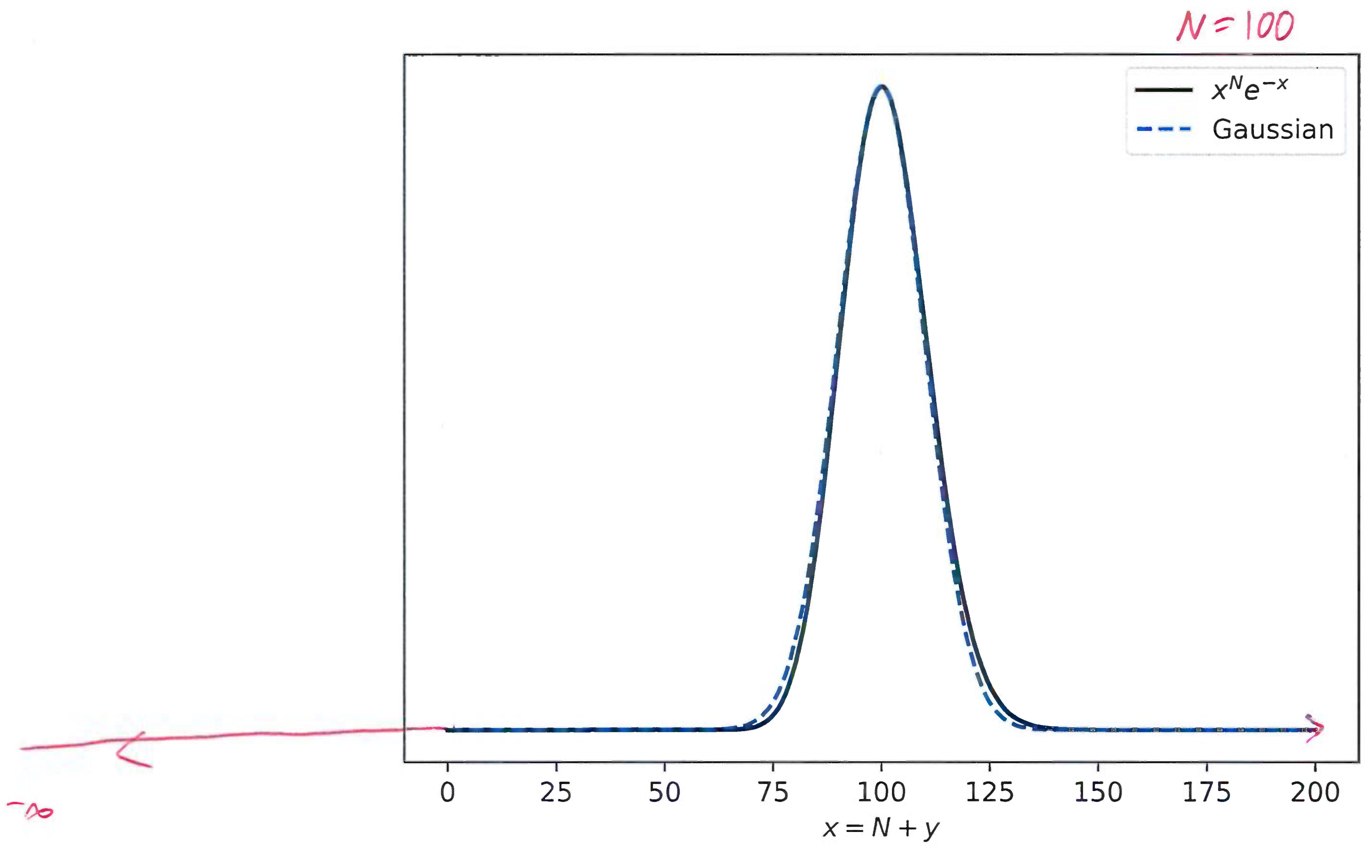
$$N! = \int_0^{\infty} \exp\left[N \log x - x \right] dx$$

$$= \int_{-N}^{\infty} \exp\left[N \log\left[N\left(1 + \frac{y}{N}\right)\right] - (N+y) \right] dy$$

$$= \int_{-N}^{\infty} \exp\left[N \log N + \cancel{N\left(\frac{y}{N}\right)} - \frac{Ny^2}{2N^2} + \mathcal{O}\left(\frac{Ny^3}{N^3}\right) - N \cancel{- y} \right] dy$$

$$\approx N^N e^{-N} \int_{-N}^{\infty} \exp\left(\frac{-y^2}{2N}\right) dy \approx \sqrt{2\pi N} \left(\frac{N}{e}\right)^N \checkmark$$

$-N \to -\infty$



N = 100
$x^N e^{-x}$
Gaussian
0
25
50
75
100
125
150
175
200
$x = N + y$
$-\infty$