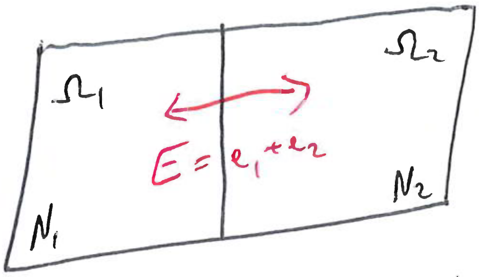
Wed 25 Feb   56 19 34



total $M = \sum_{e_1} M^{(1)}_{e_1} M^{(2)}_{E-e_1}$

Say $N_{terms} \geq 1$ in sum, largest called "max"

Simple bounds: $\quad max \leq M \leq N_{terms} \cdot max$

Micro-canonical $\quad \log(max) \leq S \leq \log(N_{terms} \cdot max)$

Reasonable behaviour: $\quad N_{terms} \sim N$

$max \sim e^N$

spin system

$M = 2^N = e^{N \log 2}$

$N! \sim N^N = e^{N \log N}$

$\longrightarrow N \lesssim S \lesssim N + \log N$

For $N \sim 10^{23}$, $\quad 10^{23} \lesssim S \lesssim 10^{23} + 50$

$S = 10^{23}$

Spin system $\quad H = 1$

$N_1 = N_2 = 10$

$E = e_1 + e_2 = -10$

$e_1 = -(2\underbrace{\cancel{}}_{n_+^{(1)}} - N_1) = -10, -8, \ldots 10$

e_1	$E-e_1$	$n_+^{(1)}$	$n_+^{(2)}$	$M_1 M_2$
-10	0	10	5	$\binom{10}{10}\cdot\binom{10}{5} = 1\cdot\frac{10\cdot9\cdot8\cdot7\cdot6}{5\cdot4\cdot3\cdot2} = 4\cdot63 = 252$
-8	-2	9	6	2100
-6	-4	8	7	$\left.\right\} \binom{10}{7}\binom{10}{8} = \frac{10\cdot9\cdot8}{3\cdot2}\cdot\frac{10\cdot9}{2} = 120\cdot45 = 5400$
-4	-6	7	8	
-2	-8	6	9	$\binom{10}{9}\cdot\binom{10}{6} = 10\cdot\frac{10\cdot9\cdot8\cdot7}{4\cdot3\cdot2} = 100\cdot21 = 2100$
0	-10	5	10	252

$N_{terms} = 6$, $\quad \max = 5400$

$$\log(5400) \leq \log(15,504) \leq \log(6\cdot5400)$$
$$8.59 < 9.65 < 10.39$$

$\log(\max)$ accounts for 89% of entropy

What happens as N_1, N_2 increase?

Stirling's formula

$$\log(N!) \approx N\log N - N \quad \text{for } N \gg 1$$
$$N! \approx \exp(N\log N - N) = N^N e^{-N} = \left(\frac{N}{e}\right)^N$$

Full form: $$N! = \sqrt{2\pi N}\left(\frac{N}{e}\right)^N\left(1 + \frac{A}{N} + \frac{B}{N^2} + \frac{C}{N^3} + \dots\right)$$

(asymptotic)

1) Simple bounds $\quad N\log N - N < \log(N!) < N\log N$

2) Find $N! \approx \sqrt{2\pi N}\left(\frac{N}{e}\right)^N$ by showing $N! = \int_0^\infty x^N e^{-x}\,dx$

approx. by gaussian

3) Compute $A, B, \dots$ by comparing $(N+1)!$

vs. $(N+1)N!$

commons.wikimedia.org/wiki/File:Stirling_error_vs_number_of_terms.svg