Wed 28 Jan

79 50 47

Recap

Many objects → stochastic analysis
→ emergent phenomena

Random experiments $\mathcal{E}$ → state $w \in \Omega = \{w\}$

Today

Prob. spaces
Tutorial setup

Measurement

$X(w)$ extract info of interest

Repeat $\mathcal{E}$ → $X(w_i)$ is random variable

$X: \Omega \to A = \{X(w)\}$ outcome space

finite, countable, continuous

Examples

$\mathcal{E}$: Roll a 6-sided die X: Measure number on top

$A = \{1, 2, 3, 4, 5, 6\}$

State w could also include position, time, temperature, ...

page 10

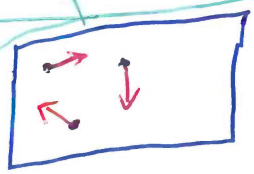
$\mathcal{E}$: Four coin flips X: Measure all H/T

$A = \{HHHH, HHHT, HHTH, \ldots\}$

Number of elements? $2^4 = 16$

page 10

$\mathcal{E}$: 10^{23} argon atoms in container



State could include 10^{23} positions velocities, isotopes, electronic states, ...

What ~~are~~ to measure? Pressure, temperature, energy, currents ...

page 11

Define event as any subset of A

Ex: Possible event From rolling die?
- Roll 6
- Roll 1-5
- Roll even number

Event space $\mathcal{F}$ is set of all events of interest

Probability is measure function $P: \mathcal{F} \to [0, 1]$
Number For each event in $\mathcal{F}$

Requirements: 1) $P(x \text{ or } y \text{ or } z) = P(x) + P(y) + P(z)$
countable, mutually exclusive

2) $P(\mathcal{F} = A) = 1$
Must have some measurable outcome

All together: probability space $(A, \mathcal{F}, P)$
prob. For subsets of outcomes

Roulette game

Events $\mathcal{F} = \{\text{red}, \text{black}, \text{green}\}$

Place $5 bet on black
Win: get back $10
Lose: get back $0

page 13

$P_{win} = P_{black} = \frac{18}{37}$

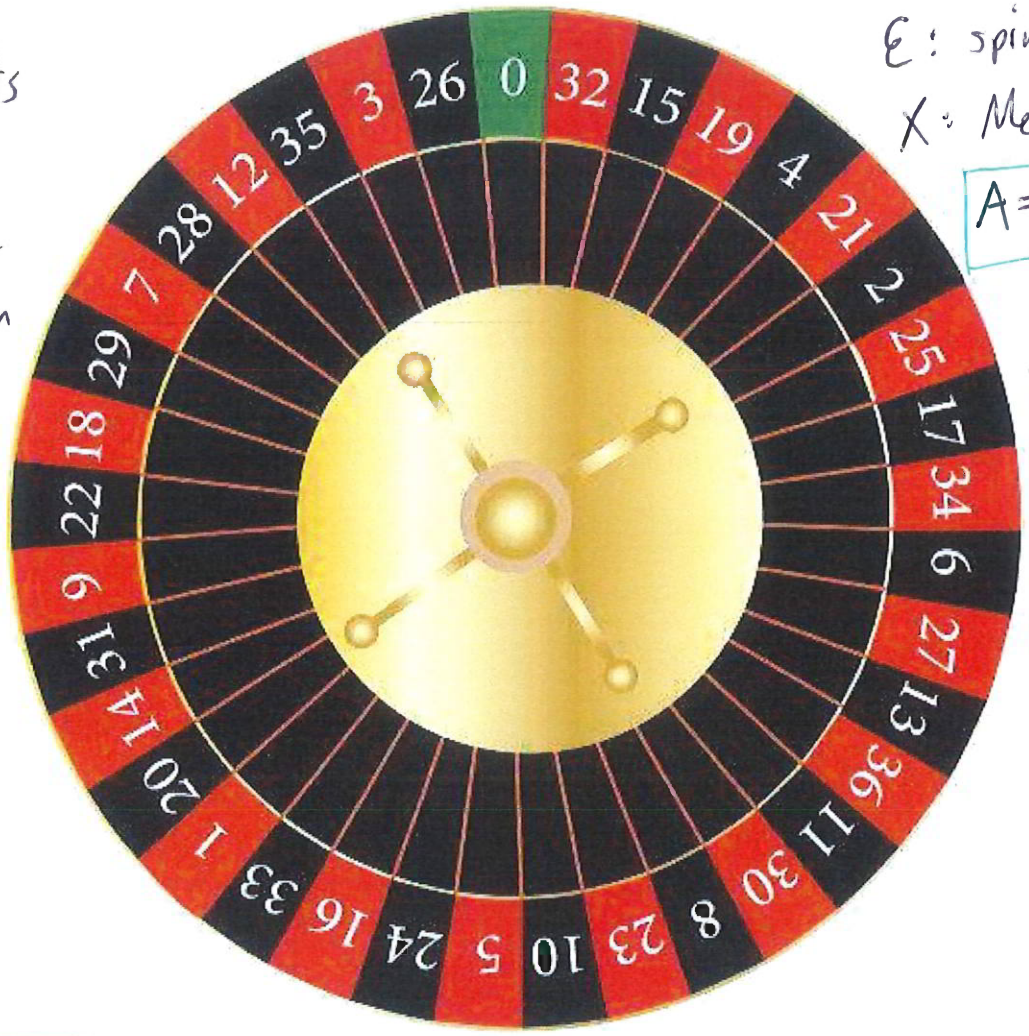
$P_{lose} = P_{red} + P_{green} = \frac{19}{37}$
$18/37 \quad 1/37$

Gain G: Money we get minus money we spend
(can be negative)

$G_{win} = 5 = G(1,1)$ ✓ $\quad G_{lose} = -5 = G(1,0)$ ✓

$G(N, W) = 10W - 5N$ For W wins in N spins

37 pockets
18 red
18 black
1 green



$\mathcal{E}$: spin wheel
X: Measure pocket

$A = \{0, 1, 2, \dots, 36\}$

page 13

page 13

Events: $\mathcal{F} = A$

$$P(A) = P(0 \text{ or } 1 \text{ or } 2 \cdots \text{ or } 36) = 1$$

$$= P(0) + P(1) + \cdots P(36) = \sum_{i=0}^{36} P_i = 37p$$

"Fair" wheel: $P_0 = P_1 = \cdots = P_{36} = p$ $\rightarrow p = \frac{1}{37}$