MATH327: StatMech and Thermo, Spring 2025

Extra practice — Moving spins

Consider N distinguishable spins that can move in a two-dimensional plane at fixed temperature $T = 1/\beta$. There is an external magnetic field of strength $H > 0$ perpendicular to the plane. Each spin has mass m and energy

$$E(\vec{p}, s) = \frac{1}{2m}p^2 - Hs$$

where $p^2 = p_x^2 + p_y^2 \geq 0$ while s can take the two values $s = 1$ (aligned with the external field) and $s = -1$ (anti-aligned against the external field). The single-particle partition function is

$$Z_1(\beta) = \sum_s \int \exp\left[-\beta E(\vec{p}, s)\right] \; d^2p$$

(a) Show that the Helmholtz free energy for this system is

$$F = -\frac{N}{\beta} \log\left[\frac{4\pi m}{\beta} \cosh(\beta H)\right].$$

(b) Calculate the internal energy $\langle E \rangle$ and the entropy S.

(c) Describe the lowest-energy state of the system. What is the corresponding ground-state energy E_0? Show that $\langle E \rangle \to E_0$ in the zero-temperature limit.