MATH327: StatMech and Thermo, Spring 2025

Extra practice — Magnetization

Consider a classical system of N distinguishable, non-interacting 'spins' in a lattice at temperature $T = 1/\beta$, where the value s_n of each spin can vary *continuously* in the range $-1 \leq s_n \leq 1$. In an external magnetic field of strength $H > 0$, the internal energy of the system is $E = -H \sum_{n=1}^{N} s_n$.

(a) By integrating over the continuous s_n, calculate the canonical partition function Z and the Helmholtz free energy F of the system, both as functions of βH.

(b) The derivative of the Helmholtz free energy with respect to the magnetic field defines the magnetization
$$\langle m \rangle = -\frac{1}{N} \frac{\partial F}{\partial H}.$$
Assuming finite $H > 0$, calculate $\langle m \rangle$ for this system as a function of βH, and determine its low- and high-temperature limits, $\lim_{T \to 0} \langle m \rangle$ and $\lim_{T \to \infty} \langle m \rangle$.

(c) For low but non-zero temperatures, expand $\langle m \rangle$ in terms of $\varepsilon \equiv e^{-\beta H} \ll 1$ to find the largest temperature-dependent term in this expansion.

(d) For high but finite temperatures, expand $\langle m \rangle$ in terms of $x \equiv \beta H \ll 1$ to find the largest temperature-dependent term in this expansion.