MATH327: StatMech and Thermo, Spring 2025

Extra practice — Barometric equation

Consider a horizontal slab of air with thickness dz. To be at rest, the pressure $P(z)$ supporting this slab from below must balance the pressure $P(z+dz)$ from above plus the weight of the slab itself. Use this and the ideal gas law to derive the **barometric equation**,

$$\frac{\partial P}{\partial z} = f(T)\, P,$$

and obtain an expression for $f(T)$.

This equation is easy to solve for $P(z)$ if we assume that the temperature of the air, T, is independent of the altitude z. This is not necessarily a great assumption. To test it, compute the relative atmospheric pressure at the top of Mount Everest (8850 m) compared to Liverpool (70 m), taking $T = 15$℃ and using $m = 4.811 \times 10^{-26}$ kg as the average mass of an air molecule, along with the unit conversion factor $k_B \approx 1.381 \times 10^{-23}$ J·K^{-1}. The true value is approximately 33.6%.