MATH327: StatMech and Thermo, Spring 2025 Extra practice — 2d bosons

Consider a gas of bosons with mass m confined to a two-dimensional surface of area A, with temperature $T=1/\beta$ and chemical potential μ . Its grand-canonical potential is

$$\Phi(T,\mu) = \frac{mAT}{2\pi\hbar^2} \int_0^\infty \log\left[1 - e^{-\beta(E-\mu)/T}\right] dE.$$

What is the average particle number $\langle N \rangle$ in the case of vanishing chemical potential, $\mu=0$?

Hint: Expect something unexpected. (You can find more information in doi:10.1016/0370-1573(77)90052-7.)