MATH327: StatMech and Thermo

Monday, 28 April 2025

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Something to consider

The essence of statistical mechanics
is the emergence of macroscopic phenomena
from systems of **many** microscopic degrees of freedom

Will a given collection of particles
always lead to the same emergent phenomena?

Logistics:

Exam 14:30 Fri 16 May

Review session Mon 12 May

Plan

Post-break review

Interacting systems

Big-picture review

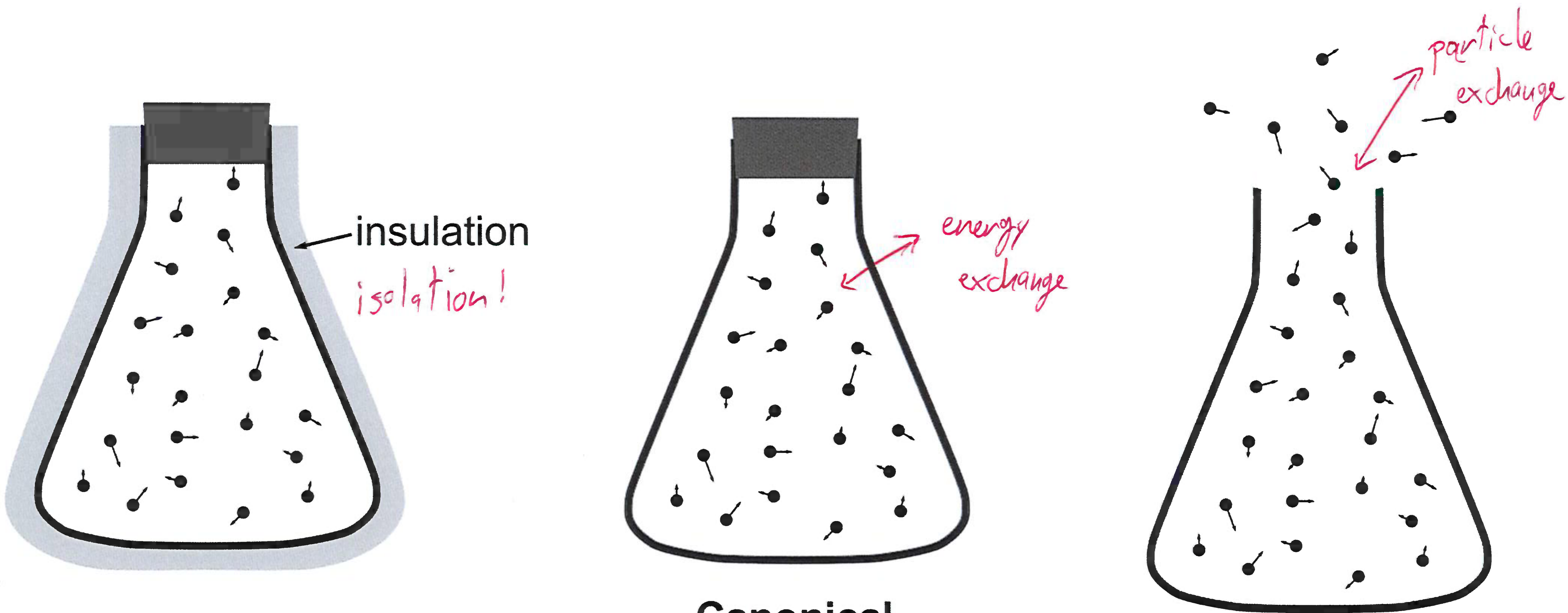
Prob. spaces → stat. ensembles

All "ideal" — non-interacting

Give some fantastic predictions (Planck, c_v)

Insufficient to describe phenomena like phase transitions

Interactions needed but much harder to analyse commons.wikimedia.org/wiki/File:Statistical_Ensembles.png



Microcanonical (const. N E)

heat exchange
entropy

Canonical (const. N T)

info
ideal gases
therm. cycles

Grand Canonical (const. μ T)

quantum gases
Planck spectrum
cv for metals

<u>Phases</u> are different emergent behaviour for same particles

Ice vs. water vs. steam all from H_2O

Quarks and gluons in plasma $\rightarrow$ nuclei in early Universe

Electrons in bilayer graphene

Insulating $\rightarrow$ super conducting at "magic" $\theta \approx 1.1°$ and $T \lesssim 1.7$ K

no energy loss!

What ~~exact~~ ~~force~~ precisely distinguishes interacting or not?

Consider N spins in d-dim'l lattices (dist'able)

at $T = 1/\beta$

H ↑↑↑↓↑↑↓↓ $s_1 = 1$, $s_N = -1$

H
↑ ↑ ↑ $s_{1,3} = 1$
↑ ↓ ↑
↑ ↓ ↓ $s_{3,3} = -1$

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Micro-states w_i defined by $\{s_n\}$, $s_n = \pm 1$

Non-interacting $E_i = -H \sum_{n=1}^{N} s_n = \sum_n \epsilon_n$ (factorization)

$\rightarrow$ very simple $Z_N = Z_1^N = (2\cosh(\beta H))^N$

More interesting $E_i = -\sum_{(jk)} s_j s_k - H \sum_n s_n$

all pairs of nearest-neighbour (n.n.) spins

Definition:

Consider change ΔE_j from alteration to jth particle

Non-interacting iff. ΔE_j independent of all particles $k \neq j$

Example: Flip spin $s_j \rightarrow -s_j$

Simple $E = -H(s_j + \sum_{k \neq j} s_k) \rightarrow -H(-s_j + \sum_{k \neq j} s_k)$

$\Delta E_j = 2Hs_j$ independent of s_k for $k \neq j$

$\rightarrow$ non-interacting ✓

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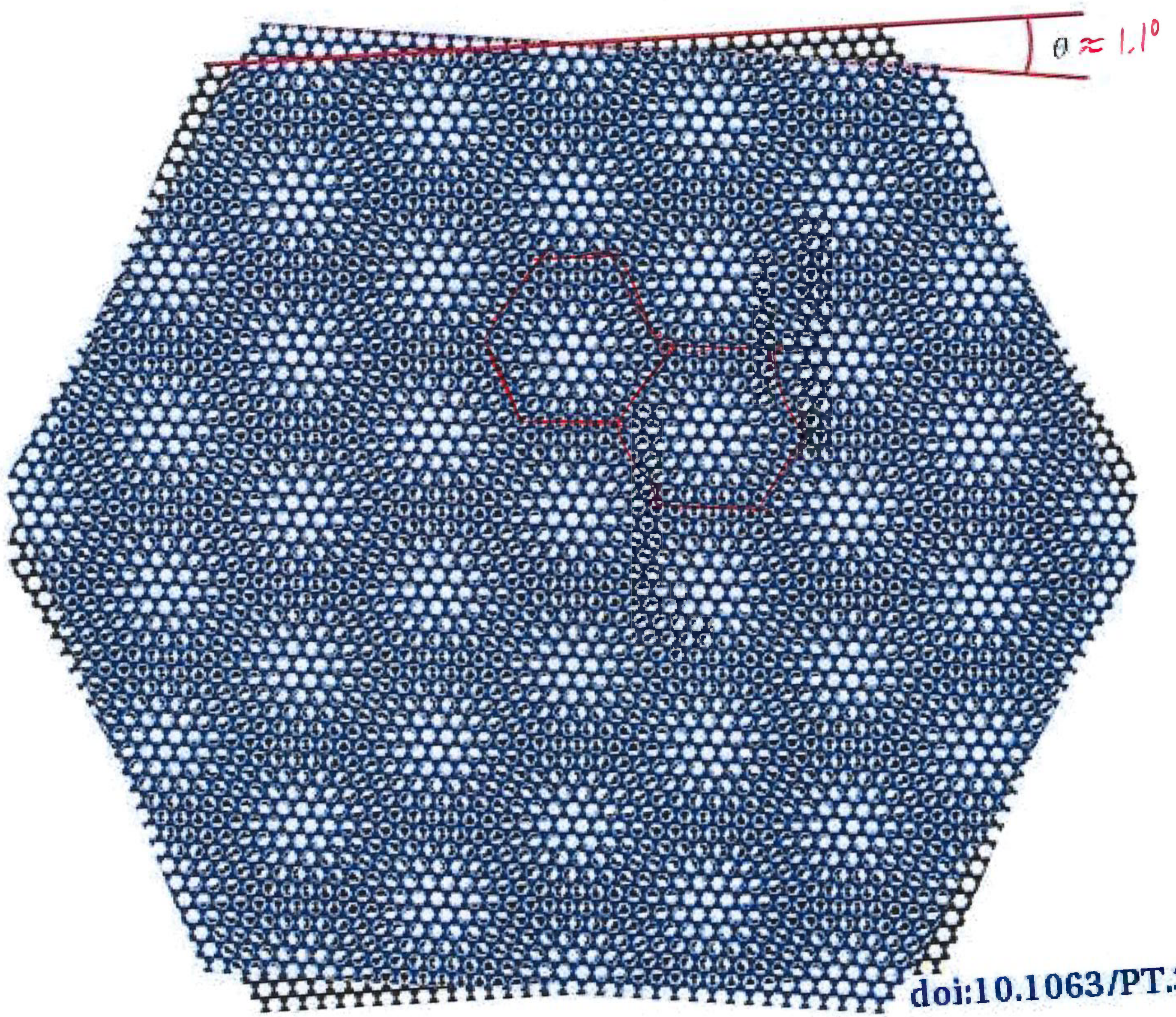

doi:10.1063/PT.3.4384

Interesting $E = -s_j \sum_{k\in(jk)} s_k - \sum_{(mk)\not\ni j} s_m s_k - H\Big(s_j + \sum_{k\neq j} s_k\Big)$

$$\to +s_j \sum_{k\in(jk)} s_k - \sum_{(mk)\not\ni j} s_m s_k - H\Big(-s_j + \sum_{k\neq j} s_k\Big)$$

$$\Delta E_j = 2 s_j \Big(H + \sum_{k\in(jk)} s_k\Big)$$

Depends on s_k with $k \neq j$ $\to$ interacting ✓

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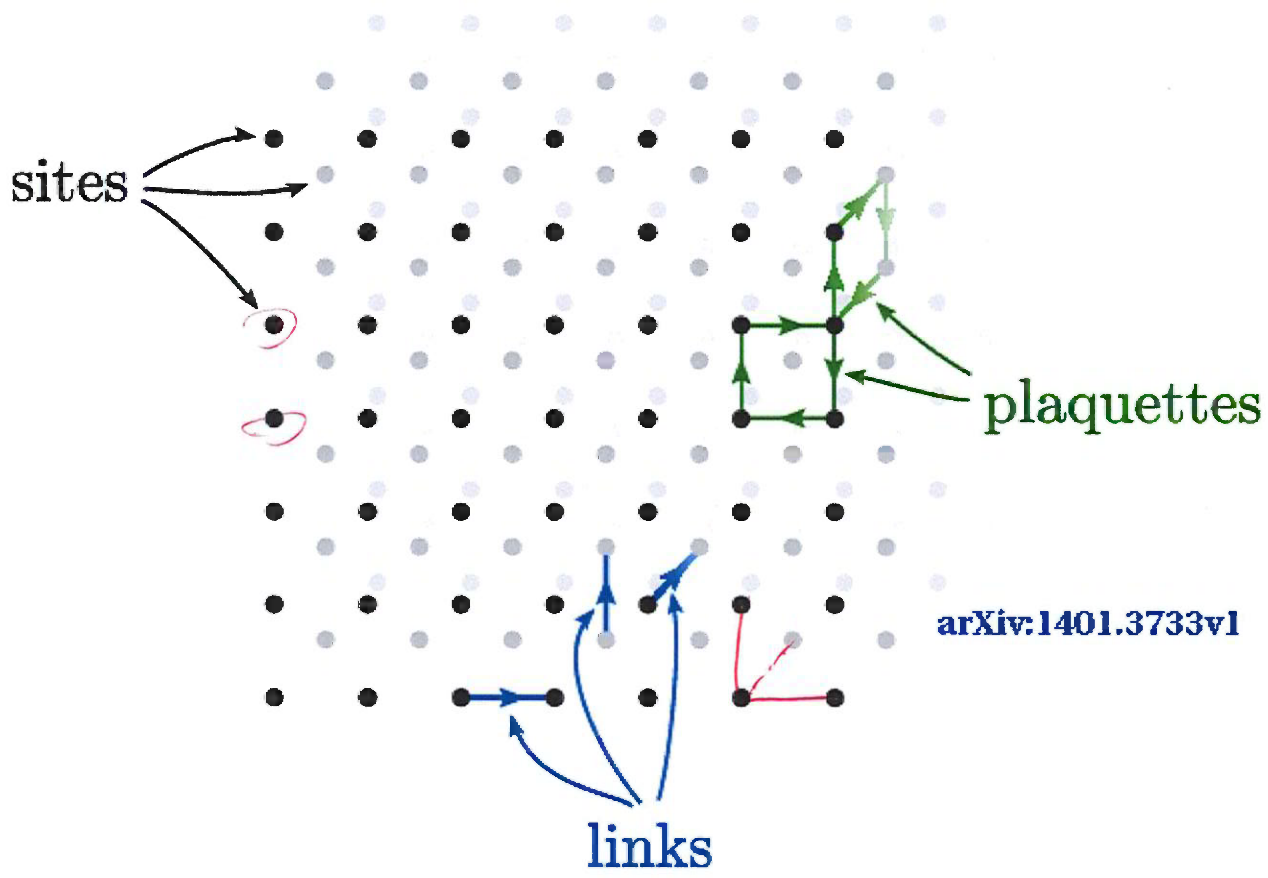
$E(s_n) \supset \sum_{(jk)} s_j s_k$ and lattice $\to$ n.n. pairs

$\to$ Ising model

d-dim'l cubic lattices

"sites" where spins are located

"links" correspond to n.n. pairs



arXiv:1401.3733v1