Thu 27 Mar 14 83 46

Plan

Photon polarizations

Dark matter & CMB

Einstein solid heat capacity → improved models

Einstein solid $S = \log \Omega(N,K)$ $E = K\hbar\omega$

N^K w/correction for over-counting → mess

K indist'able balls, N dist'able boxes } counting sequences of K balls separated by $N-1$ box walls

Example: $\bullet\bullet|\bullet||\bullet = (2,1,0,1)$

$N=4,\ K=4$

$K+N-1$ symbols, choose K balls

$$\binom{K+N-1}{K} = \Omega$$

$$\frac{1}{T} = \frac{\partial S}{\partial E} \approx \frac{1}{\hbar\omega}\frac{\partial}{\partial K}\left(\log\left(\frac{(K+N)!}{K!\,N!}\right)\right)$$

$N-1 \simeq N$ for $N \gg 1$

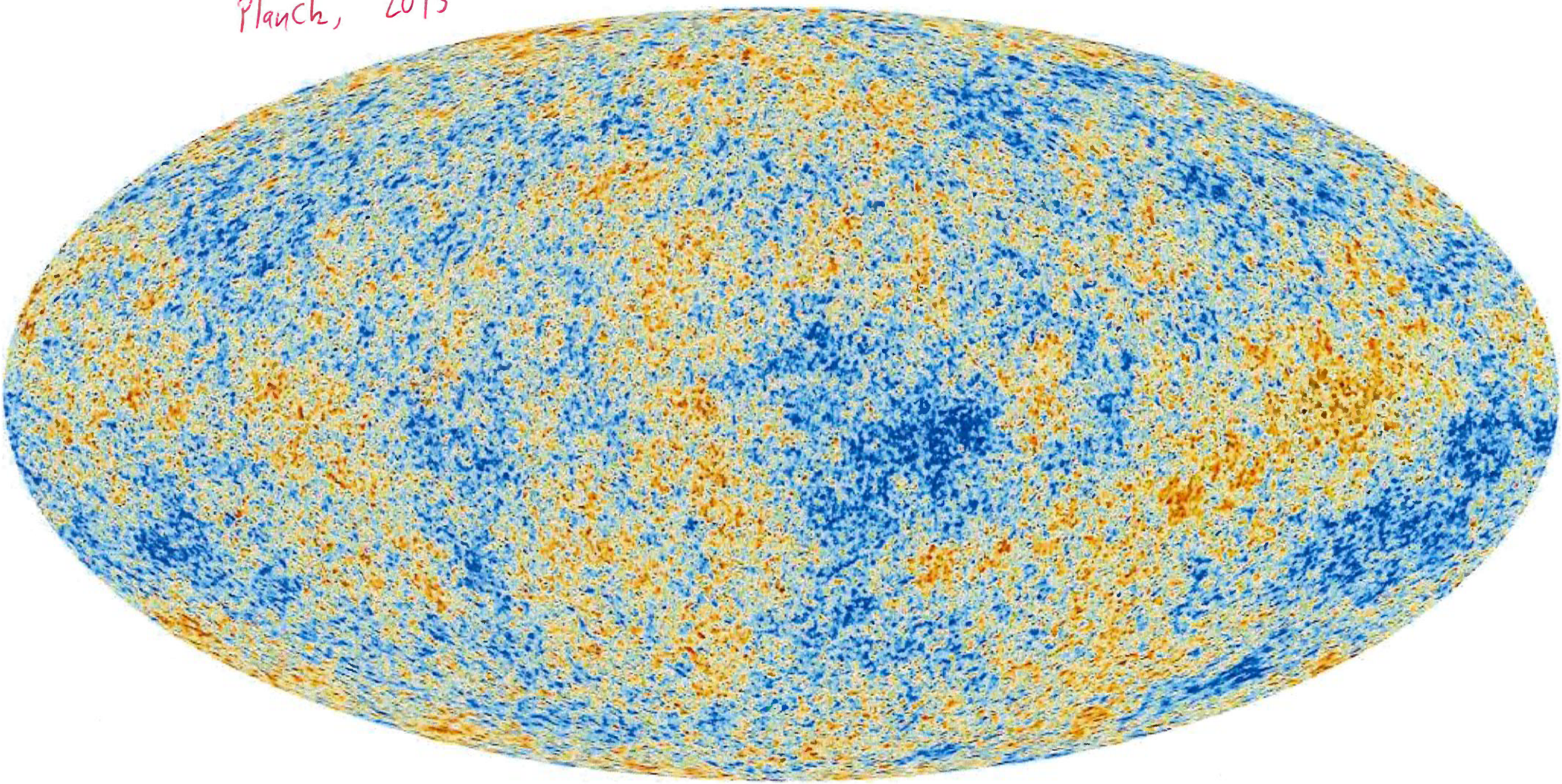
$$\frac{\hbar\omega}{T} \approx \log\left(1+\frac{N}{K}\right) = \log\left(1+\frac{N\hbar\omega}{E}\right)$$

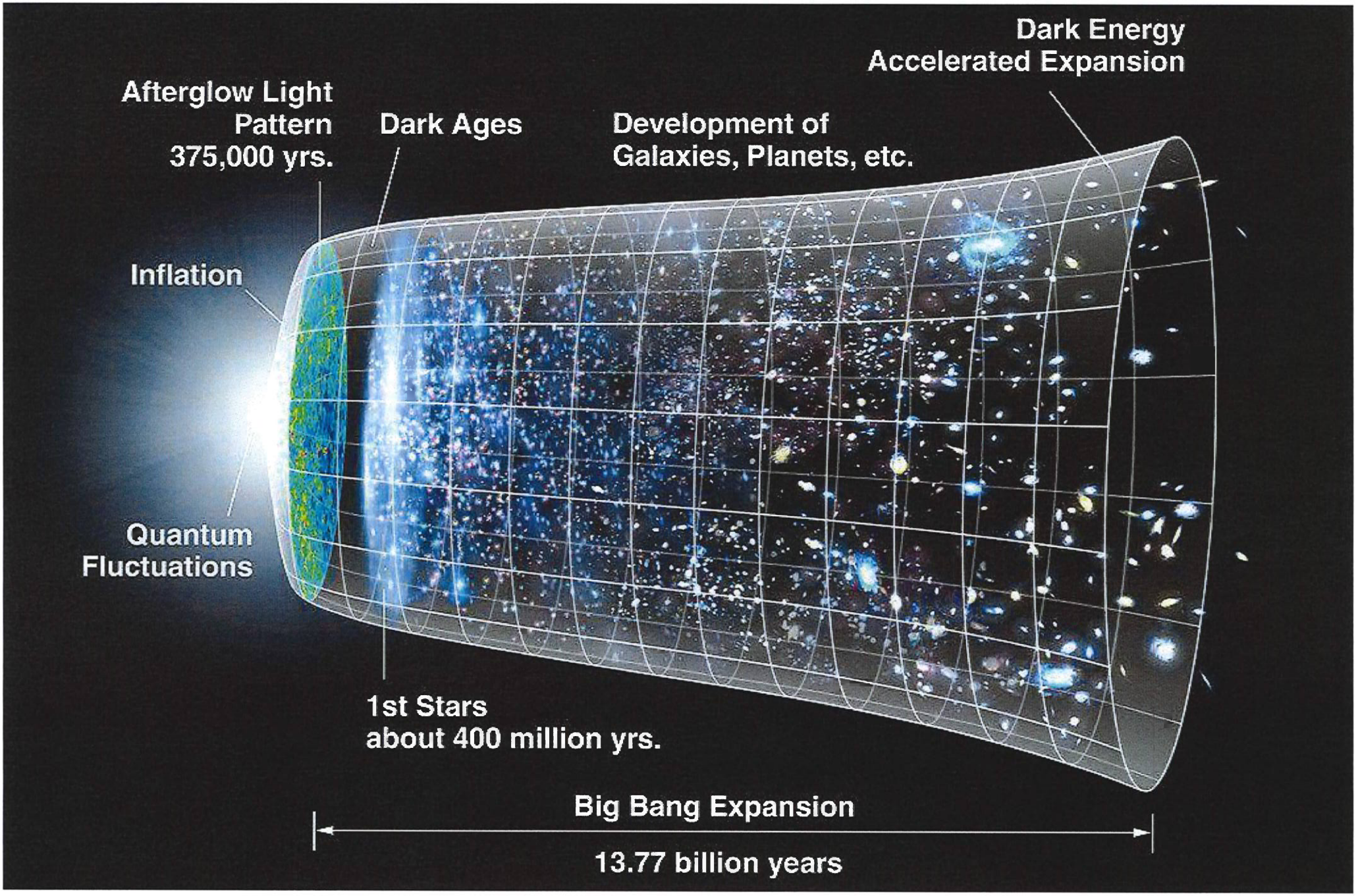
$$\frac{N\hbar\omega}{E} = e^{\beta\hbar\omega} - 1$$

$$E = \frac{N\hbar\omega}{e^{\beta\hbar\omega}-1}$$

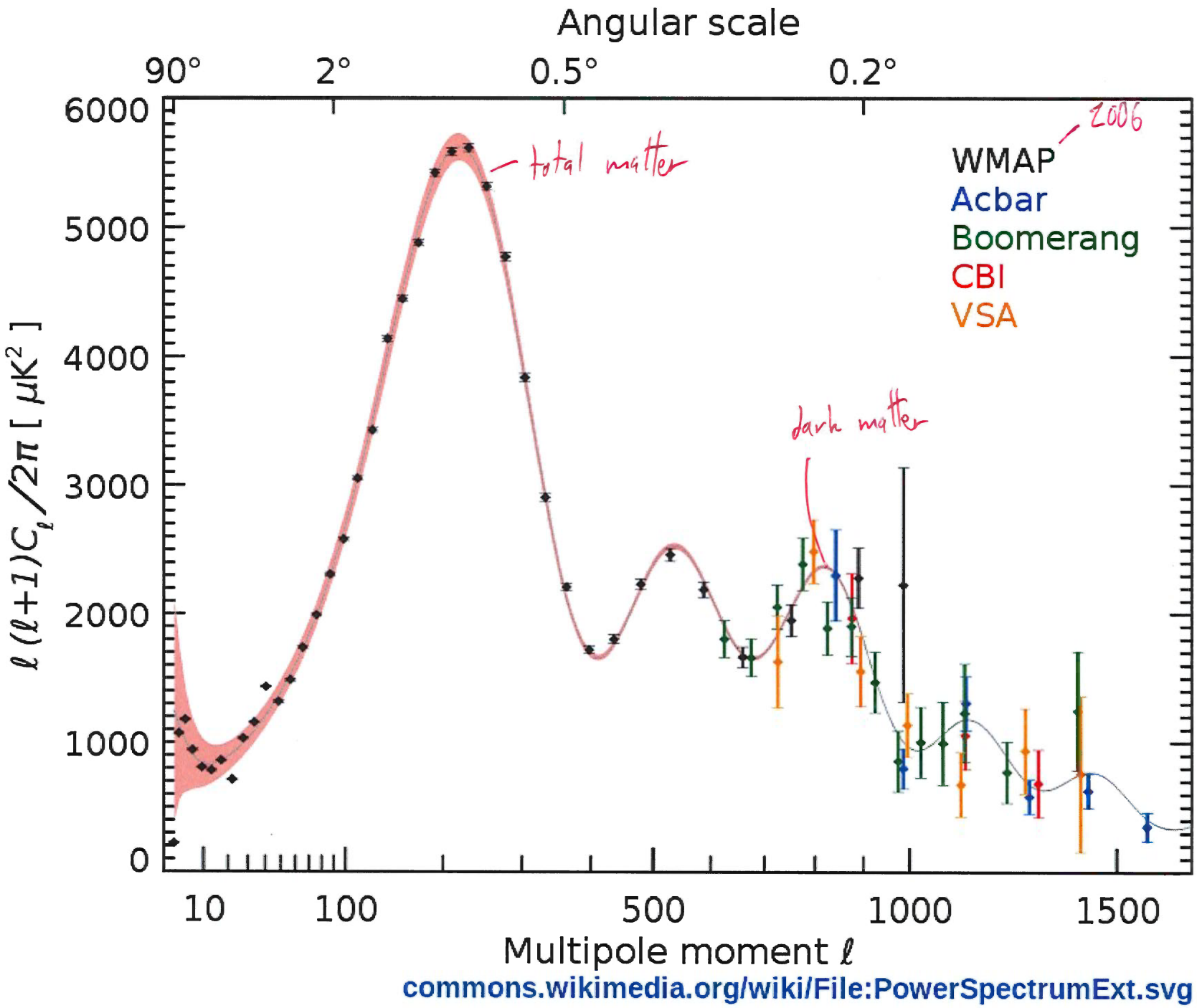
$$C_V = -\beta\frac{\partial E}{\partial \beta} = \frac{Nx^2e^x}{(e^x-1)^2}$$

$x = \beta\hbar\omega$

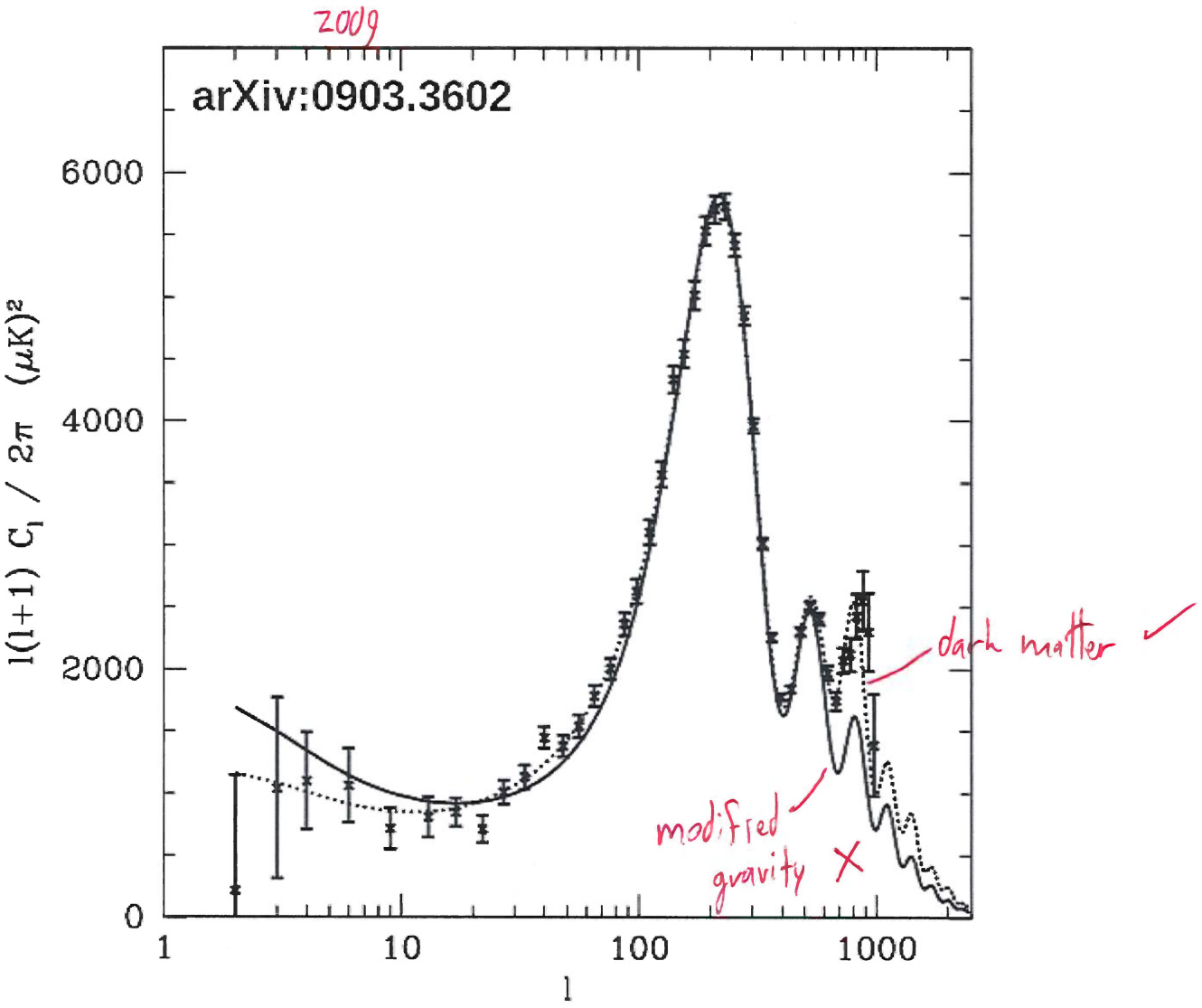

Planck, 2013



commons.wikimedia.org/wiki/File:CMB_Timeline300_no_WMAP.jpg



commons.wikimedia.org/wiki/File:PowerSpectrumExt.svg


2009
arXiv:0903.3602
6000
4000
2000
0
1
10
100
1000
l
l(l+1) C_l / 2π (μK)²
dark matter
modified gravity X

Problem: Exp. vs. polynomial approach to $T \to 0$

↳ Oscillations lead to waves in solid

↳ <u>phonons</u> inspired by photons

3, c_s vs. 2, c

Massless bosons, $E = \hbar\omega$, $\mu = 0$

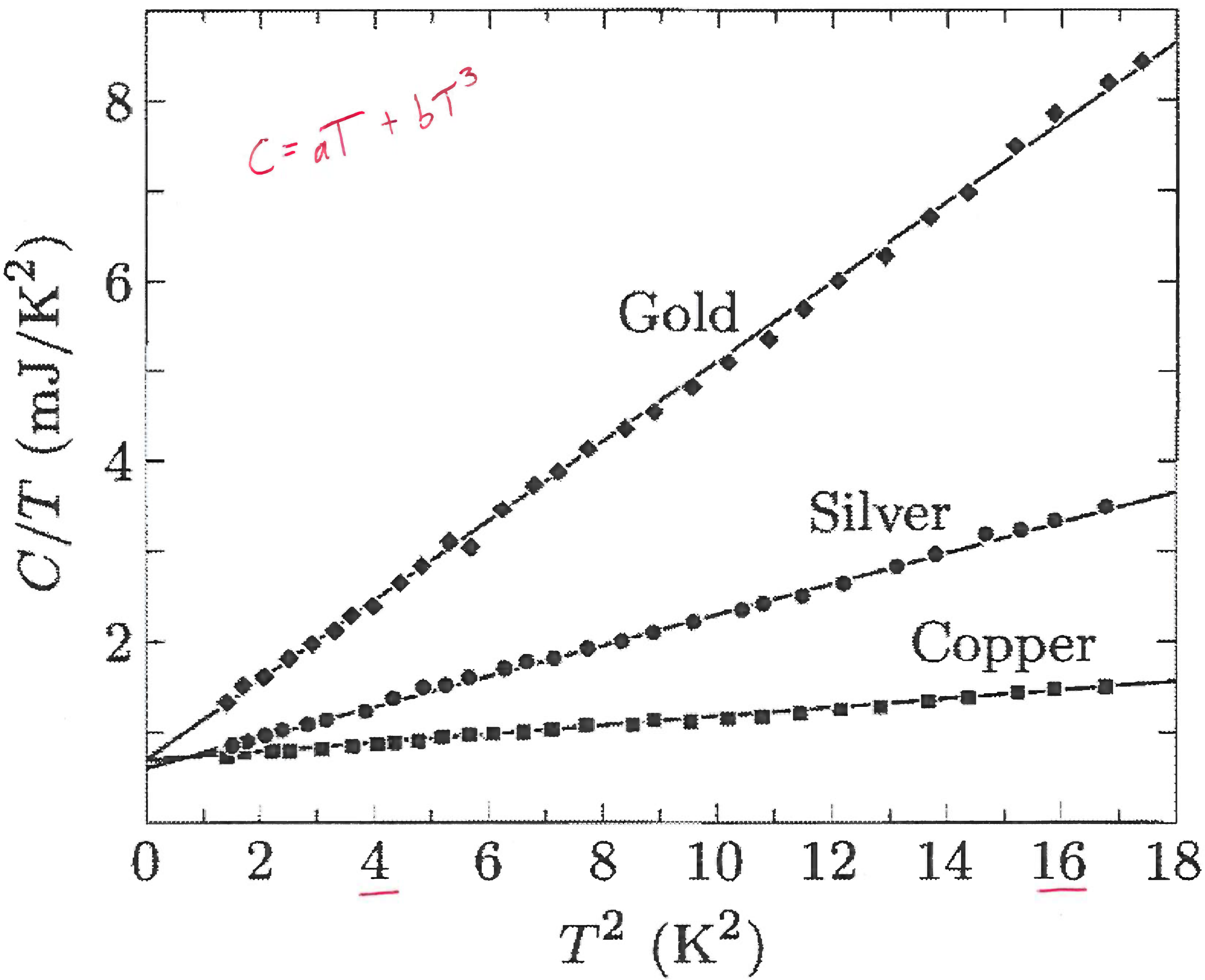
Adapt photon gas to phonons ← "Debye temp."

Key difference: $0 \leq \omega_i \leq T_D/\hbar$

$c_v \sim$ Einstein for $\frac{T}{T_D} \gg 1$

$c_v \propto T^3$ for $\frac{T}{T_D} \ll 1$

2) $c_v \propto T$ from electron gas



C=aT + bT^3
Gold
Silver
Copper
C/T (mJ/K^2)
T^2 (K^2)
0
2
4
6
8
10
12
14
16
18