Thu 27 Feb 78 54 55

Recall thermal contact → total $M = \sum_{e_1} M^{(1)}_{e_1} M^{(2)}_{E-e_1}$

Say $N_{terms} \geq 1$ in sum, largest called "max"

Simple bounds: $max \leq M \leq N_{terms} \cdot max$

Micro-canonical: $\log(max) \leq S \leq \log(N_{terms} \cdot max)$

Reasonable behaviour $N_{terms} \sim N$

$max \sim e^N$

↳ spin system $M = 2^N = e^{N \log 2}$

$N! \sim N^N = e^{N \log N}$

$N \lesssim S \lesssim N + \log N$

$N \sim 10^{23} \rightarrow 10^{23} \lesssim S \lesssim 10^{23} + 50$

↳ $S = 10^{23}$ for all practical purposes

Spin system $H = 1$

$N_1 = N_2 = 10$

$E = e_1 + e_2 = -10$

↳ $e_1 = -(2n_+^{(1)} - N_1) = -10, -8, -6, \ldots, 10$

e_1	$E - e_1$	$n_+^{(1)}$	$n_+^{(2)}$	$M_1 M_2$
-10	0	10	5	$\binom{10}{0} \cdot \binom{10}{5} = 1 \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{5 \cdot 4 \cdot 3 \cdot 2} = 4 \cdot 63 = 252$
-8	-2	9	6	2100
-6	-4	8	7	$\binom{10}{7}\binom{10}{8} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} \frac{10 \cdot 9}{2} = 120 \times 45 = 5400 = max$
-4	-6	7	8	
-2	-8	6	9	$\binom{10}{1} \cdot \binom{10}{6} = 10 \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2} = 2100$
0	-10	5	10	252

$N_{terms} = 6$

$\log(5400) \leq \log(15504) \leq \log(6 \cdot 5400)$

$8.59 \leq 9.65 \leq 10.39$

← 89%

Stirling's Formula

$$\log(N!) \approx N\log N - N \qquad N \gg 1$$

$$N! \approx \exp(N\log N - N) = N^N e^{-N} = \left(\frac{N}{e}\right)^N$$

More precise:

$$N! = \sqrt{2\pi N}\left(\frac{N}{e}\right)^N \left(1 + \frac{A}{N} + \frac{B}{N^2} + \frac{C}{N^3} + \dots\right)$$

(asymptotic)

1) Simple bounds

$$N\log N - N \leq \log(N!) < N\log N$$

2) Find $N! \approx \sqrt{2\pi N}\left(\frac{N}{e}\right)^N$

by showing $N! = \int_0^\infty x^N e^{-x}\, dx$ approx. by gaussian

3) Compute $A, B, \dots$

by comparing $N!$ and $(N+1)! = (N+1)N!$

commons.wikimedia.org/wiki/File:Stirling_error_vs_number_of_terms.svg