Thu 9 May

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Plan

Ising model ground states vs. lattice structure

Module review / exam prep

(Solve fully-connected case?)

Ground states $H=0 \rightarrow E = -J \sum_{(jk)} s_j s_k$

$J>0$: (ferromag.)

└ includes $J=1$ → all spins align, degen. 2

Ordered phase, order param. $|m|=1$

All links interchangeable — lattice structures don't matter

$J<0$: Aligned spins increase energy

Ground state maximizes pairs of opposing n.n. spin

└ Fully alternating (anti-ferromag.), degen. 2

$m = \frac{n_+ - n_-}{n_+ + n_-} = 0$ same as disordered phase

but clear order/pattern

Parameter: $\frac{1}{N}\sum_n (-1)^n s_n = m_s$ "staggered magnetization"

└ even/odd lattice labels

$|m_s| = 1$ vs. $m_s = 0$ in disordered phase

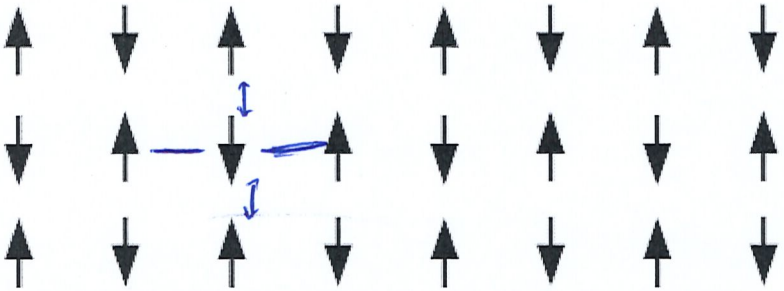
Related to $F = -T \log Z$

by generalizing $E = -J \sum_{(jk)} s_j s_k - H_s \sum_n (-1)^n s_n$

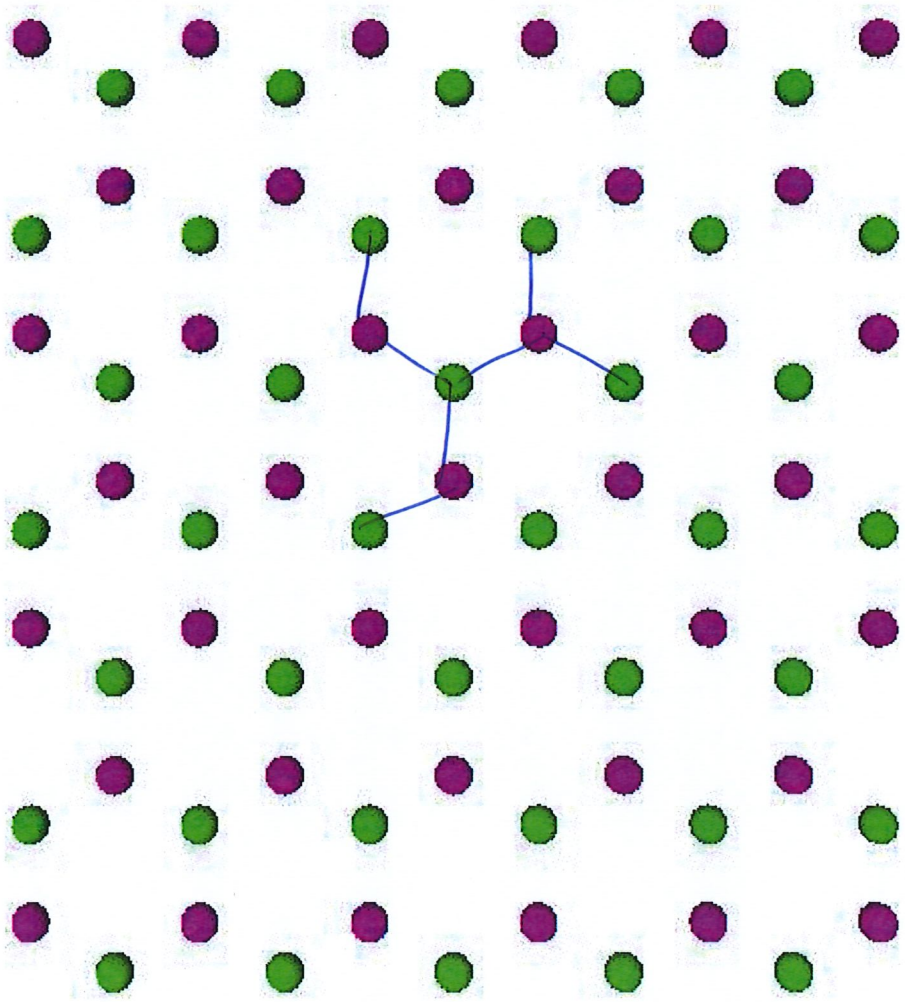
└ staggered mag. field

$\langle m_s \rangle \propto \frac{\partial}{\partial H_s} \log Z$

then set $H_s = 0$



commons.wikimedia.org/wiki/File:Antiferromagnetic_ordering.svg



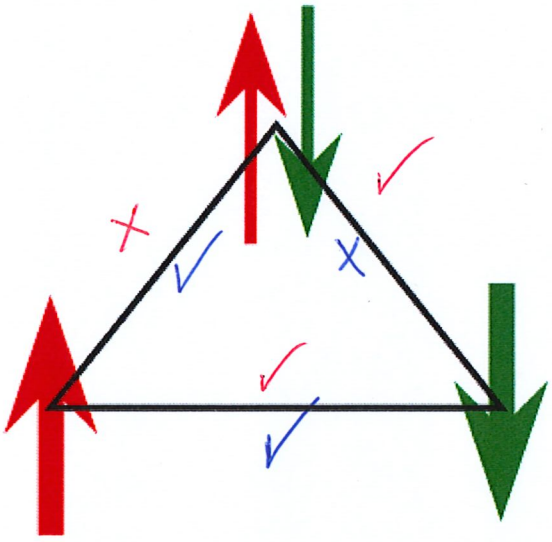
Triangular lattice $J<0$ can't have fully alternating spins
"Frustrated anti-ferromagnets" (non-bipartite)

Single unit cell → 6 degen. ~~config~~ config.
with $E = -J(\blacksquare - 2 + 1)$

With $N \gg 1$, no order, not order param.
many degenerate micro-states
↳ $S > 0$ even at absolute zero T

Finding ground states in NP-hard class
↳ quantum computing

?



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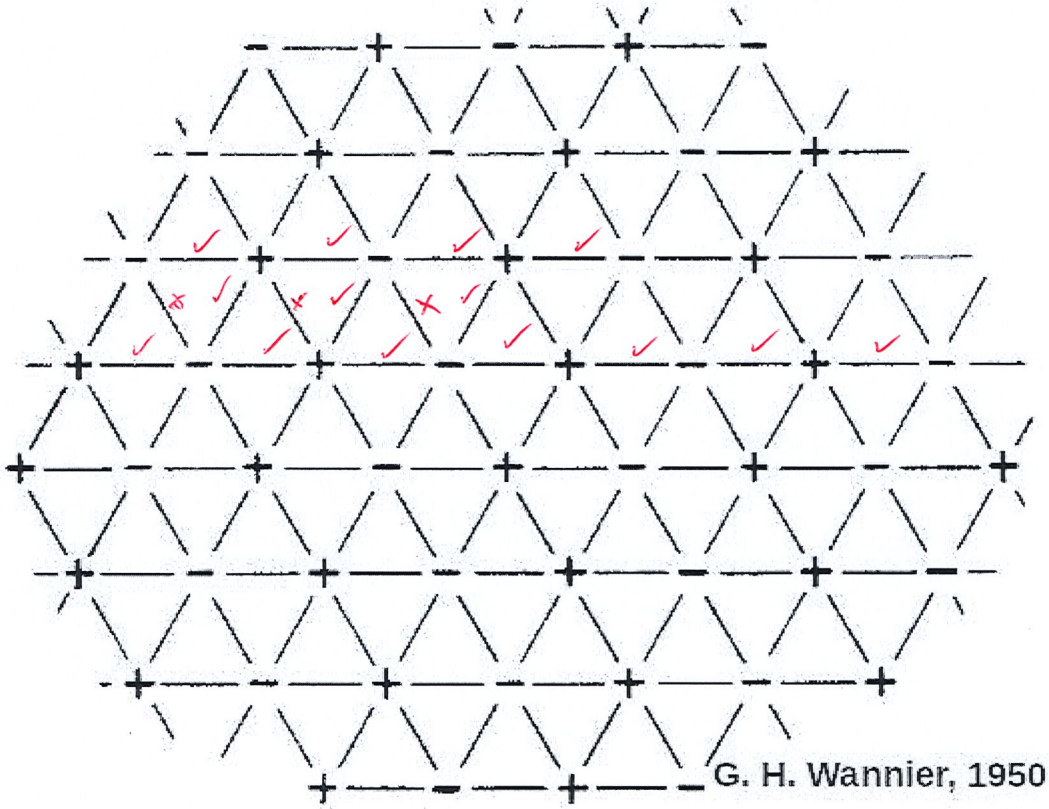

G. H. Wannier, 1950