Tue 30 Apr

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Recap

Photon gas EoS $P_{ph} V = \frac{\zeta(4)}{\zeta(3)} \langle N \rangle_{ph} T$

Non-rel. gas of fermions

Low-T simplification: Fermi function $F(E) = \frac{1}{e^{\beta(E-\mu)}+1}$

$\rightarrow$ step function

At $T=0$ all energy levels filled up to Fermi energy

$$E_F = \mu = \frac{\hbar^2}{2m} (3\pi^2)^{2/3} \rho^{2/3} > 0$$

$$\rho = \frac{\langle N \rangle_f}{V}$$

Non-zero $\langle E \rangle_f = \frac{3}{5} \mu \langle N \rangle_f$

and degeneracy pressure

$$P_f = \frac{2}{3} \frac{\langle E \rangle_f}{V} = \frac{2}{5} \mu \rho = \frac{\hbar^2}{5m} (3\pi^2)^{2/3} \rho^{5/3}$$

Degeneracy pressure matters for low temperatures $T \ll E_F$

Larger densities $\rightarrow$ larger $E_F \propto \rho^{2/3}$

Everyday metals have $\rho \sim \frac{N_A \text{ electrons}}{cc} \sim 10^{29}$ electrons/m^3

$\rightarrow E_F \sim 10^4$ K $\sim$ 1eV $\sim 10^{-19}$ J

Everyday $T \ll 10^4$K $\rightarrow$ degenerate electron gas

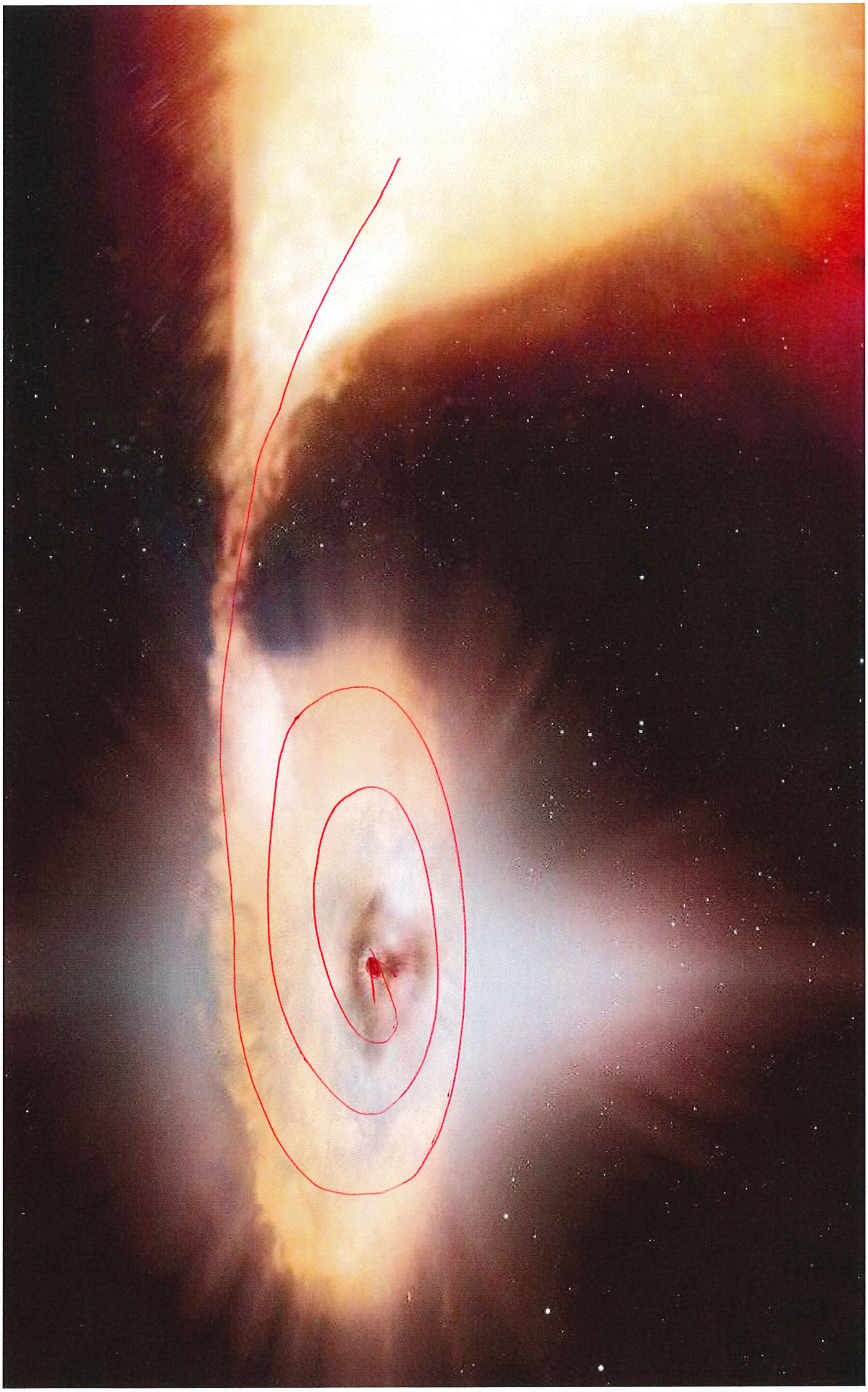
Sun (on average) has similar $\rho \sim 10^{30}$ electron/m^3 $\rightarrow E_F \sim 10^5$ K

much higher T up to $\sim 10^7$ K $\gg E_F$ in core

Fusion of hydrogen & helium heats up sun,

radiation pressure balances force of gravity, reduces ρ

After H & He 'fuel' exhausted, less radiation pressure $\rightarrow$ higher ρ



White dwarf stars have sun's mass w/earth radius (~100x smaller)

$\rho \sim 10^6 \rho_{sun} \sim 10^{36}$ electrons/m^3 (mass ~ tonne/cc)

$E_F \sim (10^6)^{2/3} E_F^{sun} \sim 10^4 \times 10^5 K \sim 10^9 K \gg T \sim 10^7 K$

↳ slowly cools to ~ 10^3 K after ~ 10^{10} years

$T \ll E_F \rightarrow$ degenerate electron gas

Degeneracy pressure prevents further collapse (to black hole)

Binary system → white dwarf can capture matter from companion star

increasing mass and density

Chandrasekhar limit $M \sim 1.4 M_{sun}$ → carbon & oxygen fusion chain reaction

→ $T \sim 10^9$ K with seconds

radiation pressure → supernova (type-Ia)

~ 5 billion times brighter than sun

~~Regularit~~

Regularity of type-Ia supernovas makes them "standard candle"

Provides distance vs. time

→ accelerating expansion of Universe (Nobel 2011)

Despite fantastic predictions from ideal system (Planck, c_V)

insufficient of describe many phenomena like phase trans.

Interactions needed but much harder to analyze

Generally no closed-form predictions

or even accurate approximations

Phases are different emergent behaviour for same particles

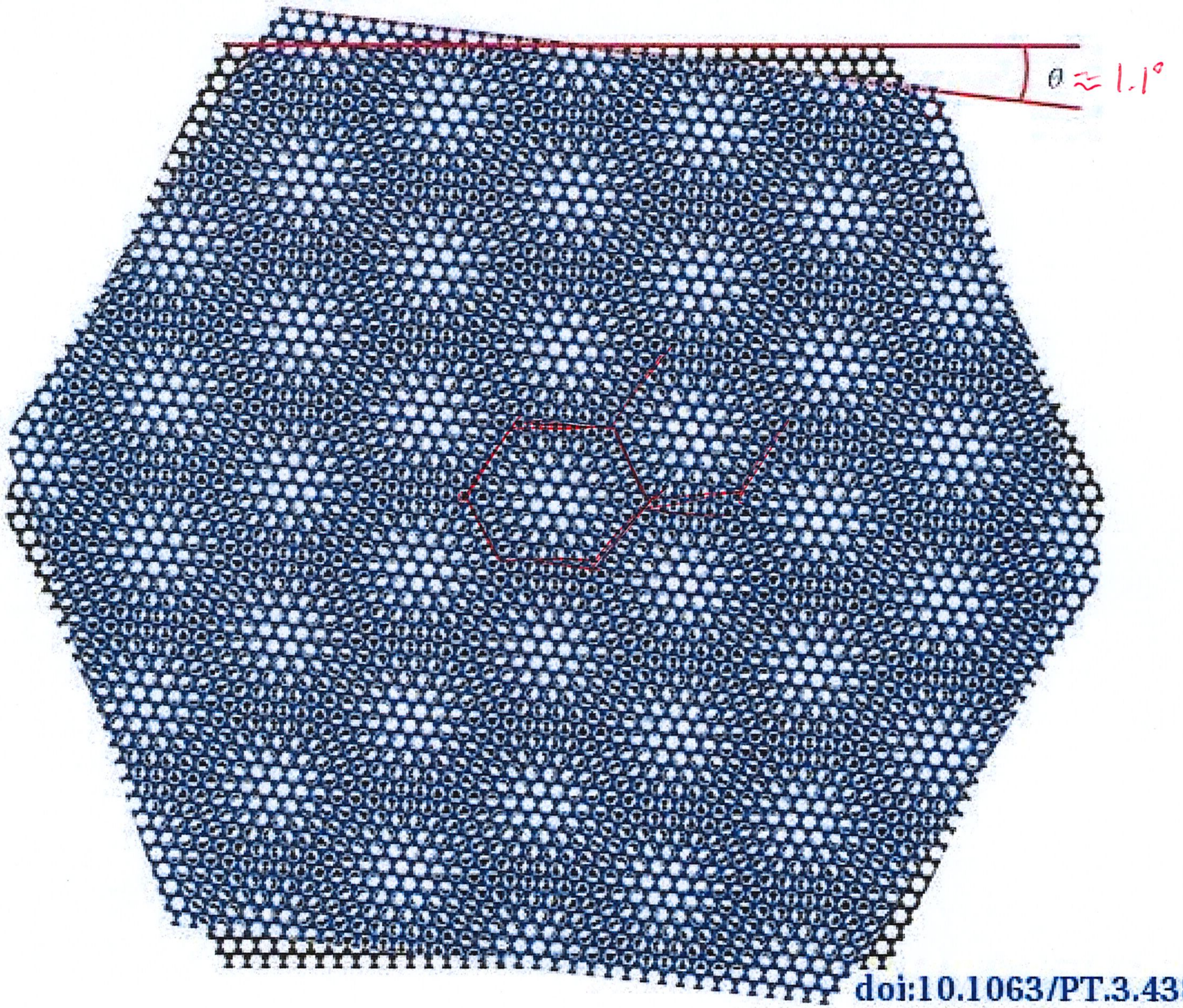
Ice vs. water vs. steam all from H_2O

Quarks and gluons in plasma → nuclei in early Universe

Electrons in bilayer graphene

Insulating → superconducting at "magic" $\theta \approx 1.1°$ and $T \lesssim 1.7K$

(superconducting: no energy loss)



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