Thu 25 Apr

57 31 56

Photons polarization

Dark matter & CMB

Results for Einstein solid → improved model

$S = \log M$ $\qquad M = \binom{K+N-1}{K} = \frac{(K+N-1)!}{K!\,(N-1)!}$

Same as K indist'able balls in N dist'able boxes

$\bullet\bullet|\cdot||\bullet = (2,1,0,1)$

Total $K+N-1$ symbols

$\binom{K+N-1}{K} = M$ ways to choose K of them to be balls □

Trying to correct N^K for over-counting → mess

Minimal check $N=3$

$K=0 \to M=1$ $\quad (0,0,0)$ $\quad \binom{2}{0}=1$ ✓

$K=1 \to M=3$ $\quad (1,0,0)$ + perms $\quad \binom{3}{1}=3$ ✓

$K=2 \to M=6$ $\quad (1,1,0)$ + perms $\quad \binom{4}{2}=6$ ✓

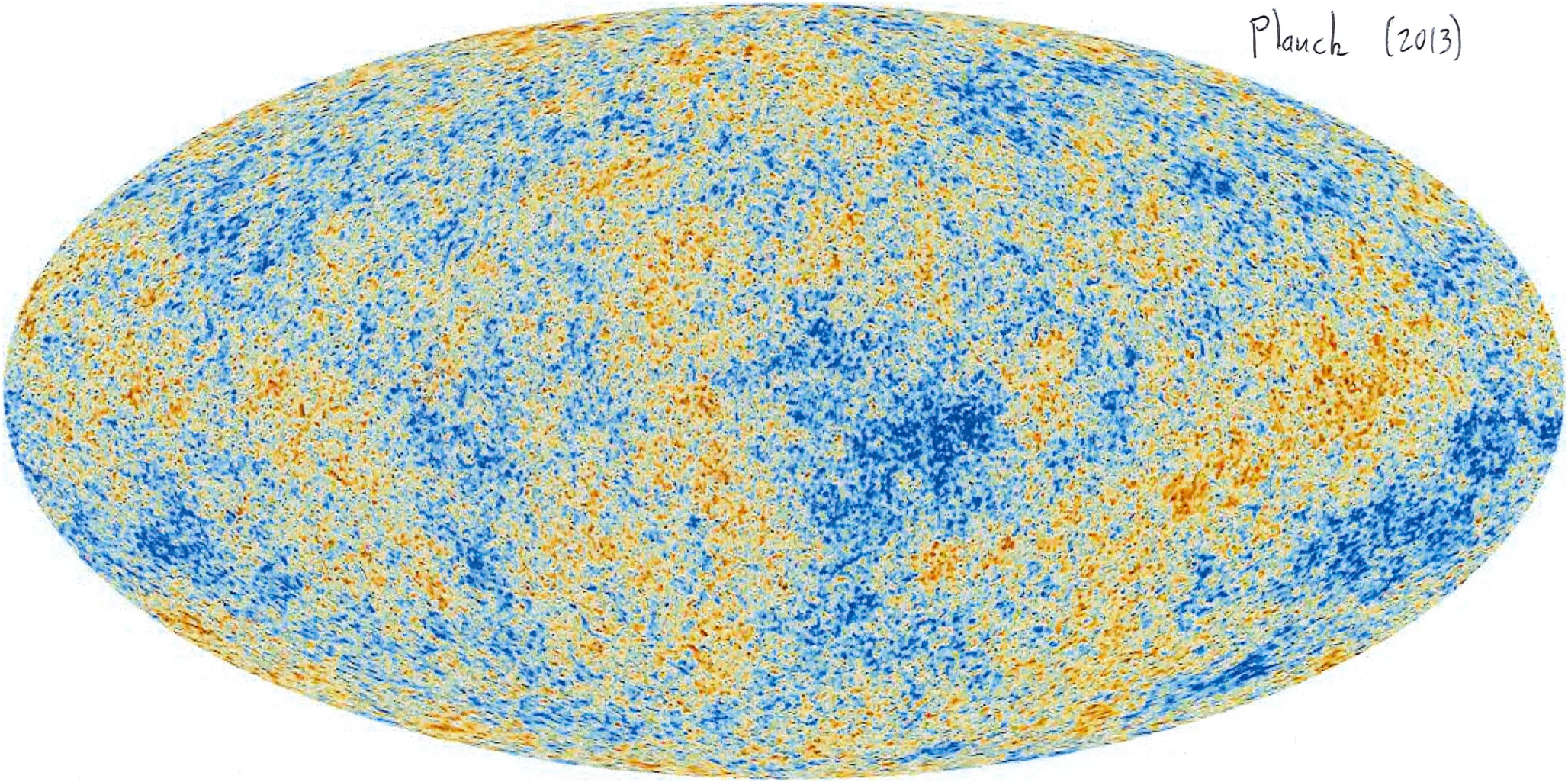
$(2,0,0)$ + perms

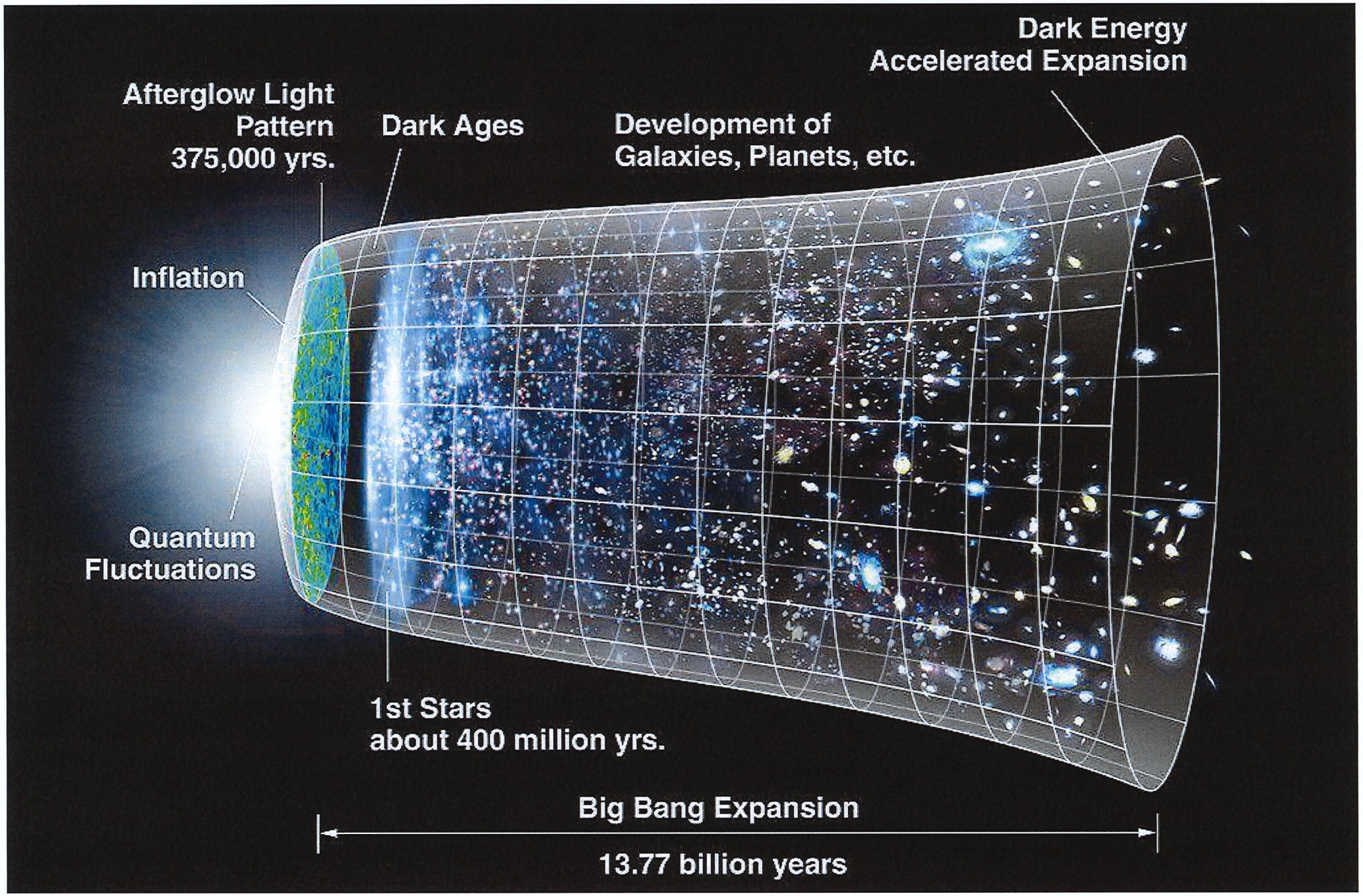
$K=3$

$(3,0,0)$ + $(2,1,0)$ + $(1,1,1)$

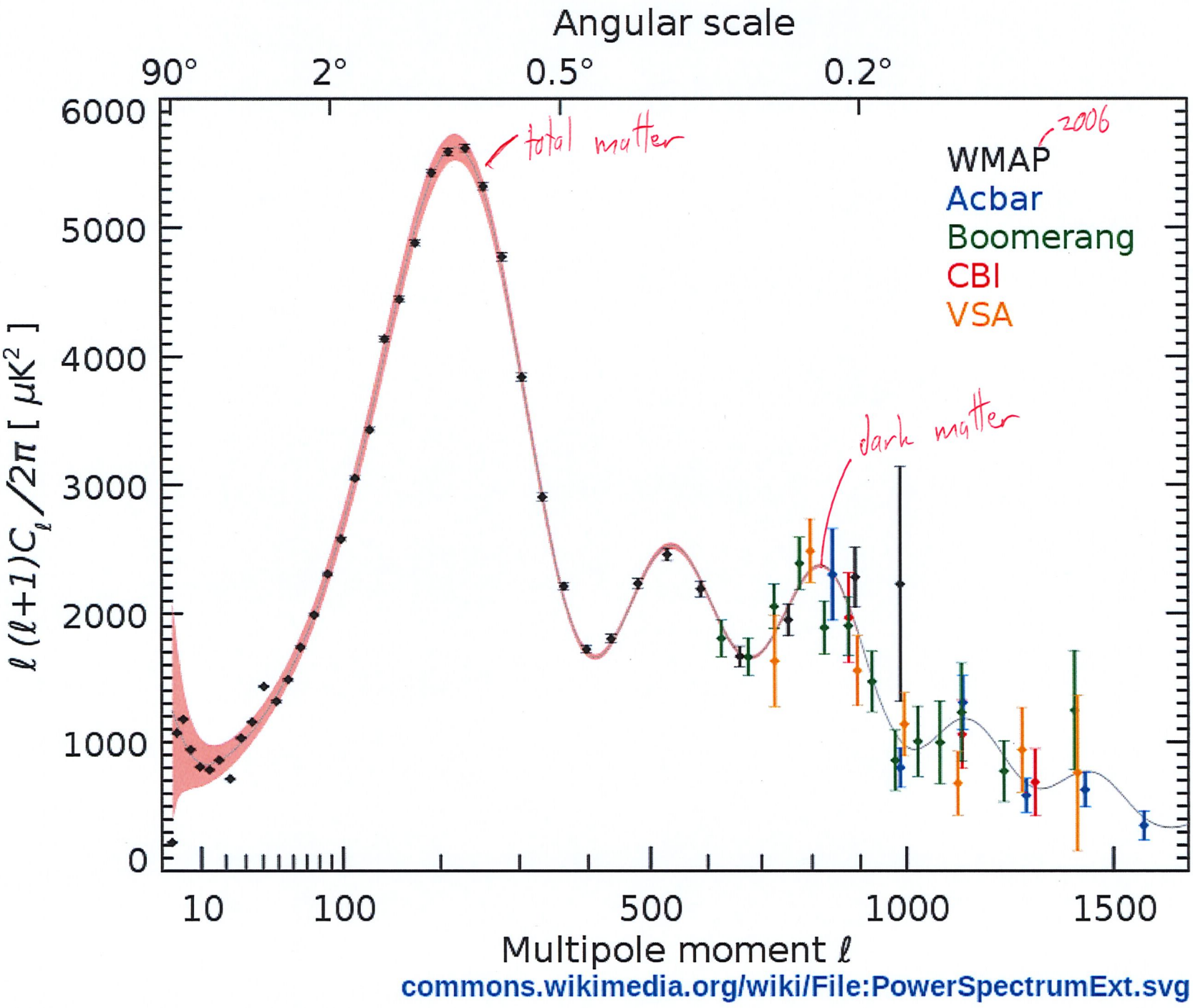
3 6 1

$M = 10 = \binom{5}{3} = \frac{5\cdot 4}{2} = 10$ ✓

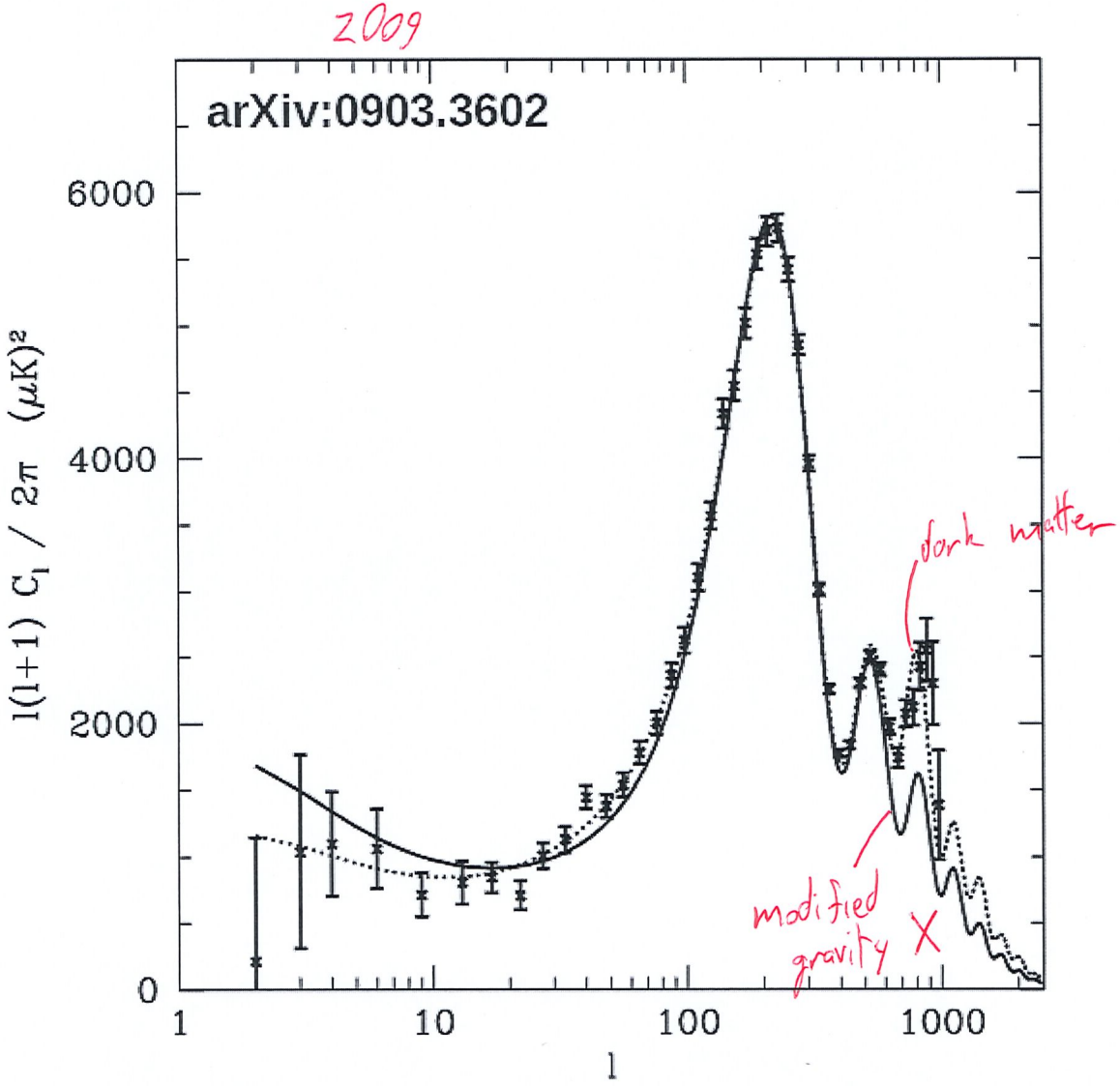

Planck (2013)



commons.wikimedia.org/wiki/File:CMB_Timeline300_no_WMAP.jpg



commons.wikimedia.org/wiki/File:PowerSpectrumExt.svg


2009
arXiv:0903.3602
6000
4000
2000
0
1
10
100
1000
l
l(l+1) C_l / 2π (μK)²
dark matter
modified gravity X

$$\beta = \frac{1}{T} = \frac{\partial S}{\partial E} = \frac{1}{\hbar\omega}\frac{\partial}{\partial K}\left((K+N)\log(K+N) - K\log K - N\log N\right)$$

$E = K\hbar\omega$ $\qquad N-1 \approx N \qquad N \gg 1$

$$\beta\hbar\omega = \log\left(1+\frac{N}{K}\right) = \log\left(1+\frac{N\hbar\omega}{E}\right)$$

$$E = \frac{N\hbar\omega}{e^{\beta\hbar\omega}-1}$$

$$C_V = -\beta^2\frac{\partial}{\partial\beta}E = +\beta^2\frac{N\hbar\omega\left(e^{\beta\hbar\omega}\right)(\hbar\omega)}{\left(e^{\beta\hbar\omega}-1\right)^2} = \frac{Nx^2e^x}{\left(e^x-1\right)^2}$$

$x = \beta\hbar\omega$

$T\to\infty \quad \beta\to 0 \quad x\to 0$

$$\frac{C_V}{N} \to \frac{x^2(1)}{(x)^2} = 1 - \frac{1}{12}\frac{\hbar^2\omega^2}{T^2} + O\left(\frac{\hbar^3\omega^3}{T^3}\right)$$

$T\to 0, \quad \beta\to\infty \quad x\to\infty$

$$\frac{C_V}{N} \to \frac{x^2e^x}{e^{2x}} = \frac{x^2}{e^x} \to 0 \checkmark$$

(third law)

Problem: neglected correlated waves

phonons inspired by photons

3, c_s 2, c

Phonons have minimum wavelength

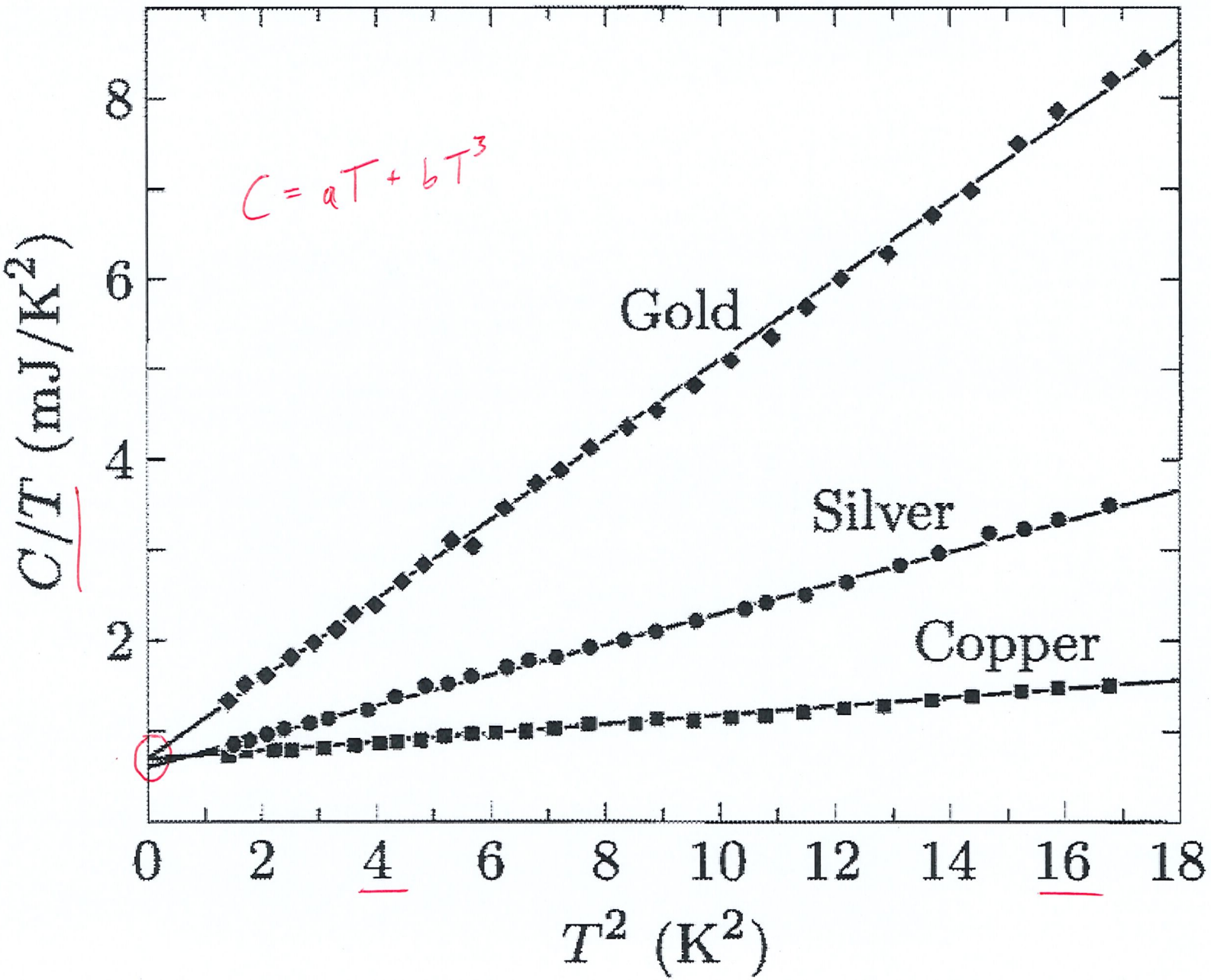
maximum freq. $\omega_{max} = T_D/\hbar$

Task: Redo photon gas for phonon

High-T C_V ~ Einstein

Low-T $C_V \sim T^3$

Task: Electron gas → $C_V \sim T$ at low T


$C = aT + bT^3$
Gold
Silver
Copper
C/T (mJ/K^2)
T^2 (K^2)
0
2
4
6
8
10
12
14
16
18
2
4
6
8