Tue 23 Apr   19 69 85

Recap

BE & FD $\to$ classical MB for $-\mu \gg T \gg E_\ell$

Quantum gas of ultra-relativistic (massless) bosons
(photons, $E_{ph} = \hbar\omega$, $\mu = 0$)

Quantized wavelength & frequencies $c = \frac{\lambda\omega}{2\pi}$

Planck spectrum $\frac{\langle E \rangle_{ph}}{V} = \int_0^\infty P(\omega)\, d\omega = \int_0^\infty P(\lambda)\, d\lambda$

$$P(\omega) = \left(\frac{\hbar}{c^3 \pi^2}\right) \frac{\omega^3}{e^{\beta\hbar\omega} - 1}$$

$$P(\lambda) = \left(\frac{16\pi^2 \hbar c}{\lambda^5}\right) \frac{1}{e^{2\pi\beta\hbar c/\lambda} - 1}$$

As always consider limiting behaviour

UV: $\lambda \to 0$

Exponential factor dominates, $P(\lambda) \to 0$

IR: λ large compared to $\beta\hbar c$

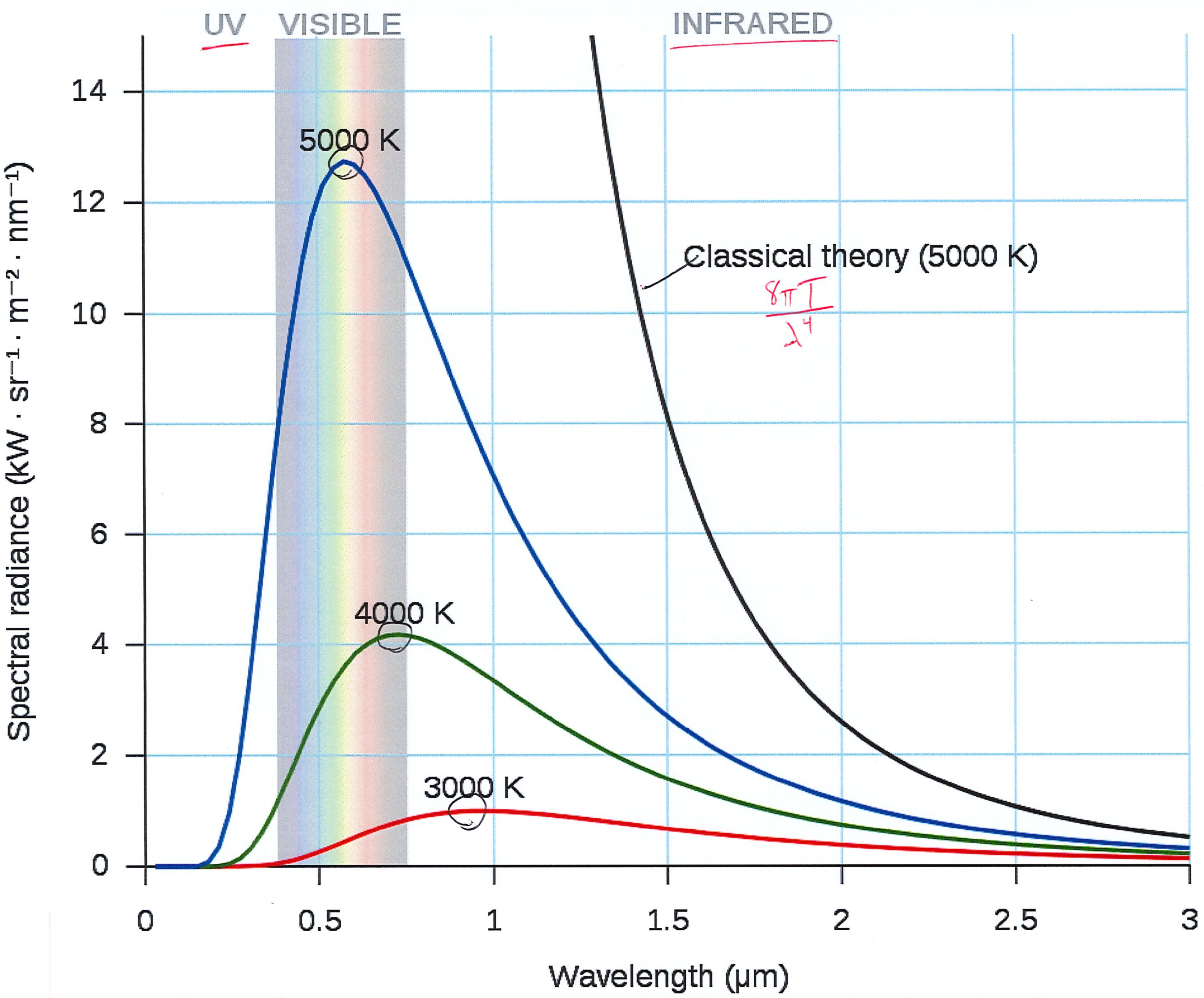
$$e^{2\pi\beta\hbar c/\lambda} - 1 \approx \frac{2\pi\beta\hbar c}{\lambda} \quad \cancel{\to P(\lambda)}$$

$$\to P(\lambda) \approx \left(\frac{16\pi^2 \cancel{\hbar c}}{\lambda^5}\right)\left(\frac{\lambda}{2\pi\beta\cancel{\hbar c}}\right) = \frac{8\pi T}{\lambda^4}$$

Rayleigh-Jeans spectrum
(classical)

Classical spectrum diverges $\sim \frac{1}{\lambda^4}$ as $\lambda \to 0$

"ultraviolet catastrophe" $\to$ quantum


UV
VISIBLE
INFRARED
5000 K
4000 K
3000 K
Classical theory (5000 K)
$\frac{8\pi T}{\lambda^4}$
Spectral radiance (kW · sr⁻¹ · m⁻² · nm⁻¹)
Wavelength (μm)
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3

Full Planck $P(\lambda)$ has temperature-dependent peak

Visible light around peak for $T \sim 5000K$

Because sunlight has $T \approx 5778$ K

Determine effective surface temperature by fitting $P(\lambda)$

Red stars $\rightarrow$ $T \lesssim 3500$ K

Blue stars $\rightarrow$ $T \gtrsim 10,000$ K

Intergalactic space $\rightarrow$ $T \approx 2.725$ K

Cosmic Microwave Background left over from Big Bang

~13.7 Gyr

T at each point in sky

From photons after subtracting galaxies

Blue/red small fluctuations around average $T_{CMB} \approx 2.725K$

$\Delta T \approx 0.0002$ K

Pattern of fluctuations $\rightarrow$ dark matter

Amazingly accurate descriptions

given assumption of non-interacting ideal gas

Finish integrating Planck's spectrum!

$$\langle E \rangle_{ph} = \frac{V\hbar}{c^3 \pi^2} \int_0^\infty \frac{\omega^3}{e^{\beta\hbar\omega} - 1} d\omega$$

$$x = \beta\hbar\omega = \frac{\hbar\omega}{T}$$

$$d\omega = \frac{T}{\hbar} dx$$

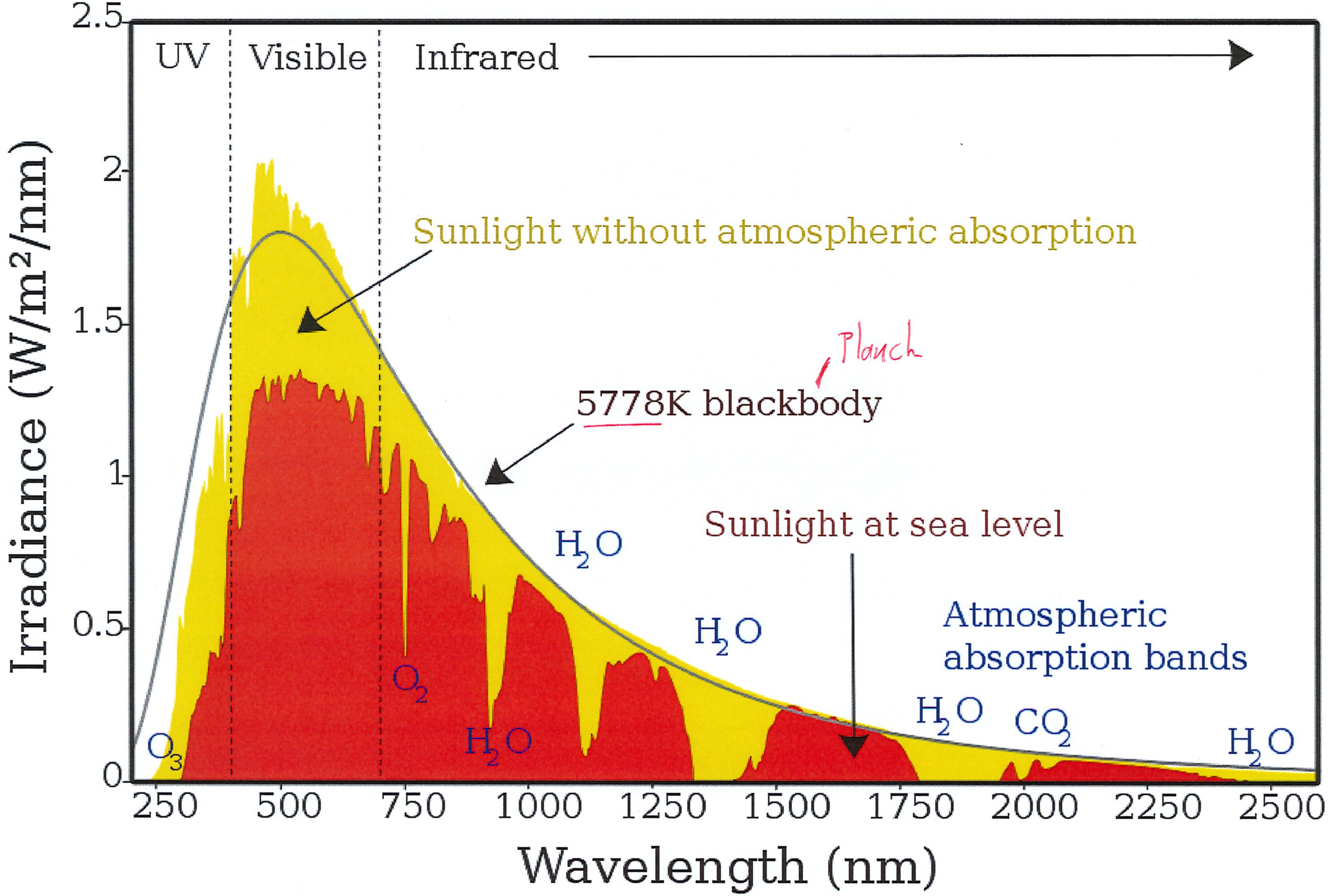
$$= \frac{V\hbar}{c^3 \pi^2} \left(\frac{T}{\hbar}\right)^4 \int_0^\infty \frac{x^3}{e^x - 1} dx$$

$$\Gamma(4)\zeta(4) = 6 \cdot \frac{\pi^4}{90} = \frac{\pi^4}{15}$$

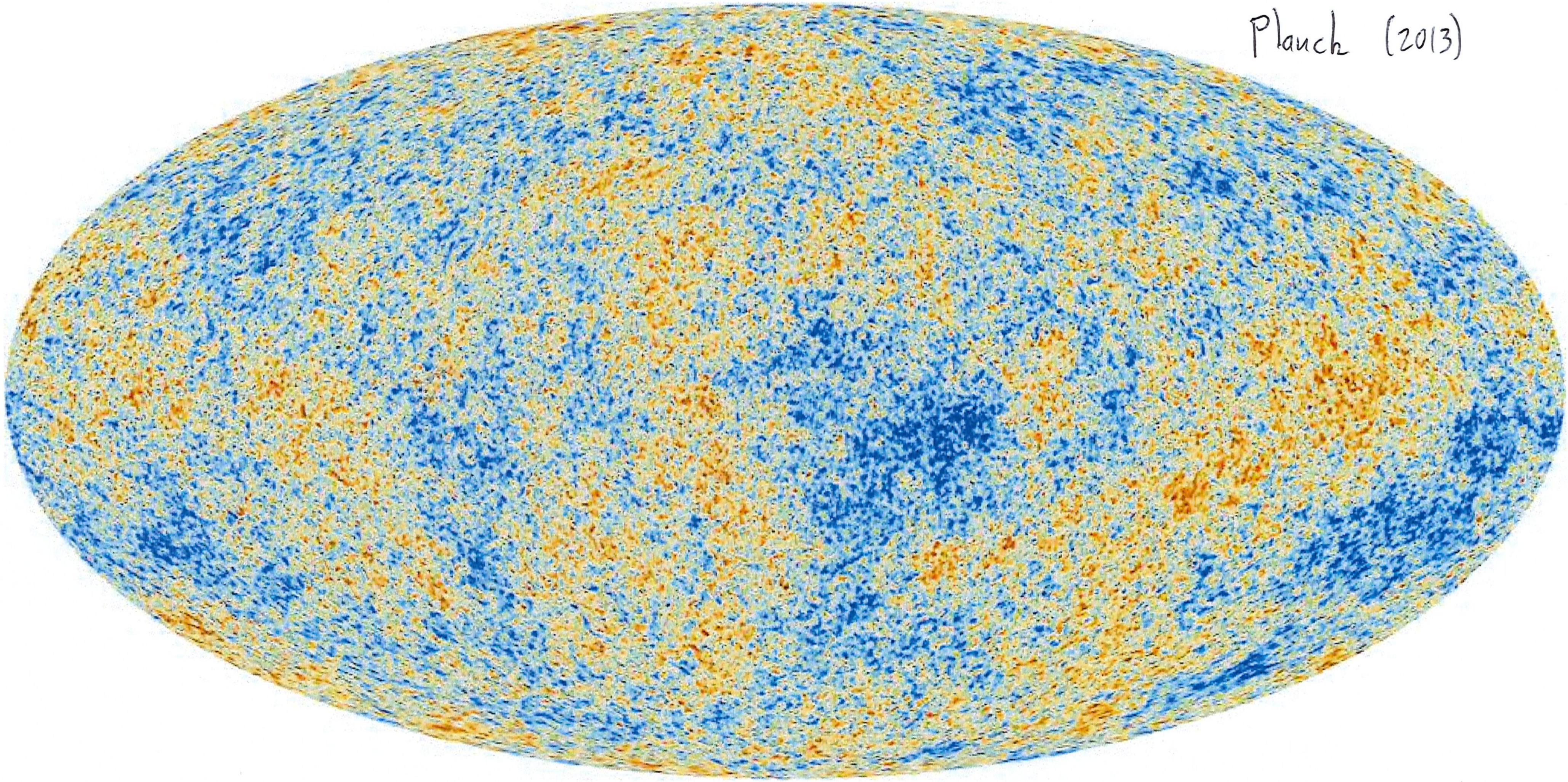
Riemann Zeta func.

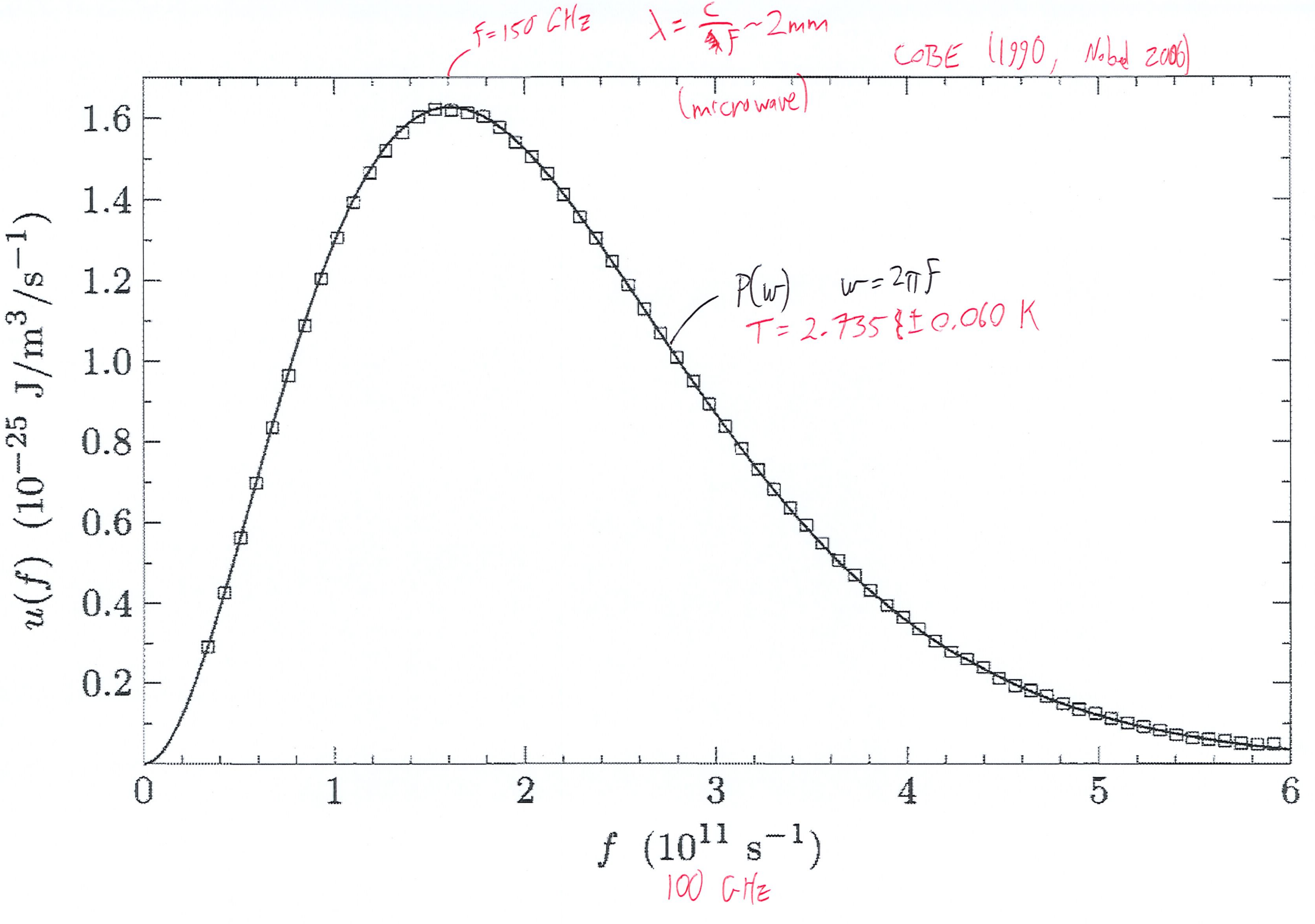
$$\langle E \rangle_{ph} = \frac{\pi^2}{15 \hbar^3 c^3} V T^4$$

Spectrum of Solar Radiation (Earth)



Planck (2013)




f=150 GHz
$\lambda = \frac{c}{f} \sim 2$mm
(microwave)
COBE (1990, Nobel 2006)
$P(\omega)$ $\omega = 2\pi f$
T = 2.735 ± 0.060 K
$u(f)$ $(10^{-25}$ J/m^3/s$^{-1})$
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
0
1
2
3
4
5
6
f $(10^{11}$ s$^{-1})$
100 GHz