Thu 18 Apr 26 38 04

Plan

First HW Feedback
Second HW due next week

Heat capacities & Einstein solid

HW 1) Key is 1% probability $\leftrightarrow$ integral of probability distribution

Faster first-~~process~~ passage process because no chance of walking back out

2) "Heat exchange moves temperatures closer together" X
Actually move farther apart (unnatural)

4) $C_V = \frac{\partial E}{\partial T} = \beta^2 \langle (E - \langle E \rangle)^2 \rangle \geq 0$

Don't forget $Z = \sum_i e^{-E_i/T}$ depends on T

$\frac{\partial Z}{\partial T} \neq 0$

Heat capacity

Ideal gas $E = \frac{3}{2} NT$ $\quad C_V = \frac{3}{2} N$ X

Spin system (dist'able) $E = -NH \tanh(\beta H)$ $\quad H > 0$

$$C_V = -\beta^2 \frac{\partial E}{\partial \beta} = N\beta^2 H \left(\frac{H \cosh(\beta H)}{\cosh(\beta H)} - \frac{H \sinh^2(\beta H)}{\cosh^2(\beta H)} \right)$$

$$= N\beta^2 H^2 / \cosh^2(\beta H) > 0 \quad \checkmark$$

Low-T $\beta \to \infty$ $\quad \cosh^2(\beta H) \approx e^{2\beta H}$ $\quad C_V \to 0$ ✓

High-T $\beta \to 0$ $\quad \cosh^2(\beta H) \to 1$ $\quad C_V \to 0$ X

Einstein solid

Atoms in solid held in place by interacting w/neighbours

Focus on oscillators → non-interacting

suppose energies $\varepsilon_i = 0, \hbar\omega, 2\hbar\omega, \ldots$

Consider micro-canonical. N oscillators

$E = K\hbar\omega$ $\qquad K = \sum_{i=1}^{N} k_i$

$E = \sum_i k_i \hbar\omega = \sum_i \varepsilon_i$

K units of energy

to assign to N oscillators

Find $\Omega \to S = \log \Omega \to T = \left(\frac{\partial S}{\partial E}\right)^{-1}$

Invert to find (canonical) $E(T) \to c_V = \frac{\partial E}{\partial T}$

$N = 3$

$K = 0, 1, 2, 3 \ldots$

$$\Omega = \frac{(K+N-1)!}{K!\,(N-1)!} = \binom{K+N-1}{K}$$