Thu 14 Mar

104 781

Plan

Mixing entropy

Therm. cycles

Otto cycle ~ petrol engine

Useful trick: $\Delta S_{mix} = S_c - S_0 = \frac{\partial}{\partial T}\left(T \log Z_c - T \log Z_0\right)$

$= \frac{\partial}{\partial T}\left(T \log\left(\frac{Z_{0c}}{Z_0}\right)\right)$

$$\frac{Z_c}{Z_0} = \frac{\frac{1}{(N!)^2}\left(\frac{2V}{\lambda_{th}^3}\right)^{2N}}{\left[\frac{1}{N!}\left(\frac{V}{\lambda_{th}^3}\right)^N\right]^2} = 2^{2N} \quad (T\text{-indep.})$$

$\Delta S_{mix} = 2N \log 2$

↳ same as fully dist'able

Less info but same relative increase upon mixing

$S_F = (S_F - S_c) + S_c = \frac{\partial}{\partial T}\left(T \log \frac{Z_F}{Z_c}\right) + S_c$

Gibbs approx. of N particles on each side

r red on left → $N-r$ blue

$N-r$ red & r blue on right

$$Z_F = \sum_{r=0}^{N} Z_r = \sum_r \left[\frac{1}{r!}\left(\frac{V}{\lambda_{th}^3}\right)^r \frac{1}{(N-r)!}\left(\frac{V}{\lambda_{th}^3}\right)^{N-r}\right]^2$$

$$Z_F = \left(\frac{V}{\lambda_{th}^3}\right)^{2N} \sum_\nu \frac{1}{(\nu!)^2([N-\nu]!)^2} = \left(\frac{V}{\lambda_{th}^3}\right)^{2N} \frac{1}{(N!)^2} \sum_\nu \binom{N}{\nu}^2$$

$$\uparrow \frac{1}{(N!)^2}\binom{N}{\nu}^2$$

$$Z_F = \frac{1}{(N!)^2}\left(\frac{V}{\lambda_{th}^3}\right)^{2N}\binom{2N}{N}$$

$$\frac{Z_F}{Z_C} = \frac{1}{Z^{2N}}\binom{2N}{N} \rightarrow S_F = S_C + \log\left(\frac{(2N)!}{(N!)^2}\right) - 2N\log 2$$

$N \gg 1$: $S_F \approx S_C + 2N\log 2N - 2N - 2(N\log N - N) - 2N\log 2$

$= S_C$

$S_F \approx S_C > S_0$ consistent w/second law ✓

Next correction: $\log(N!) \approx N\log N - N + \log(\sqrt{2\pi N})$

$$\frac{1}{2}\log(2\pi(2N)) - 2\left(\frac{1}{2}\log(2\pi N)\right) = \log\left(\frac{\sqrt{4\pi N}}{2\pi N}\right)$$

$$= -\log(\sqrt{\pi N}) < 0$$

$\rightarrow S_F \approx S_C - \log(\sqrt{\pi N}) < S_C$

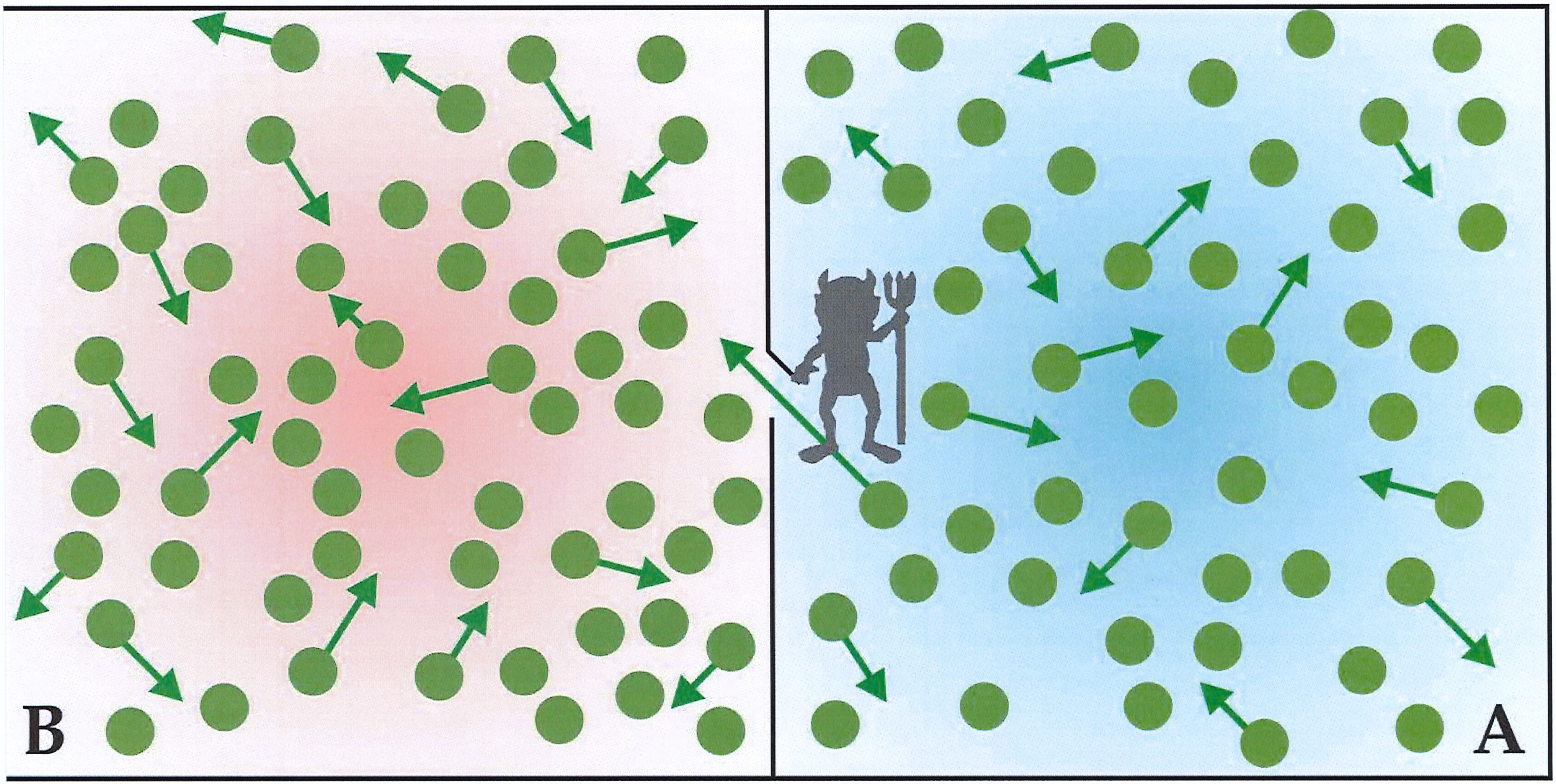
Solution: Go beyond Gibbs approx $(N \pm h)$

to see second law obeyed

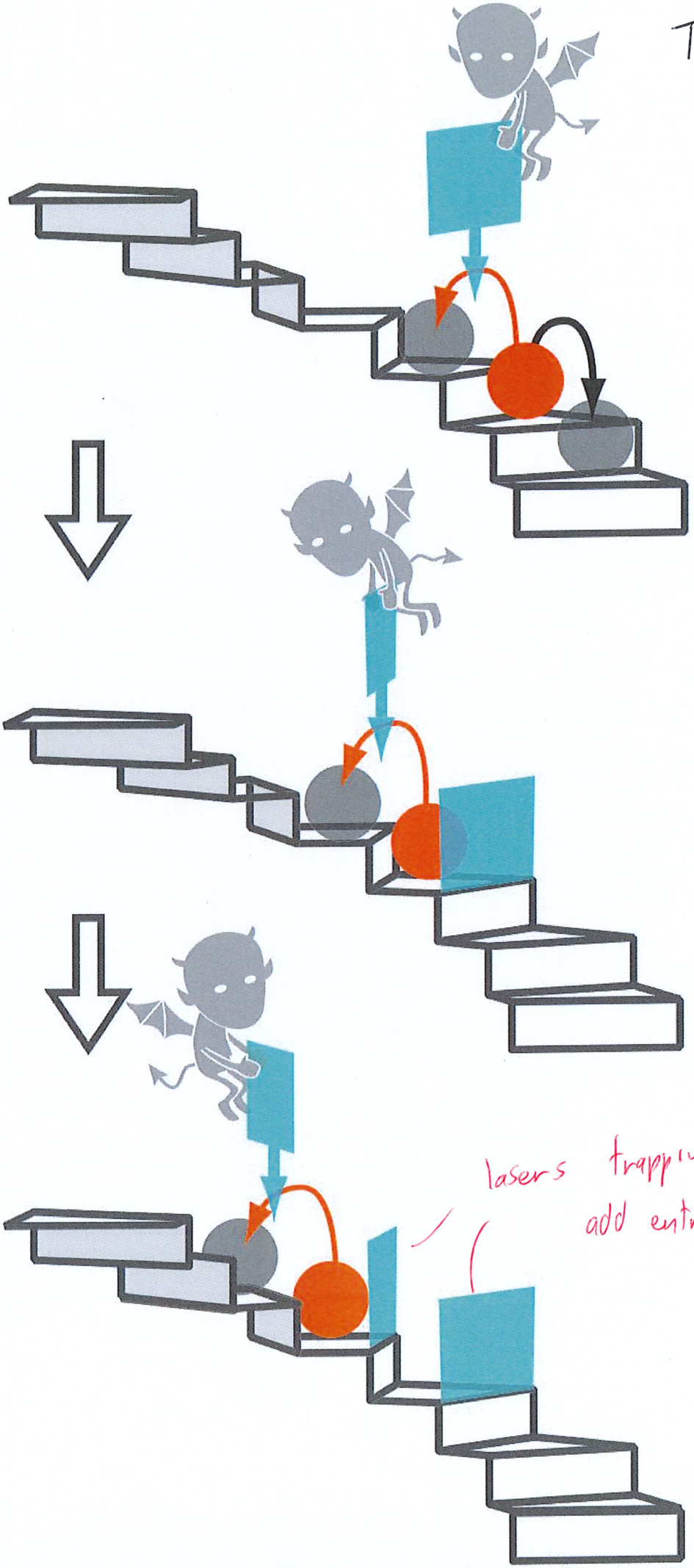
Maxwell's demon (1867) and 2010 experiment (below)

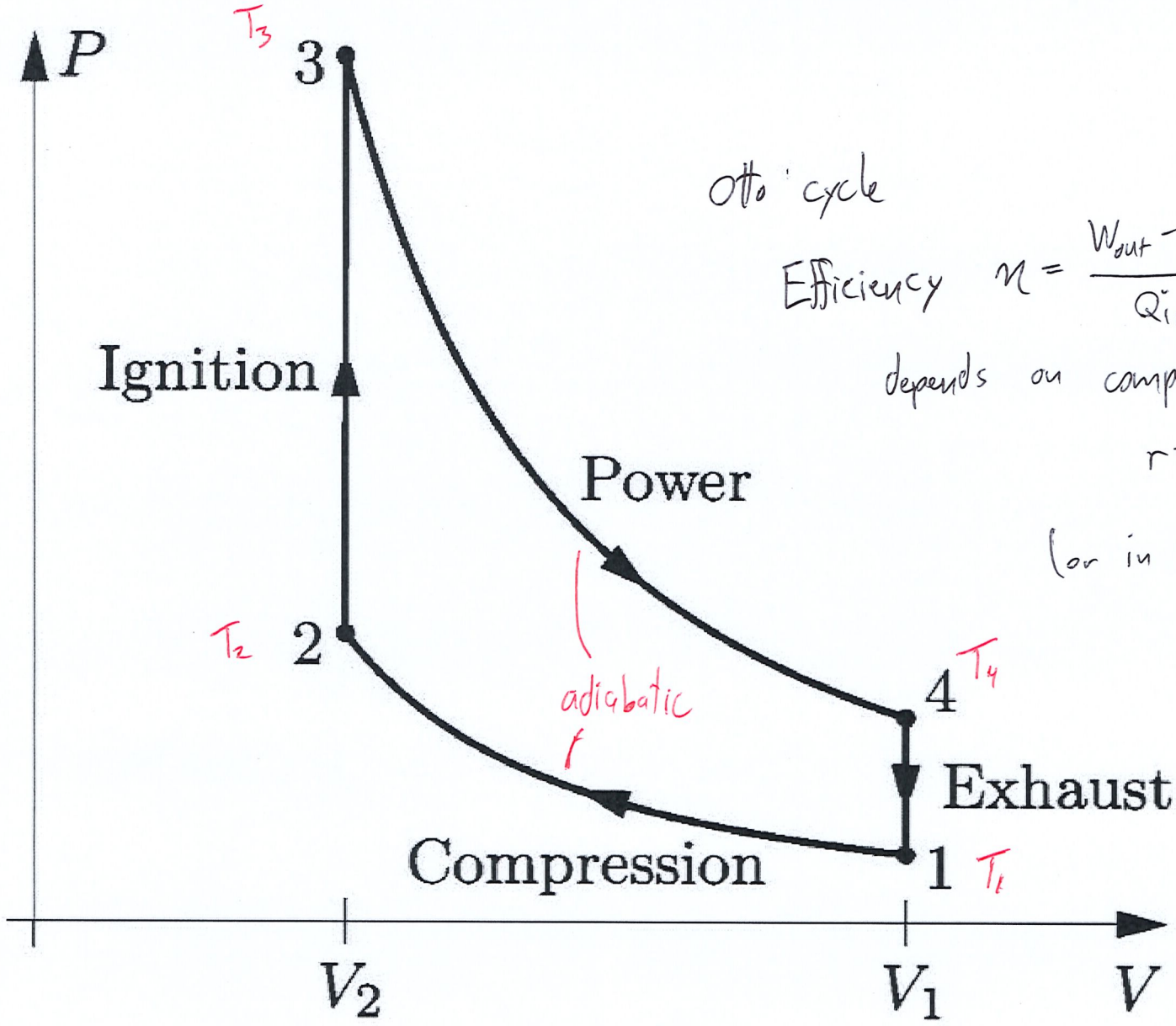
Therm. cycles – two "reservoirs", hot (T_H) and cold (T_L)

otto cycle ~ petrol engine (below)



$T_B > T_A$


Toyabe et al.
2010
lasers trapping particles
add entropy to
universe



T_3

Otto cycle

Efficiency $\eta = \frac{W_{out} - W_{in}}{Q_{in}}$

depends on compression ratio

$r = \frac{V_1}{V_2} > 1$

(or in terms of T_i)

T_2

adiabatic

T_4

T_1

– swap w/ fresh