Thu 7 Mar 66 87 09

Plan

Entropy bounds

Stirling's formula

Mixing entropy

Stirling

$$N \log N - N < \log(N!) < N \log N \qquad N \gg 1$$

$$N! = N(N-1)(N-2)\cdots 1 < N\cdot N\cdot N \cdots N = N^N$$

$$\log(N!) < N\log N$$

$$e^N = \sum_{k=0}^{\infty} \frac{N^k}{k!} > \frac{N^N}{N!} \qquad N! > \left(\frac{N}{e}\right)^N$$

$$\log(N!) > N\log N - N$$

$$\frac{d}{da}\left(\int_0^\infty e^{-ax}\,dx\right) = \frac{d}{da}\left(a^{-1}\right)$$

$$\int_0^\infty + x e^{-ax}\,dx = + a^{-2}$$

$$\frac{d^N}{da^N} \rightarrow \int_0^\infty (+x)^N e^{-ax}\,dx = \cancel{(-1)^N}\, N!\, a^{-(N+1)}$$

$$N! = \int_0^\infty x^N e^{-x}\,dx \qquad \square$$

$$\frac{d}{dx} x^N e^{-x} = 0 = N x^{N-1} e^{-x} - x^N e^{-x} \rightarrow N x^{N-1} = x^N$$

$$\underline{N = x}$$

$$y = x - N \qquad \left|\frac{y}{N}\right| \ll 1$$

$$N! = \int_0^\infty \exp\left[N \log x - x\right] dx$$

$$= \int_{-N}^{\infty} \exp\left[N \log\left(N\left(1+\frac{y}{N}\right)\right) - (N+y)\right] dy$$

$$= \int_{-N}^{\infty} \exp\left[N \log N + N\left(\cancel{\frac{y}{N}} - \frac{y^2}{2N^2} + \mathcal{O}\left(\frac{y^3}{N^3}\right)\right) - N - \cancel{y}\right] dy$$

$$\approx N^N e^{-N} \int_{-N}^{\infty} \exp\left(-y^2/2N\right) dy = \sqrt{2\pi N}\left(\frac{N}{e}\right)^N \qquad \checkmark$$

$-N \to -\infty$

~~Mixing~~ Mixing

$$\Omega_0 = \Omega_A \otimes \Omega_B \qquad S_0 = S_A + S_B$$

$$\downarrow$$

$$\Omega_C \qquad S_C$$

$$\downarrow$$

$$\Omega_A' + \Omega_B' \qquad S_F = S_A' + S_B'$$

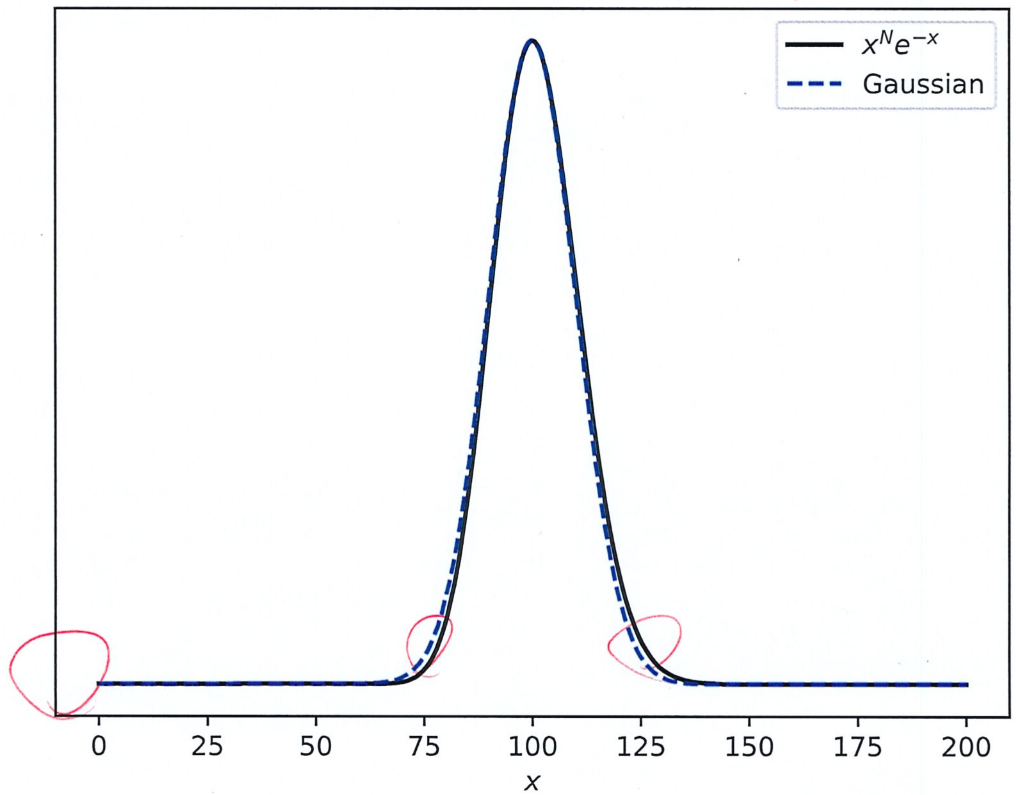
$$S_0 \geq S_C \geq S_F$$

$$S_0 = 2 S_D(N, V, T) = 3N + 2N \log\left(\frac{V}{\lambda_{th}^3}\right)$$

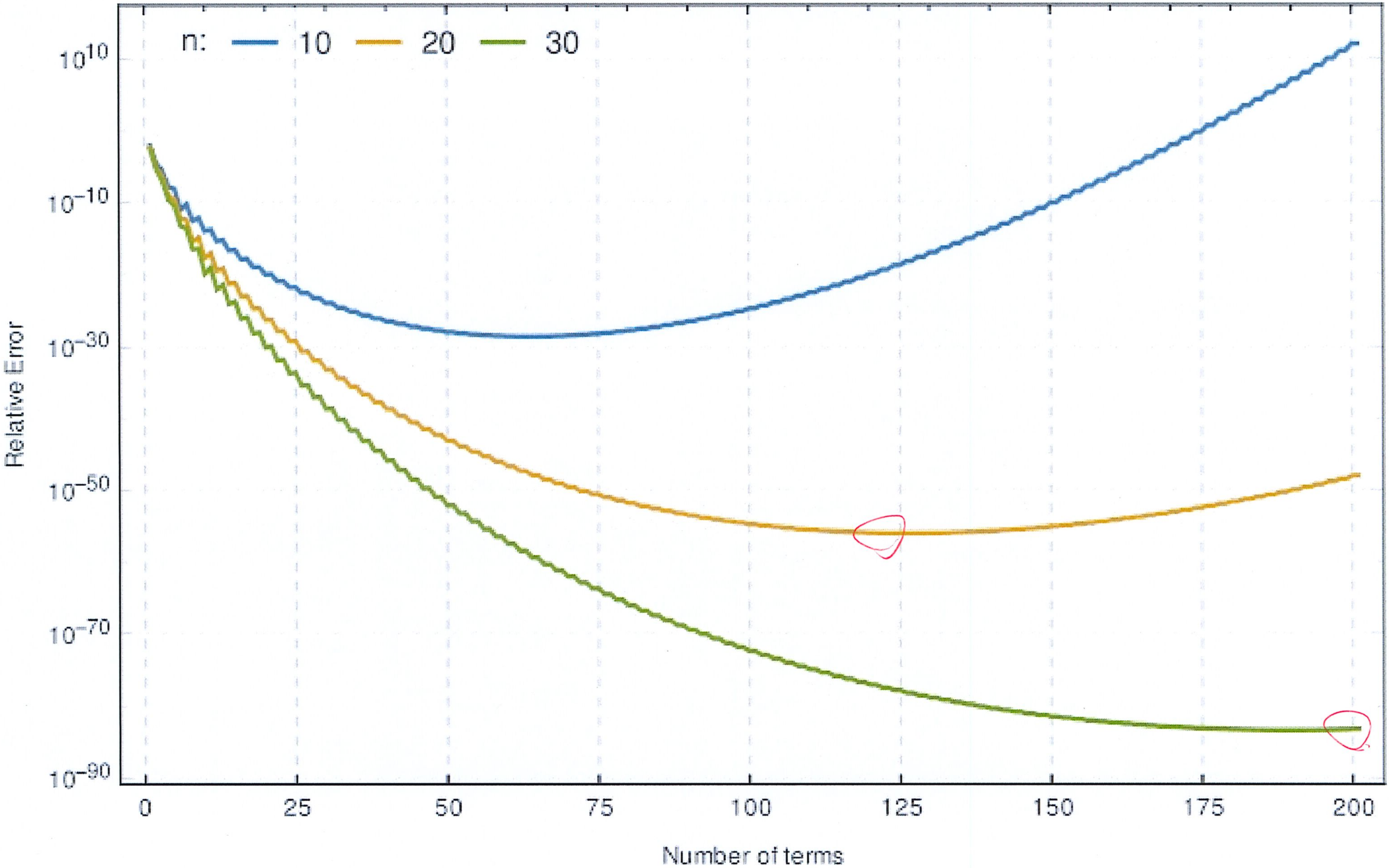
$$S_C = S_D(2N, 2V, T) = 3N + 2N \log\left(\frac{2V}{\lambda_{th}^3}\right)$$

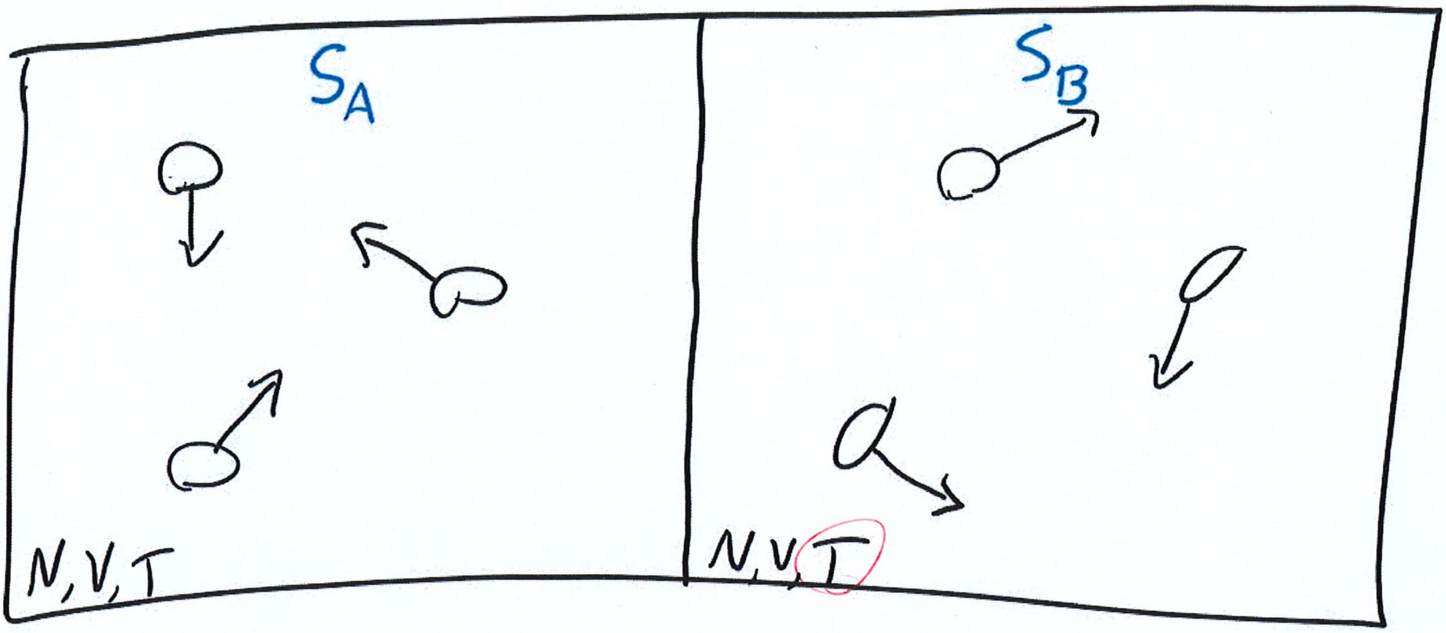
$$\Delta S_{mix} \equiv S_C - S_0 = 2N \log 2 > 0$$

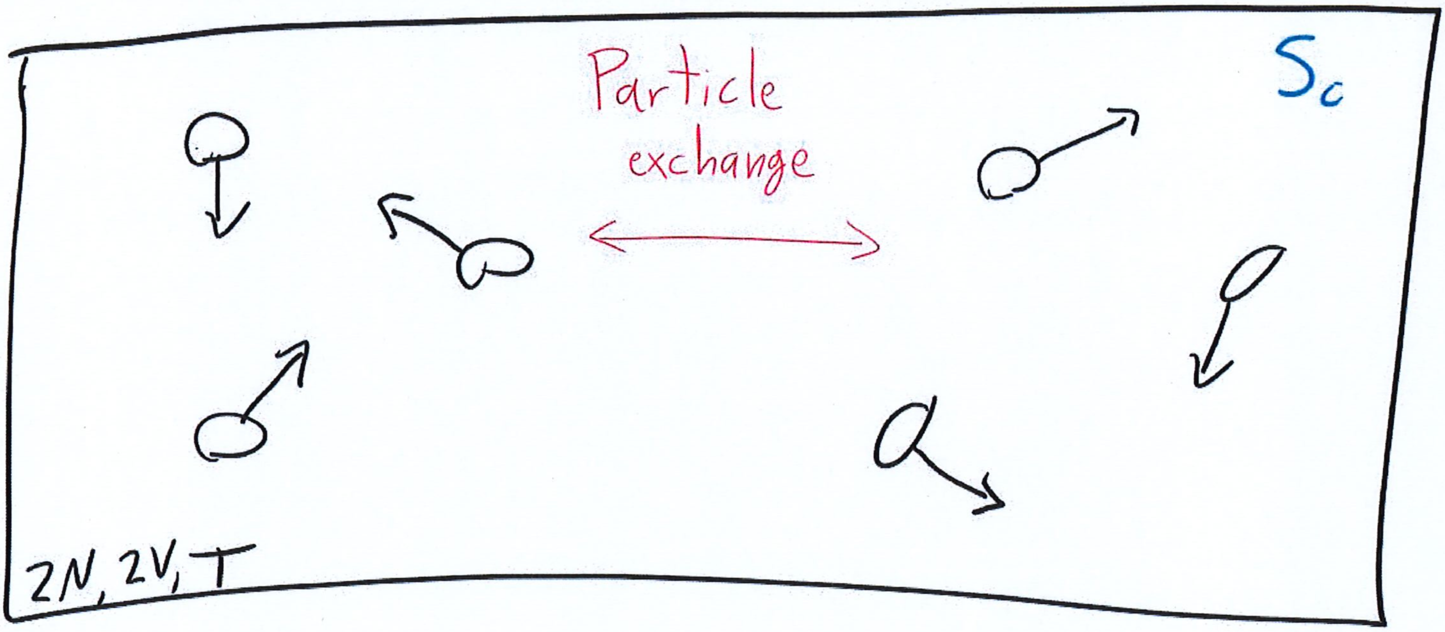
$N = 100$

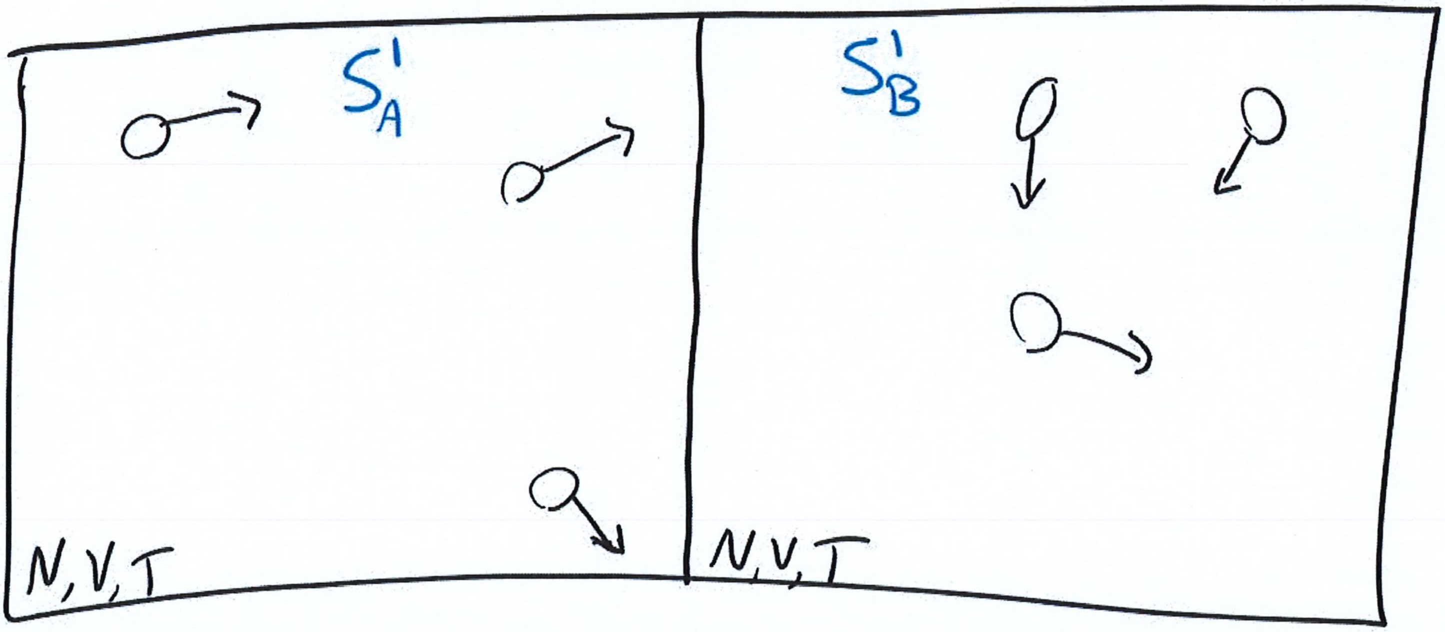


commons.wikimedia.org/wiki/File:Stirling_error_vs_number_of_terms.svg




S_A
S_B
N, V, T
N, V, T


Particle exchange
S_c
2N, 2V, T


S'_A
S'_B
N, V, T
N, V, T