Thu 29 Feb

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Thermal contact → total $M = \sum_{e_1} M^{(1)}_{e_1} M^{(2)}_{E-e_1}$

Say $N_{terms} \geq 1$ in sum, largest called "max"

Simple bound $\quad max \leq M \leq N_{terms} \cdot max$

Micro-canonical: $\quad \log(max) \leq S \leq \log(N_{terms} \cdot max)$

Illustrative example $N \sim 10^{23}$

$N_{terms} \sim N$

$max \sim e^N$

$M = 2^N = e^{N \log 2}$

$N! \sim \left(\frac{N}{e}\right)^N \sim N^N$

$$N \lesssim S \lesssim N + \log N$$

$$10^{23} \lesssim S \lesssim 10^{23} + 50$$

Concrete example

$H = 1 \quad N_1 = N_2 = 10 \qquad E = e_1 + e_2 = -10$

$e_1 = -(2n_+^{(1)} - N_1) = -10, -8, -6, \cdots 10$

e_1	$E-e_1$	$n_+^{(1)}$	$n_+^{(2)}$	$M_1 M_2$
-10	0	10	5	$\binom{10}{0}\cdot\binom{10}{5} = 1 \quad \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2} = 4\cdot 63 = 252$
-8	-2	9	6	2100
-6	-4	8	7	$\binom{10}{8}\cdot\binom{10}{7} = \frac{10\cdot 9}{2} \times \frac{10\cdot 9\cdot 8}{3\cdot 2} = 45\cdot 120 = 5400 = max$
-4	-6	7	8	
-2	-8	6	9	2100
0	-10	5	10	252

$max = 5400$

$N_{term} = 6$

$\log(5400) \leq \log(15,504) \leq \log(6 \cdot 5400)$

$8.59 \lesssim 9.65 \lesssim 10.39$

$\log(max)$ gives 89% of entropy

Stirling's Formula

$$\log(N!) \approx N\log N - N \quad \text{for} \quad N \gg 1$$

$$N! \approx \exp(N\log N - N) = N^N e^{-N} = \left(\frac{N}{e}\right)^N$$

More precise:

$$N! = \sqrt{2\pi N}\left(\frac{N}{e}\right)^N \left(1 + \frac{A}{N} + \frac{B}{N^2} + \frac{C}{N^3} + \dots\right)$$

(asymptotic)

1) Simple bounds

$$N\log N - N < \log(N!) < N\log N$$

2) Find $N! \approx \sqrt{2\pi N}\left(\frac{N}{e}\right)^N$

by showing $N! = \int_0^\infty x^N e^{-x}\, dx$ approximate by gaussian

3) Compute $A, B, \dots$

by compare $N!$ and $(N+1)! = (N+1)N!$

commons.wikimedia.org/wiki/File:Stirling_error_vs_number_of_terms.svg