Thu 1 Feb 56 38 68

Recap

Emergence From many particles

Experiment $\mathcal{E}$ → states $\Omega = \{w\}$

Measurement X → outcomes $A = \{X(w)\}$

Examples

$\mathcal{E}$: rolling a die X: Measures number on top

page 10

$A = \{1, 2, 3, 4, 5, 6\}$ State w could also include position, time, temperature ...

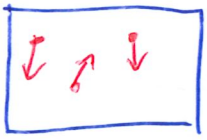
$\mathcal{E}$: Four coin flip X: Measure H or T each time

page 10

$A = \{HHHH, TTTT, HHHT, \ldots\}$

# of elements? $2^4 = 16$, all distinct

$\mathcal{E}$: 10^{23} argon atoms in box



page 10

State could include 10^{23} positions velocities, electronic states, ionization isotopes

Measure?

Pressure, temperature, energy, heat capacity, currents

Define event as any subset of outcome space A

$A = \{1, 2, 3, 4, 5, 6\}$ Possible events: Rolling 6, rolling even #

Rolling 1-5

Define $\mathcal{F}$ event space as set of all events of interest

Finally probability is measure function $P: \mathcal{F} \to [0,1]$
Number for each event in $\mathcal{F}$

Requirements:

1) $P(x \text{ or } y \text{ or } z) = P(x) + P(y) + P(z)$
   (countable, mutually exclusive)

2) $P(\mathcal{F}=A) = 1$ (must have some measurable outcome)

Putting it all together: probability space $(A, \mathcal{F}, P)$
prob. for each subset of outcomes

Roulette game

Events $\mathcal{F} = \{\text{red}, \text{black}, \text{green}\}$ (mutually exclusive)

$P_{red} = \frac{18}{37}$ $P_{black} = \frac{18}{37}$ $P_{green} = \frac{1}{37}$

page 13

Place £5 bet on black

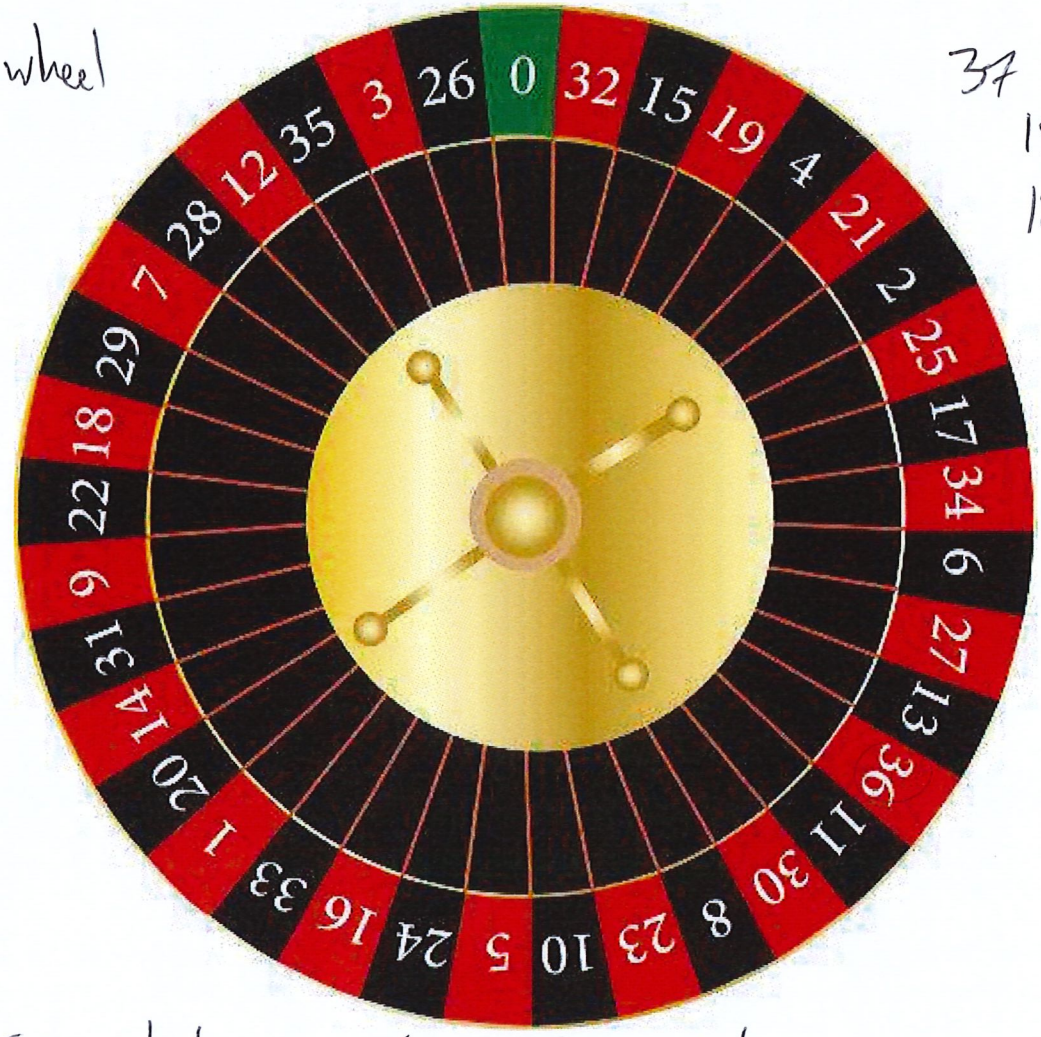
Win: get back £10 $P_{win} = \ldots$

Lose: get back £0 $P_{lose} = \ldots$

Gain: Money back minus money spent (can be negative)
(G)

N spins, W wins, $G_W = \ldots$

Roulette wheel



37 pockets
18 red
18 black
1 green

$\mathcal{E}$: Spin wheel X: Measure pocket

$A = \{0, 1, 2, \ldots 36\}$ w/ corresponding colour

Events: $\mathcal{F} = A$ $P(A) = P(0 \text{ or } 1 \text{ or } 2 \cdots \text{ or } 36) = 1$

$= P(0) + P(1) + \cdots + P(36) = \sum_{i=0}^{36} p_i = 37p$

"Fair" wheel: $p_0 = p_1 = p_2 = \cdots = p_{36} = p$ $p = 1/37$