Thu 20 Apr 669067

Plan

Photon polarization

Dark matter in CMB

Einstein solid & Debye solid

---

Heat capacity $c_V = \frac{\partial E}{\partial T}\Big|_V$

Ideal gas $\langle E \rangle = \frac{3}{2} N T \rightarrow c_V = \frac{3}{2} N$ ✗

Dist'able spins $c_V = \frac{N \beta^2 H^2}{\cosh^2(\beta H)} > 0$

Low-T $\beta \to \infty$ $\cosh^2(\beta H) \simeq e^{2\beta H} \to \infty$ $c_V \to 0$ ✓ (third law)

High-T $\beta \to 0$ $\cosh^2(\beta H) \to 1$ $c_V \to 0$ ✗

---

Einstein solid

Fix atoms in solid by interacting w/ neighbours

↳ energy in oscillators rather than atoms

Quantized energies for oscillators $\varepsilon_i = 0, \hbar\omega, 2\hbar\omega, \ldots$

Micro-canonical: Fix $N$ oscillators sharing $K$ units of energy

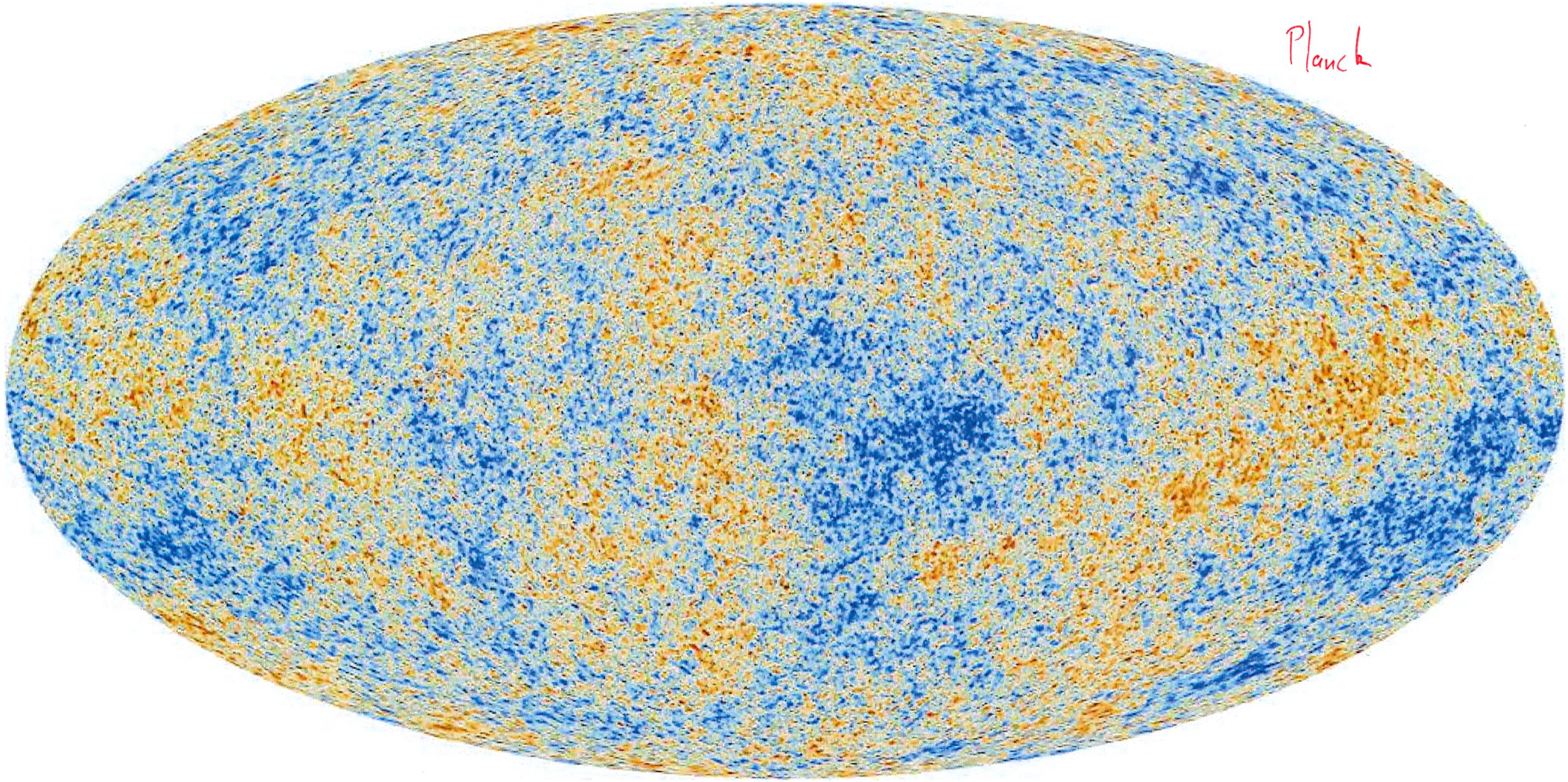
1) $\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{\partial}{\partial E} \log M$

2) $E(T)$

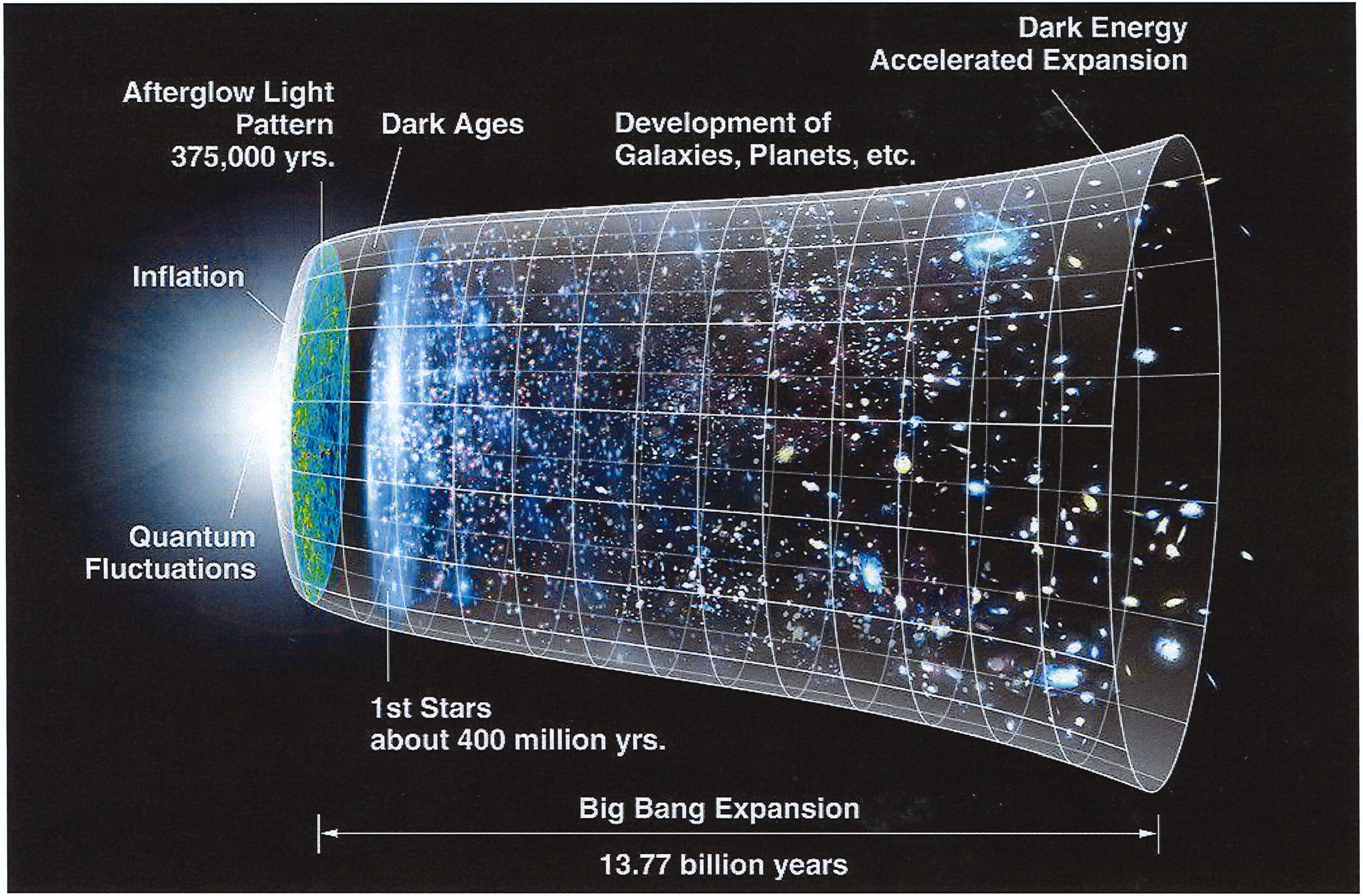
3) $c_V = \frac{\partial E}{\partial T}$

$K = \sum_{i=1}^{N} k_i$

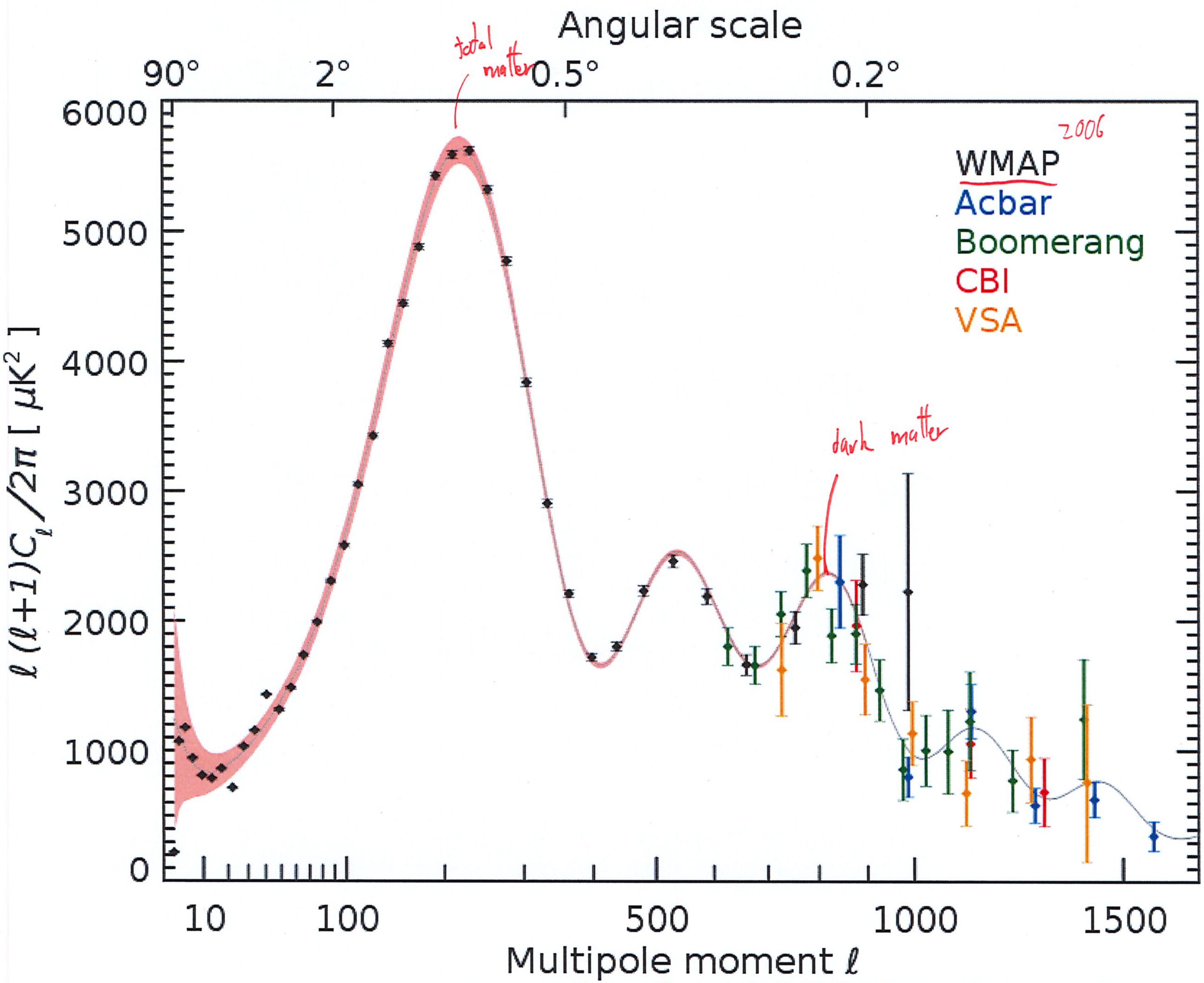
$E = \sum_i k_i \hbar\omega$



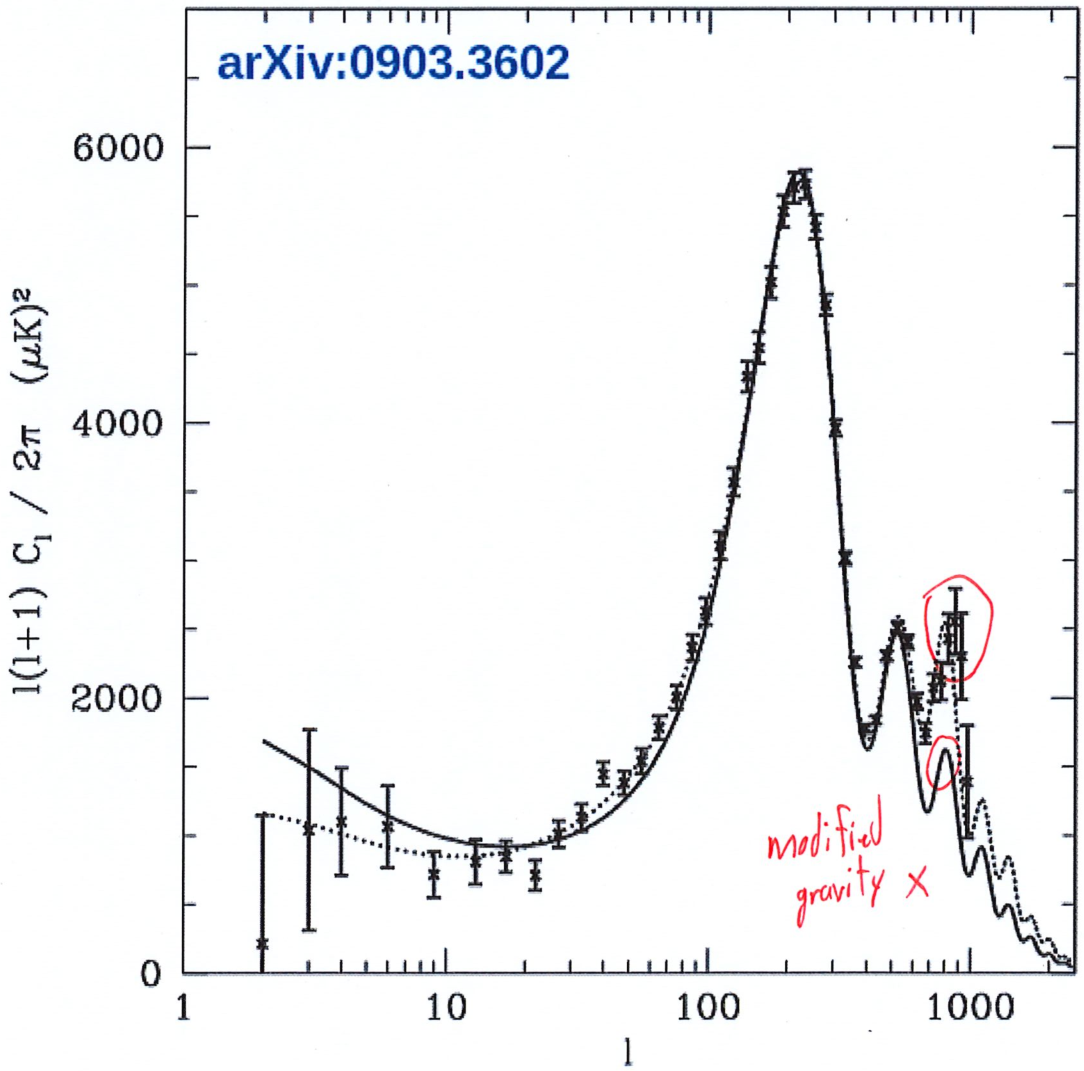
Planck



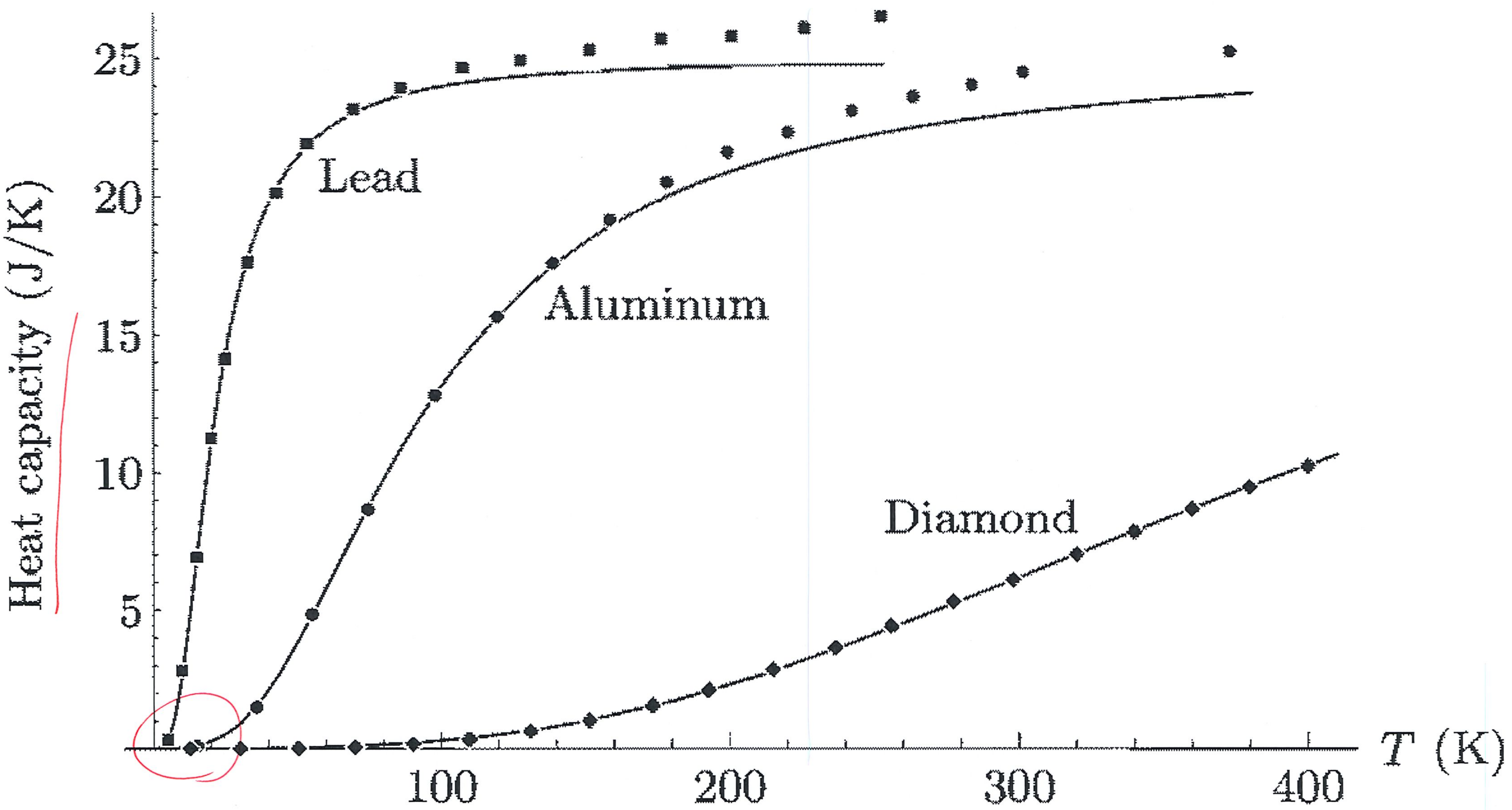

commons.wikimedia.org/wiki/File:CMB_Timeline300_no_WMAP.jpg



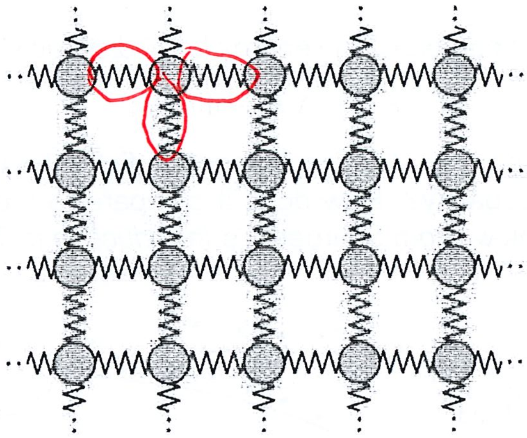

commons.wikimedia.org/wiki/File:PowerSpectrumExt.svg



arXiv:0903.3602
6000
4000
2000
0
1
10
100
1000
l
l(l+1) C_l / 2π (μK)²
modified gravity ✗


Heat capacity (J/K)
25
20
15
10
5
Lead
Aluminum
Diamond
100
200
300
400
T (K)

their layout. In this two-dimensional square lattice, $N$ oscillators would correspond to $N/2$ atoms in the solid. In a three-dimensional cubic lattice, $N$ oscillators would correspond to $N/3$ atoms.



Our goal is to compute the heat capacity for an Einstein solid. Let's begin by working in terms of the micro-canonical ensemble, fixing the total energy

$$E = \sum_i \varepsilon_i = \sum_i k_i \hbar\omega \equiv K\hbar\omega$$

where $K \equiv \sum_i k_i$ is the integer number of energy 'units' available to be distributed among the $N$ oscillators. Each different way of distributing these $K$ units of energy among the $N$ (distinguishable) oscillators defines a unique micro-state.

M

- What is the total number of micro-states in terms of $N$ and $K$? Check your result for a minimal three-oscillator system when it has $K = 0, 1, 2$ or $3$ units of energy.
- Now consider $K \gg 1$ and $N \gg 1$, so that we can apply Stirling's formula and also approximate $N-1 \approx N$ and $K-1 \approx K$. What is the corresponding entropy?
- What is the temperature of the Einstein solid? Is it a 'natural' system with $T > 0$?
- To find the heat capacity, we need to find the energy in terms of the temperature, then differentiate. What is the resulting heat capacity of the Einstein solid, in terms of $x \equiv \hbar\omega/T$? How does it compare to the experimental data points we began by considering?

While the Einstein solid provides a significantly better description of the experimental data, room for improvement is still possible...

$N=3$

$$\left.\begin{array}{ll} K=0 & M=1 \\ K=1 & M=3 \\ K=2 & M=3+3=6 \\ K=3 & M=3+6+1=10 \end{array}\right\} \quad M=\frac{(K+N-1)!}{K!\,(N-1)!}=\binom{K+N-1}{K}$$