MATH327: Statistical Physics

Monday, 30 January 2023

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Something to consider

The existence of atoms was established long before technology could 'see' individual atoms. 1926 Nobel

What sort of debates might this have involved?

Are there similarities with ongoing debates about string theory?

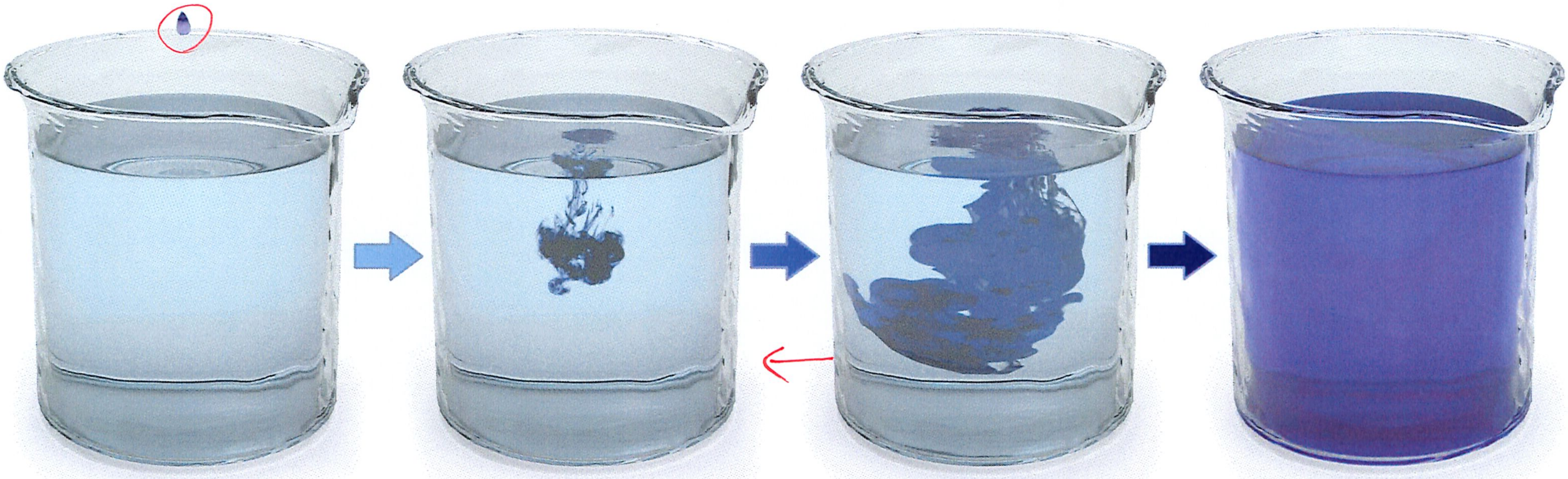
Plan

Big picture

Logistics

Probability Foundations

Arrow of time



commons.wikimedia.org/wiki/File:Blausen_0315_Diffusion.png

THE NOBEL PRIZE IN PHYSICS 2021

Illustrations: Niklas Elmehed

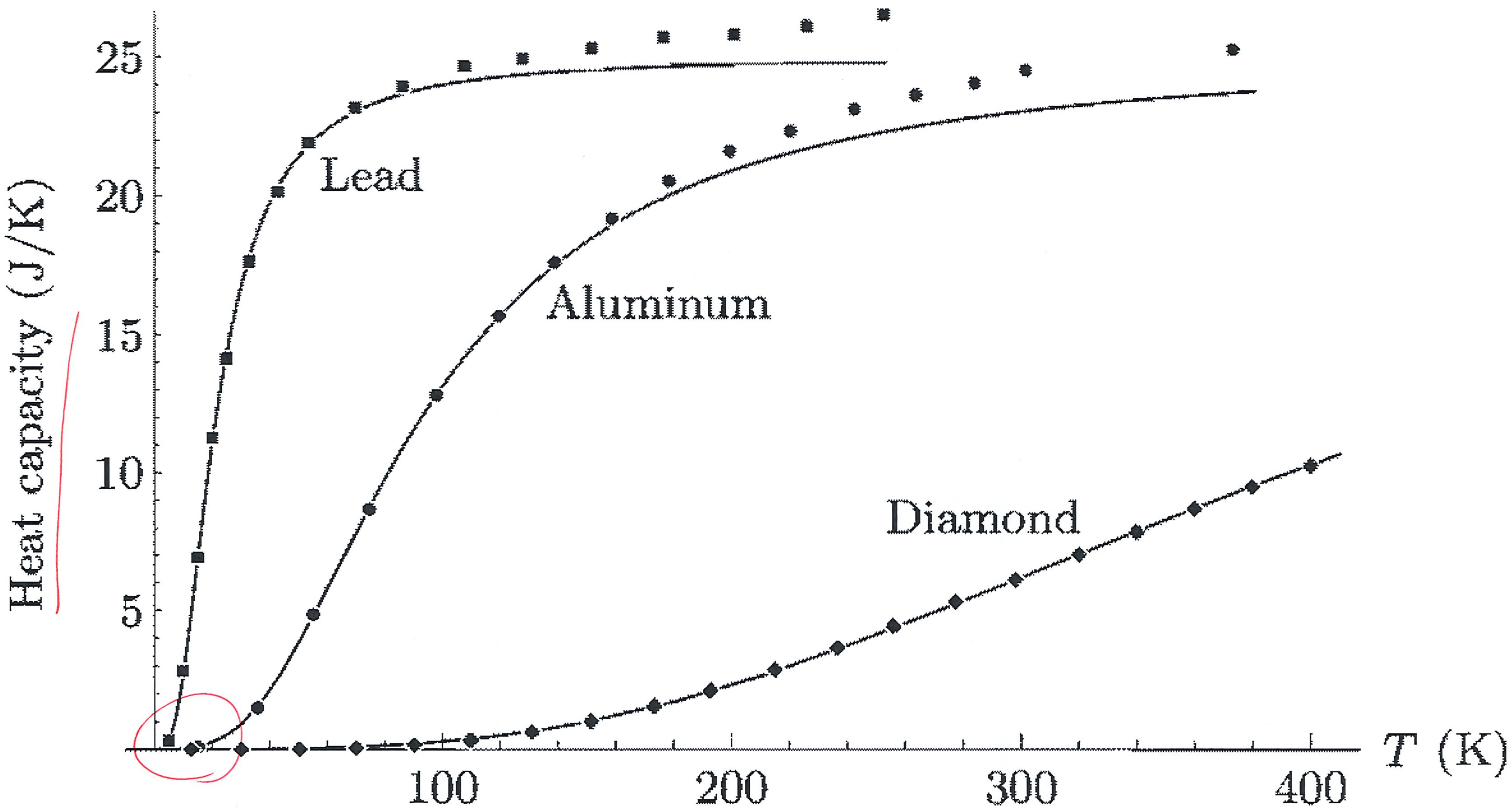
Syukuro Manabe **Klaus Hasselmann**

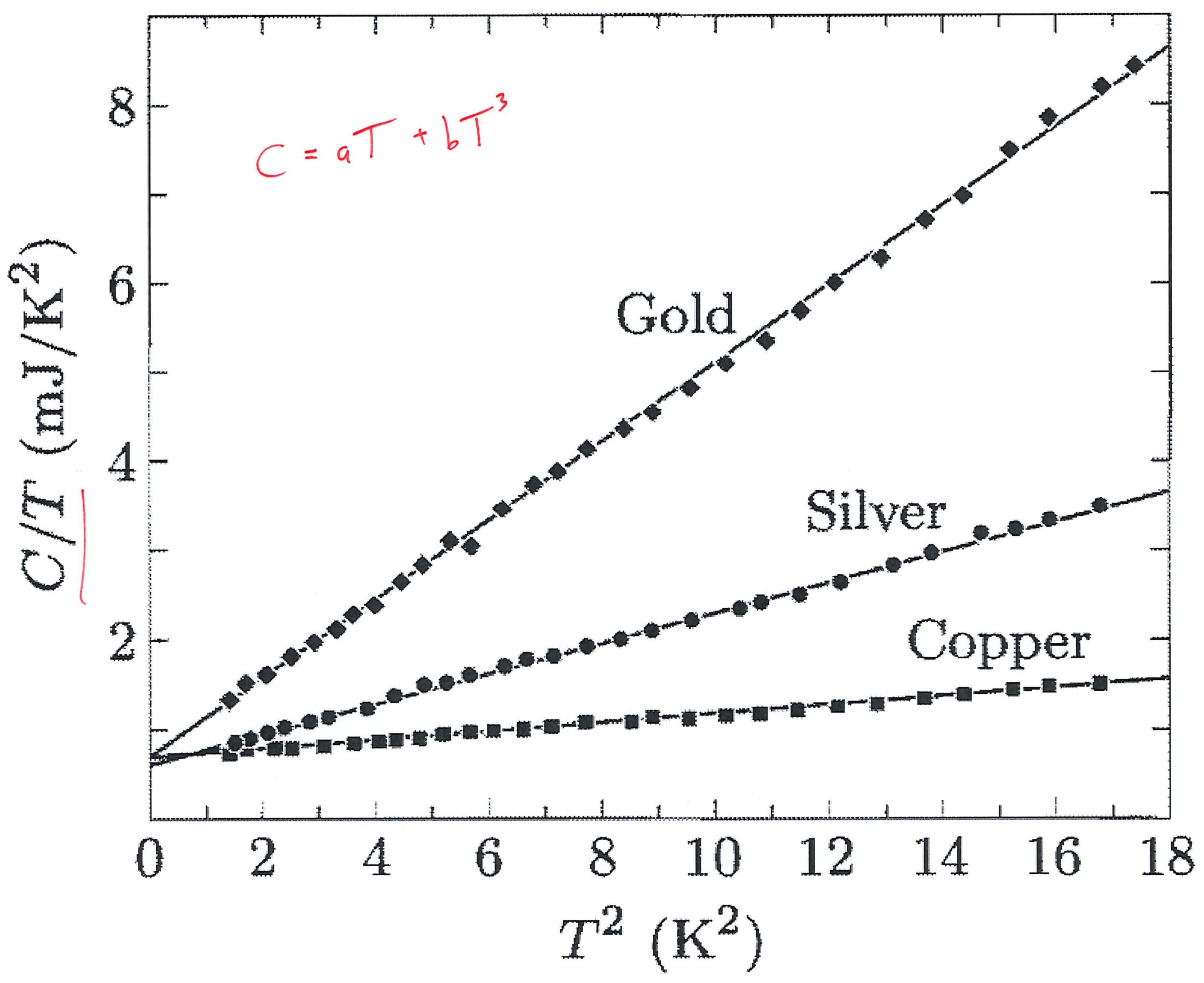
"for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming"

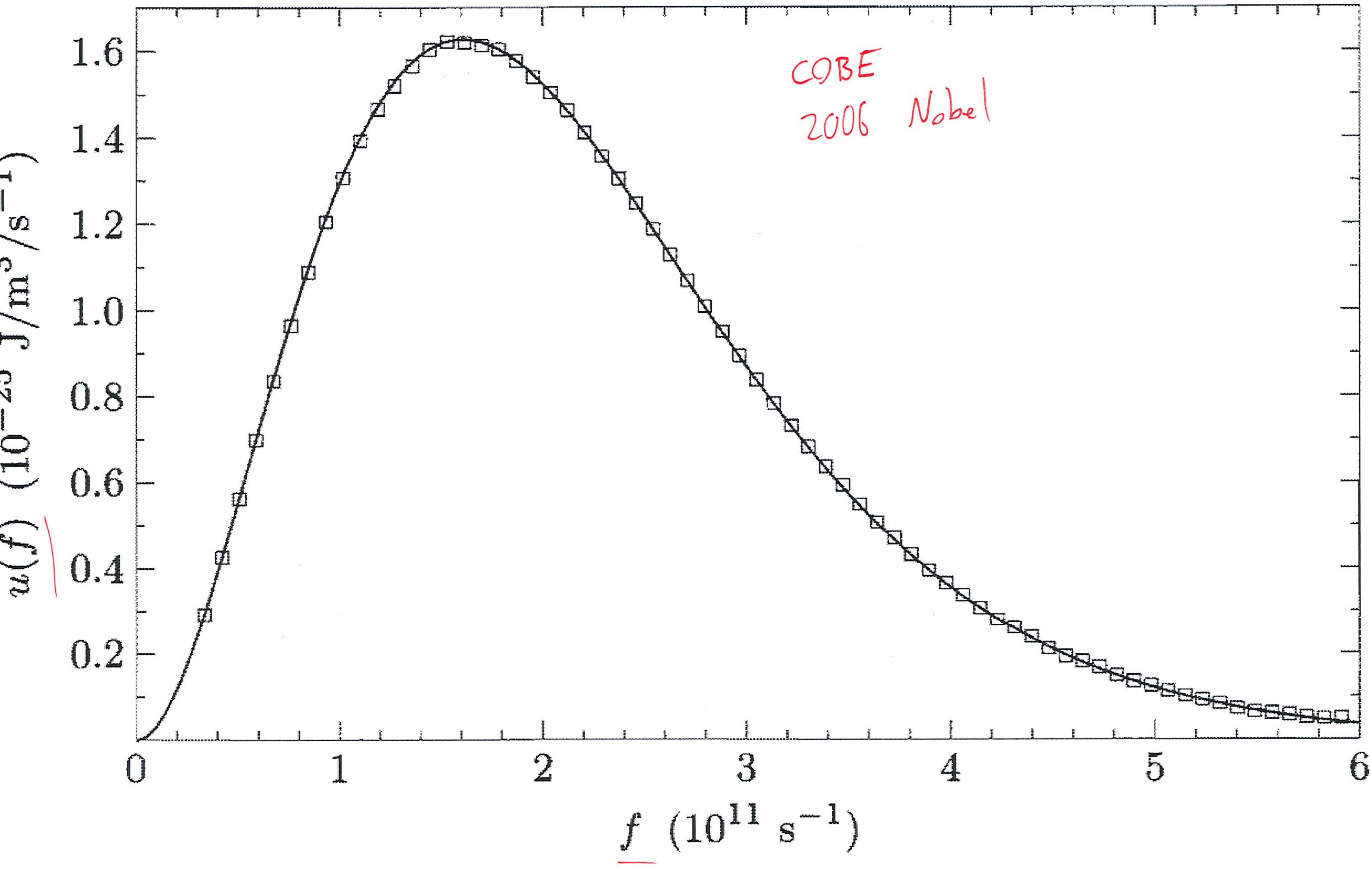
Giorgio Parisi

"for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales"

THE ROYAL SWEDISH ACADEMY OF SCIENCES


Heat capacity (J/K)
25
20
15
10
5
Lead
Aluminum
Diamond
100
200
300
400
T (K)


C = aT + bT^3
Gold
Silver
Copper
C/T (mJ/K^2)
T^2 (K^2)
0
2
4
6
8
10
12
14
16
18


$u(f)$ $(10^{-25}$ J/m^3/s$^{-1})$
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
f $(10^{11}$ s$^{-1})$
COBE
2006 Nobel

Big picture

Stat Phys + Quantum + Relativity

$\downarrow$

LARGE #s of 'particles'

1 cubic centimetre water $\sim 10^{22}$ H_2O molecules

$\sim 10^{22}$ coupled equations

$3 \times \{\text{position, velocity}\} \times 4B \rightarrow 10^{23}$ bytes of info

10^{11} TB

hundred billion!

Probability Foundations

"Emergence" from law of large numbers
central limit theorem

Random experiment $\mathcal{E}$ observe world

→ state w (omega)

(heads/tails + more)

$\Omega = \{w\}$ set of all states for $\mathcal{E}$

Measurement $X(w)$ extract info of interest (heads/tails)

Repeat $\mathcal{E}$ → random variable $X(w_i)$

Measure all states → all outcomes $X : \Omega \to A$

outcome space

finite, countable, continuous

Examples

Roll a dice, measure number on top

$\mathcal{E}$ X

$A = \{1, 2, 3, 4, 5, 6\}$

state w could also include position, orientation, time, temperature ...

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~~Roll~~ Flip a coin four times, measuring H vs. T each time
(single $\mathcal{E}$) (X)

$A = \{$ HHHH
HHHT
HHTH $\ldots \}$

$\#A = 2^4 = 16$

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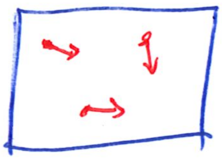
$\sim 10^{23}$ argon atom

$\mathcal{E}$: observe



state could include $\sim 10^{23}$ positions
velocities
electronic states
spin orientations

Measure: Temperature, pressure
energy, heat capacity
currents

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Can label each state with unique $L(\omega) \longleftrightarrow \omega$

Generalizes non-unique measurement $X(\omega)$

Only care about what we measure $\rightarrow A$, not Full Ω

Event is any subset of outcome space A

$A = \{1, 2, 3, 4, 5, 6\}$

Event: rolling 6 rolling even

rolling 1-5

Event space $\mathcal{F}$ is set of all events of interest

<u>Probability</u> is measure function $P : \mathcal{F} \to [0, 1]$

Number for each event in $\mathcal{F}$

Requirements: $P(x \text{ or } y \text{ or } z) = P(x) + P(y) + P(z)$

'mutually exclusive

For any countable union

$$P(\mathcal{F} = A) = 1$$

must have some measureable outcome

Putting it all together $\rightarrow$ <u>probability space</u> $(A, \mathcal{F}, P)$

prob. for each subset of outcome

<u>Example</u>

Finite $\Omega = \{w_1, w_2, \dots w_N\}$ $\quad \#\Omega = N$

Measurement X can give same outcome for different states

$$X(w_i) = X(w_j) \qquad w_i \neq w_j$$

Fewer outcomes $A = \{X_1, X_2, \dots X_n\}$ $\quad \#A = n \leq N$

all distinct

Choose $\mathcal{F} = A = \{X_1, X_2, \dots X_n\}$

$$P(X_i \text{ or } X_j) = P(X_i) + P(X_j) = P_i + P_j \qquad i \neq j$$

$$P(A) = P(X_1 \text{ or } X_2 \text{ or} \dots X_n)$$

$$= \sum_{i=1}^{n} P_i = 1$$

"Fair" die $\quad A = \{1, 2, 3, 4, 5, 6\}$

$\hookrightarrow \; p_1 = p_2 = \ldots = p_6 = p$

$$\sum_{i=1}^{6} p = 6p = 1 \quad \rightarrow \quad p = 1/6$$

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Four fair coin flip $\quad p_1 = p_2 = \ldots = p_{16} = p = \underline{1/16}$

$$p(\mathcal{F} = A) = \sum_{i=1}^{16} p = 16p = 1$$

$\mathcal{F} = \{$ equal H/T, different H/T $\}$

$\{HHTT, \; HTHT, \; HTTH, \; TTHH, \; THTH, \; THHT\}$

$$p_{equal} = \frac{6}{16} = \frac{3}{8}$$

$$p_{diff} = \frac{10}{16} = \frac{5}{8}$$

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Assigning probabilities to events $\rightarrow$ <u>modelling</u>

Can be fixed by symmetries

(6 sides of fair die)

Data driven modelling more general

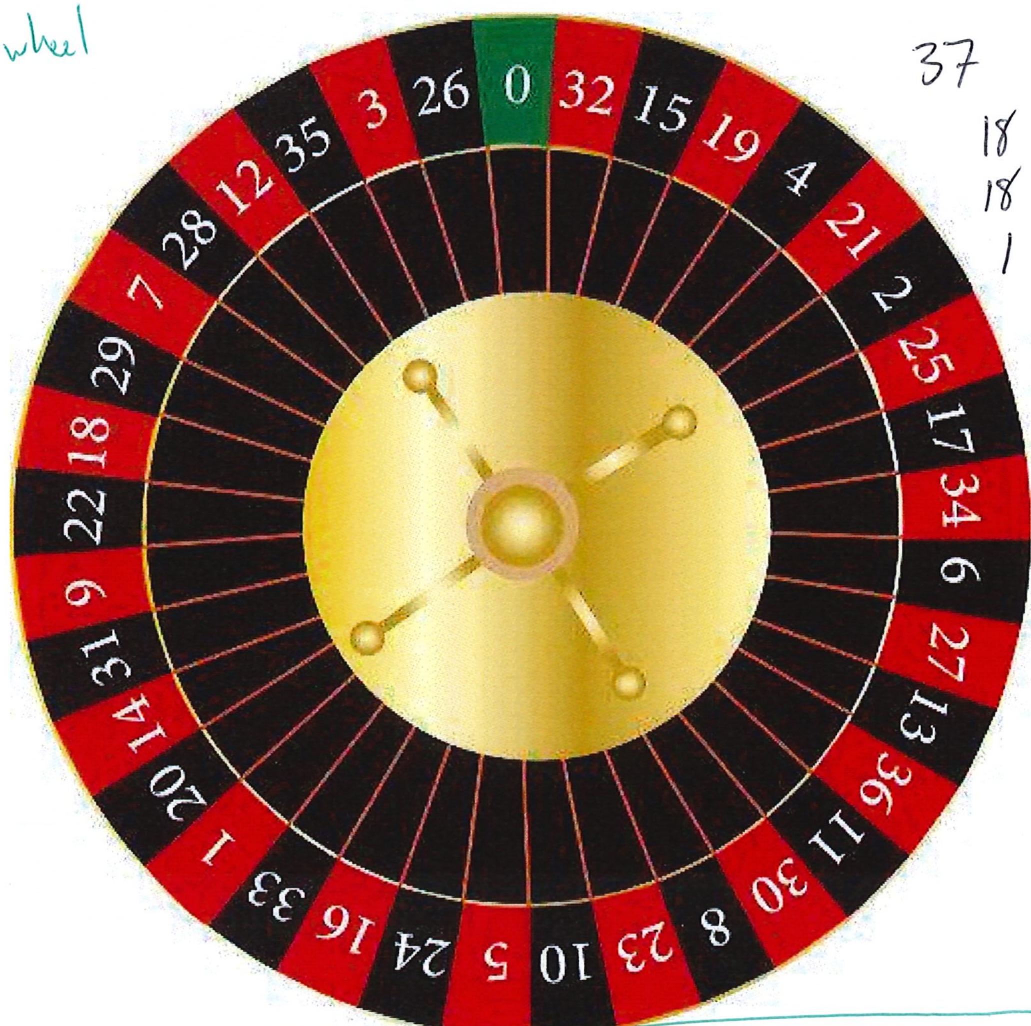
Repeat experiment many times

Monitor outcomes X_0

Infer probabilities p_i relying on

<u>law of large numbers</u>

Roulette wheel



37 pockets
18 red
18 black
1 green

Spin wheel, measure pocket $\rightarrow$ $A = \{0, 1, 2, \ldots 36\}$

Fair wheel $\rightarrow$ $\sum_{i=0}^{36} p = 1$ $\quad p = 1/37$ for each pocket

$\mathcal{F} = \{\text{red, black, green}\}$ (mutually exclusive)

$P_{red} = \frac{18}{37}$ $\quad P_{black} = \frac{18}{37}$ $\quad P_{green} = 1/37$

Consider $\mathcal{F} = A = \{X_1, X_2, \dots X_n\}$

$$P(X_i) = P_i \in [0,1] \qquad \sum_{i=1}^{n} P_i = 1$$

Or say $\sum_{X \in A} P(X) = 1$

Mean of prob. space $\quad \mu = \langle X \rangle = \sum_{X \in A} X \, P(X)$

Variance $\quad \sigma^2 = \langle (X-\mu)^2 \rangle = \sum_{X \in A} (X-\mu)^2 \, P(X)$

General expectation value $\quad \langle f(X) \rangle = \sum_{X \in A} f(X) \, P(X)$

linear operation

Standard deviation

$$\sigma = \sqrt{\sigma^2}$$