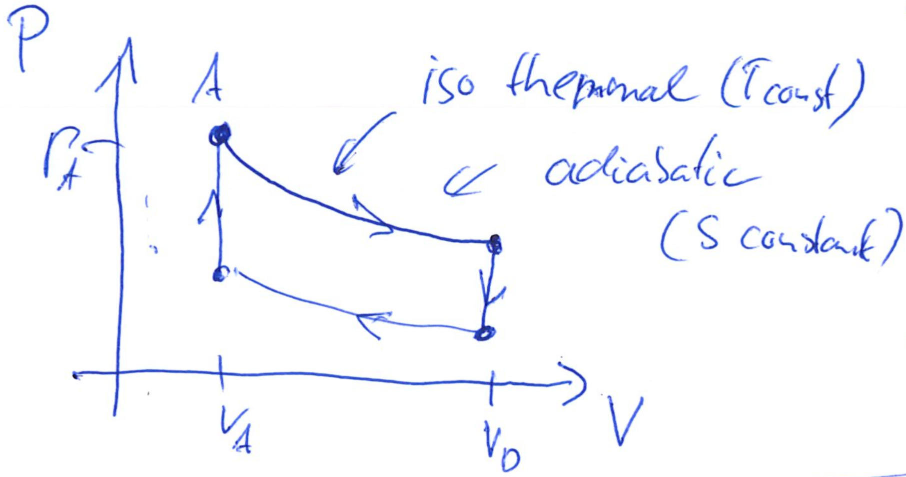
Recap lecture 28/2

Thermodynamical Cycles:



- equ. of state:

$$\boxed{p \cdot V = N \times T}$$

- change in internal energy

$$\boxed{dE_i = dQ - p\,dV}$$

heat: $dQ = T\,dS$

- adiabatic:
  entropy is constant

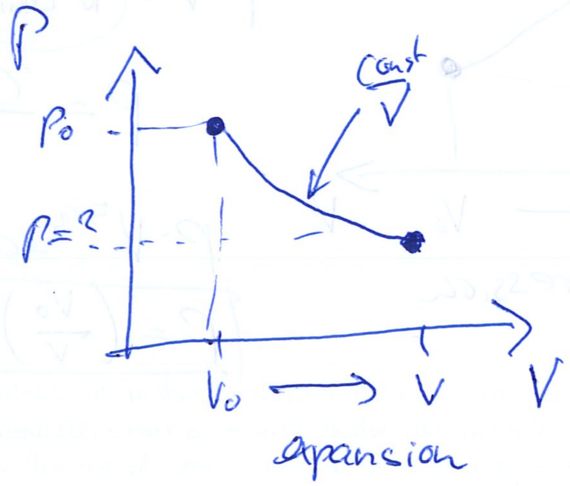
  ideal gas: $\boxed{V T^{3/2} = \text{const}}$

  $\Rightarrow S = \text{const.}$

A change of the state of the gas by a change in external parameters hence can be describes as a line in the p-V-diagramme.

WORKED EXAMPLES

Visualise an isothermal (constant T) expansion and calculate the change in pressure.



$$p \cdot V = N \cdot T = \text{const.}$$

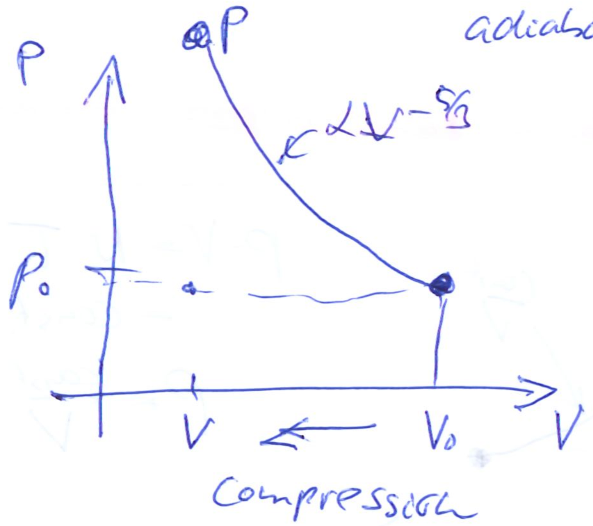
$$p = \frac{\text{const.}}{V}$$

(true for any point on the pV-diag)

$$p \cdot V = \overbrace{NT}^{\text{const}} = p_0 V_0$$

$$\boxed{p = \frac{V_0}{V} \cdot p_0}$$

Visualise an adiabatic (constant S) compression and calculate the changes in pressure and temperature.



adiabatic → $V T^{3/2} = \text{Const}$

$p \cdot V = N T$

$p \cdot V = (N \cdot \text{const.}) V^{-\frac{2}{3}}$

$$p = \frac{\text{Constant}}{V^{5/3}}$$

$p \cdot V^{5/3} = \text{constant} = p_0 V_0^{5/3}$

$$\boxed{p = \left(\frac{V_0}{V}\right)^{5/3} \cdot p_0}$$

After a sequence of these changes, we could find ourselves *back at the starting point* in the p-V-diagramme. We call the whole process a thermodynamic cycle, since we could repeat the process over an over again. As we will see below, thermodynamical cycle can take heat from a hot reservoirs to a cold reservoir and convert part of it into work. This is e.g. used in car engines, refrigerators or heat pumps.

5.5.3 The Carnot cycle

The proto-type of a thermodynamic cycle was proposed by the French physicist Sadi Carnot in 1824, and is nowadays know as the Carnot process. Our case container has access to two different heat reservoirs, one 'hot' reservoir with temperature T_H and one 'cold' with T_L.

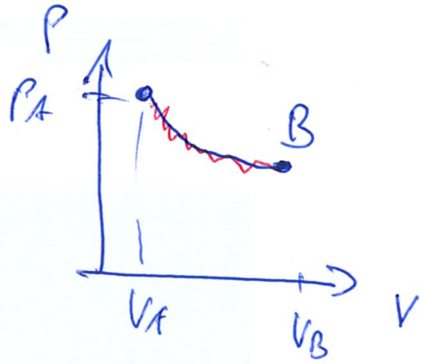
e.o.s: $pV = N.T \rightarrow p = \frac{(NT)}{V}$

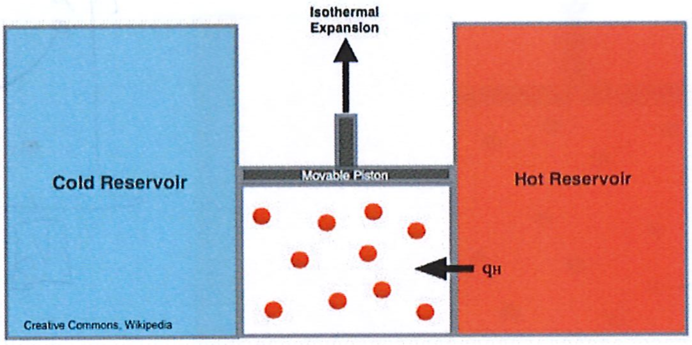
- Heated ideal gas particle

q_H - heat from hot reservoir

Stage One:

At this stage heat is released from the hot reservoir and is absorbed by the ideal gas particles within the system. Thus, the temperature of the system rises. The high temperature causes the gas particles to expand; pushing the piston upwards and doing work on the surroundings.

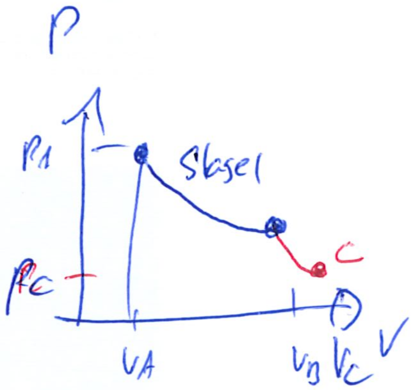


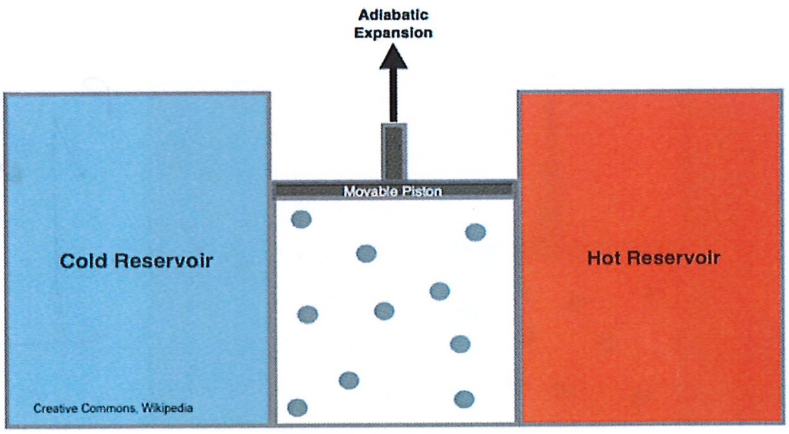


- Ideal gas particle that is cooled.

Stage Two:

At this stage expansion continues, however there is no heat exchange between system and surroundings. Thus, the system is undergoing adiabatic expansion. The expansion allows the ideal gas particles to cool, decreasing the temperature of the system.



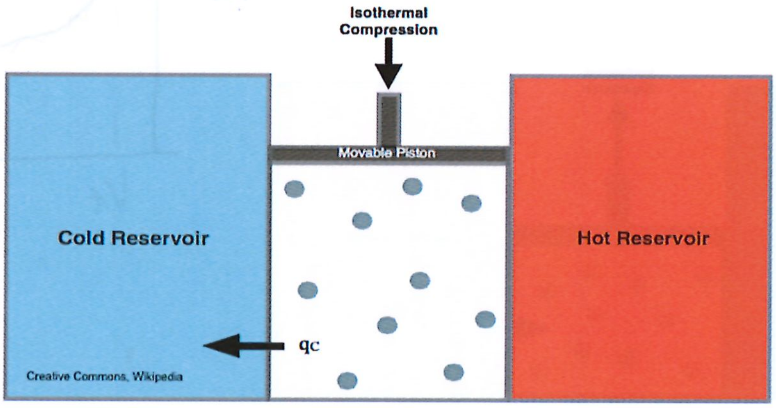


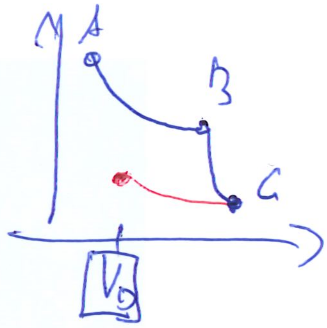
qc - heat released from system to cold reservoir

- Ideal gas particle that is cooled.

Stage Three:

At this stage the surroundings do work on the system which causes heat to be released (q_C). The temperature within the system remains the same. Thus, isothermal compression occurs.

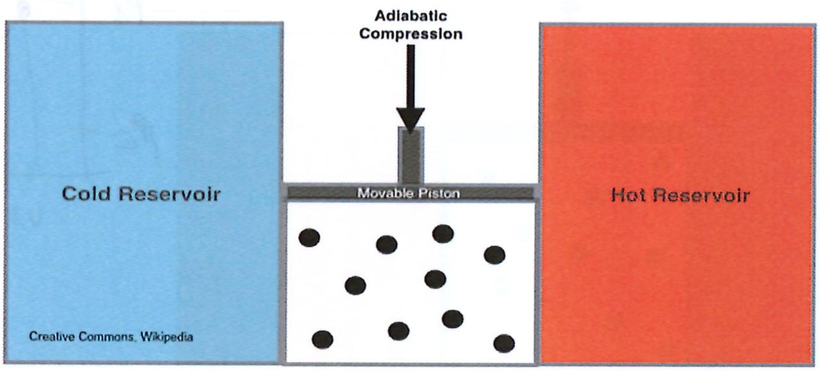


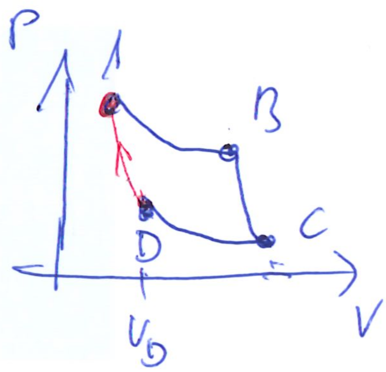


- Ideal gas particle at normal temperature prior to Carnot Cycle commencement.

Stage Four:

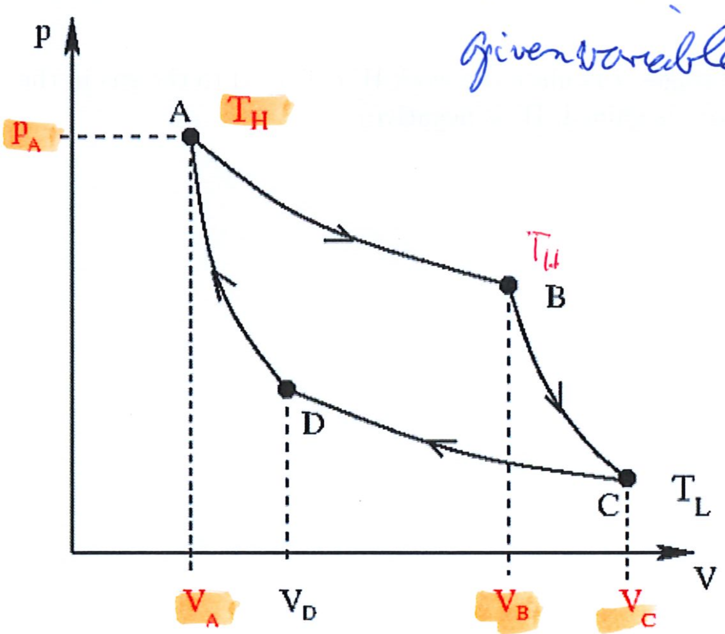
No heat exchange occurs at this stage, however, the surroundings continue to do work on the system. Adiabatic compression occurs which raises the temperature of the system as well as the location of the piston back to its original state (prior to stage one).





The figure below shows the Carnot Cycle in the p-V-diagramme.

given variables are in "red"



expand to V_B expand to V_c

WORKED DETAILS:

Consider p_A, T_H and the volumes V_A, V_B and V_c as given. Assigning input to variables is quite tedious, since picking the wrong combination of variables could easily lead to inconsistencies. The above choice is inspired bya real experiment: we start at state A at normal condition with p_A the atmospheric pressure, T_H the temperature of the surrounding and V_A our volumen initially. We then slowly expand (isothermal) until we reach V_B. We then rapidly expand (adiabatic) until we reach V_C. The temperature and pressure at B and C is what we could measure, and, hence, we should be able to calculate those. We then slowly compress (isothermal) until we reach V_D and then rapidly back to V_A. We cannot just give a value for V_D, since if we pick this value wrong, we would not getting back to V_A with the final compression. This is telling us that we need to calculate V_D in order to complete the cycle.

- Calculate at each of the points A,B,C,D whatever is missing from: volume, pressure, temperature.

- For each of the stages, calculate the heat transfer Q in or out the gas container.
- For each of the stages, calculate the work W delivered to the gas in the container (if work is gained, W is negative).

Stage 1: from $A \to B$ (T constant)

point B: $\boxed{V_B}$ $P_B = ?$ $T_B = ?$

isothermal: $A \to B$: T is constant

$$\boxed{T_B = T_A}$$

equ. of state:

$$P_A V_A = \underbrace{N T_H}_{\text{constant}} = P_B \cdot V_B \Rightarrow \boxed{P_B = \frac{V_A}{V_B} P_A}$$

Stage 2: from $B \to C$ adiabatic

$V T^{3/2} = \text{const.}$ $\boxed{V_C}$

$$V_B T_H^{3/2} = V_C T_L^{3/2} \Rightarrow \boxed{T_L = \left(\frac{V_B}{V_C}\right)^{2/3} \cdot T_H}$$

pressure?

e.o. state: $\frac{P_A V_A}{T_H} = N$ $P_C V_C = N \times T_L$

$$P_C V_C = \frac{P_A V_A}{T_H} \times \left(\frac{V_B}{V_C}\right)^{2/3} T_H \Rightarrow \boxed{P_C = \frac{V_A}{V_C} \left(\frac{V_B}{V_C}\right)^{2/3} P_A}$$

Stage 3+4: from $C \to D \to A$

Point D: adiabatic from D to A

$VT^{3/2} = \text{const.}$ $\quad V_A T_H^{3/2} = V_D \cdot T_L^{3/2}$

(process C to D is isothermal)

T_L: we have calculated

$$V_D = \left(\frac{T_H}{T_L}\right)^{3/2} \cdot V_A = \left[\left(\frac{V_C}{V_B}\right)^{\frac{2}{3}}\right]^{3/2} V_A$$

$$\boxed{V_D = \frac{V_C}{V_B} V_A} \qquad \boxed{T_D = T_L = \left(\frac{V_B}{V_C}\right)^{2/3} T_H}$$

pressure? e.o. state:

$$\frac{P_A V_A}{T_H} = N = \frac{P_D V_D}{T_L}$$

$$\boxed{\frac{P_D}{P_A} = \frac{T_L}{T_H}\frac{V_A}{V_D} = \left(\frac{V_B}{V_C}\right)^{2/3} \frac{V_B}{V_C} = \left(\frac{V_B}{V_C}\right)^{5/3}}$$