New strong dynamics beyond the standard model

Lecture 5 28 November 2017

Last time

- Experimentally test composite Higgs models by direct searches for new particles and indirect constraints from precision measurements
- Clear targets for direct searches are heavy vectors, extra PNGBs, and top partners
- Diboson bounds up to ~ 3 TeV and top partner bounds up to ~ 800 GeV lead to $\xi \lesssim 0.2$ (or $f \gtrsim 550$ GeV), with some caveats (e.g., narrow width approx.)
- Indirect constraints from Higgs couplings, electroweak S and T parameters, and lower-energy flavor physics (e.g., D–$\overline{D}$ mixing, kaon CP violation)
- Indirect constraints currently prefer $\xi \lesssim 0.1$ (or $f \gtrsim 800$ GeV)
- Significantly more precise Higgs coupling measurements possible in the future

Lattice gauge theory as QFT regularization

- Lattice gauge theory is a non-perturbative regularization for QFT
- Observable expectation values are formally infinite-dimensional integrals:

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [dX]\, \mathcal{O}\, e^{-S[X]} \qquad \mathcal{Z} = \int [dX]\, e^{-S[X]} \qquad \begin{pmatrix} X \text{ are fields} \\ S \text{ is action} \end{pmatrix}$$

  (Aside: Wick rotate to euclidean space-time $\longrightarrow$ SO(4) Lorentz invariance)

- Regularize by replacing continuous space-time $\longrightarrow$ grid of discrete points
  Non-zero lattice spacing 'a' $\longrightarrow$ UV cutoff $\Lambda_{UV} = 1/a$
  Finite lattice extent $L = Na \longrightarrow$ IR cutoff $\Lambda_{IR} = 1/L$
  (anticipating numerical calculations rather than perturbation theory)
- Both regulators must be removed $\longrightarrow$ **continuum limit** $a \to 0$ (with $Na \to \infty$)
- Here only consider hypercubic lattice (potentially with temporal extent $N_t > N_s$)
  Discrete hypercubic rotations $\longrightarrow$ Lorentz invariance automatic in continuum limit
- Fixed $M_{\text{phys}} = 1/\left(\xi_{\text{lat}} a\right) \Longrightarrow$ divergent correlation length $\xi_{\text{lat}} \to \infty$ as $a \to 0$
  $\Longrightarrow$ Continuum limit is second-order critical point
  (asymptotic freedom $\longrightarrow$ free gaussian fixed point with $g^2(\Lambda_{UV}) \to 0$)
- Must know fundamental lagrangian at cutoff scale a^{-1}, not just low-energy EFT

Discretization of fields and gauge action

- For free fermions in discrete space-time $\quad \overline{\psi}(x)\gamma_\mu\partial_\mu\psi(x) \longrightarrow \overline{\psi}(n)\gamma_\mu\frac{\psi(n+\widehat{\mu})-\psi(n-\widehat{\mu})}{2a}$
  (derivative $\longrightarrow$ finite-difference operator)

  (Aside: In momentum space $\longrightarrow \frac{\gamma_\mu}{2a}\left(e^{ip\cdot\widehat{\mu}} - e^{-ip\cdot\widehat{\mu}}\right) = \frac{i\gamma_\mu}{a}\sin(ap_\mu)$)

- With gauge fields A_μ, fermion moving from $n+\widehat{\mu}$ to n picks up Wilson line,
  $$\psi(x) = \mathcal{P}\exp\left[\int_{n+\widehat{\mu}}^{n} igA_\mu(y)dy_\mu\right]\psi(n+\widehat{\mu})$$
  $\Longrightarrow$ Include gauge d.o.f. as **gauge links** $U_\mu(n) = \exp\left[-igaA_\mu(n+\widehat{\mu}/2)\right]$

- $\therefore$ Gauge-covariant derivative D (or $\not{D}$) becomes
  $$\overline{\psi}(x)D\psi(x) \longrightarrow \frac{1}{2a}\overline{\psi}(n)\gamma_\mu\left[U_\mu(n)\psi(n+\widehat{\mu}) - U_\mu^\dagger(n-\widehat{\mu})\psi(n-\widehat{\mu})\right]$$
  Check gauge invariance for fermions in fundamental rep:
  $$\overline{\psi}(n)\to\overline{\psi}(n)G^\dagger(n) \qquad U_\mu(n)\to G(n)U_\mu(n)G^\dagger(n+\widehat{\mu}) \qquad \psi(n+\widehat{\mu})\to G(n+\widehat{\mu})\psi(n+\widehat{\mu})$$
  (Aside: For higher reps with generators T^A, $\quad U^{jk} \longrightarrow V^{AB} = \mathrm{Tr}\left[T_{ij}^A U^{jk} T_{ki}^B\right]$)

- Gauge-invariant pure-gauge terms must be traces over closed loops of links
  Simplest is plaquette $\quad P_{\mu\nu}(n) = U_\mu(x)U_\nu(x+\widehat{\mu})U_\mu^\dagger(x+\widehat{\nu})U_\nu^\dagger(x)$
  Consider abelian case (merge exp's) and identify finite differences as derivatives:
  $$P_{\mu\nu}(n) = \exp\left[-iga\left(A_\mu(x+\frac{\widehat{\mu}}{2}) + A_\nu(x+\widehat{\mu}+\frac{\widehat{\nu}}{2}) - A_\mu(x+\widehat{\nu}+\frac{\widehat{\mu}}{2}) - A_\nu(x+\frac{\widehat{\nu}}{2})\right)\right]$$
  $$\longrightarrow \exp\left[-iga^2\left\{(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{a^2}{12}\left(\partial_\mu^3 A_\nu - \partial_\nu^3 A_\mu\right) + \ldots\right\}\right]$$
  $$= 1 - iga^2 F_{\mu\nu}(x) - \frac{g^2a^4}{2}F_{\mu\nu}(x)F^{\mu\nu}(x) + \mathcal{O}(a^6)$$

- Generalizes to non-abelian $\quad \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{2N}{g^2a^4}\sum_{\mu<\nu}\left(1 - \frac{1}{N}\mathrm{ReTr}\left[P_{\mu\nu}\right]\right) + \mathcal{O}(a^2)$

Fermion operator and lattice chiral symmetry

- Chiral transformation $\Psi \to e^{i\alpha\gamma_5}\Psi$ and Baker–Campbell–Hausdorff
  give $\overline{\Psi}D\Psi \to \overline{\Psi}e^{i\alpha\gamma_5}De^{i\alpha\gamma_5}\Psi = \overline{\Psi}D\Psi + \overline{\Psi}\left\{\gamma_5, D\right\}\Psi$
  $\Longrightarrow$ Chiral symmetry means $\{\gamma_5, D\} = 0$ for fermion operator D

- $\{\gamma_5, \gamma_\mu\} = 0 \Longrightarrow$ free propagator $D^{-1}(p) = \left[\frac{i\gamma_\mu}{a}\sin(ap_\mu)\right]^{-1}$ is chirally symmetric
  But single lattice field has poles for $ap_\mu \in \{0, \pi\}$ in Brillouin zone $-\frac{\pi}{a} < p_\mu \leq \frac{\pi}{a}$
  $\longrightarrow$ 16 fermions in continuum limit, with $\mathrm{U}(4)_V \times \mathrm{U}(4)_A$ chiral symmetry

- (Aside: No anomalies with finite number of d.o.f.
  $\Longrightarrow$ any anomalous continuum symmetry must be broken explicitly on the lattice)

- **Nielsen–Ninomiya no-go theorem** (schematically):
  '$\nexists$ any lattice fermion operator that is local, chirally symmetric,
  and produces a single continuum Dirac fermion'

Relevant lattice fermion formulations

- **Wilson fermions** sacrifice chiral symmetry by adding dimension-5 term
$$a\overline{\psi}(x)\nabla_\mu^2\psi(x) = \frac{1}{2a}\overline{\psi}(n)\left[U_\mu(n)\psi(n+\widehat{\mu}) - 2\psi(n) + U_\mu^\dagger(n-\widehat{\mu})\psi(n-\widehat{\mu})\right]$$
Vanishes in continuum limit, where chiral symmetry can be restored

- In momentum space, free $D_W^{-1}(p) = \left[\frac{i\gamma_\mu}{a}\sin(ap_\mu) + \frac{1}{a}\sum_\mu(1-\cos(ap_\mu))\right]^{-1}$
$\longrightarrow$ divergent fermion mass $\propto \frac{1}{a}$ for the 15 "doublers" with any $ap_\mu \approx \pi$

- Explicit breaking $\longrightarrow$ **relevant** (dim-3) fermion mass operator $\overline{\psi}\psi$ not protected
$\Longrightarrow$ need to fine-tune fermion mass to cancel large additive renormalization

- **Staggered fermions** use spin-diagonalization trick, $\psi(x) = \left(\prod_i \gamma_i^{x_i}\right)\chi(x)$
$\longrightarrow D_{\text{stag}}(n) = \overline{\chi}(n)\,(-1)^{\sum_{i<\mu}x_i}\left[U_\mu(n)\chi(n+\widehat{\mu}) - U_\mu^\dagger(n-\widehat{\mu})\chi(n-\widehat{\mu})\right]$

- γ matrices replaced by phases $\Longrightarrow$ four spinor components of χ decouple
Only keep one $\longrightarrow \mathrm{U}(1)_V \times \mathrm{U}(1)_A$ chiral symmetry and 4 continuum fermions

- Fermion d.o.f. distributed around unit hypercube, mixing spin and flavor

- **Domain wall fermions** have $\{\gamma_5, D\} = aD\gamma_5 D$ (Ginsparg–Wilson relation)
Preserve lattice *remnant* of $\mathrm{SU}(F)_V \times \mathrm{SU}(F)_A$ symmetry, $\gamma_5 D + D\gamma_5(1-aD) = 0$

- Qualitatively, set up N_5 copies of the gauge field as a 'fifth direction'
Doublers propagate in fifth direction $\longrightarrow$ decouple with large Wilson-like mass M_5

- Chiral fermions localized on 5th dim. 'walls' $\longrightarrow$ small 'residual mass' for finite N_5
In $N_5 \to \infty$ limit, reproduce 4d **overlap fermions**, $(H_5 \equiv \gamma_5(D_W - M_5))$
$$D_{\text{DWF}} = 1 + \gamma_5\frac{(1+H_5)^{N_5} - (1-H_5)^{N_5}}{(1+H_5)^{N_5} + (1-H_5)^{N_5}} \longrightarrow 1 + \gamma_5\frac{H_5}{\sqrt{H_5^\dagger H_5}} = 1 + \gamma_5\,\text{sign}[H_5]$$

- (Aside: Different symmetries may lead to different universality classes in IR)

Brief computational/algorithmic remarks

- First generate ensemble of gauge fields $\{U_k\}$ with Boltzmann distribution $\frac{1}{\mathcal{Z}}e^{-S[U]}$,
typically using hybrid Monte Carlo (HMC) Markov chain algorithm

- Then can measure many quantities on saved configurations, $\langle\mathcal{O}\rangle = \frac{1}{N}\sum_{k=1}^N \mathcal{O}(U_k)$

- Swap Grassmann fermion fields ψ for complex-number 'pseudofermions' Φ,
$$\int[dU][d\overline{\psi}][d\psi]e^{-S_G-\overline{\psi}D\psi} = \int[dU]e^{-S_G}\det D = \int[dU][d\Phi^*][d\Phi]e^{-S_G-\Phi^*D^{-1}\Phi}$$

- $\Longrightarrow$ Need many inversions of large sparse matrix D (rank up to $\sim 10^9$)
Useful to consider pairs of degenerate fermions $\longrightarrow$ real positive $(\det D)^2$ in e^{-S}

Composite spectrum from lattice calculations

- Spectrum of composite particle masses comes from two-point correlation function

$$C(t) = \langle 0 | \mathcal{O}_f(t) \mathcal{O}_i(0) | 0 \rangle = \sum_n \frac{\langle 0 | \mathcal{O}_f | n \rangle \langle n | \mathcal{O}_i | 0 \rangle}{E_n} e^{-E_n t} \longrightarrow A e^{-Mt}$$

  Only the ground state (with smallest mass M) remains for sufficiently large $t \gg 1$

- Here $\mathcal{O}(t)$ is a lattice interpolating operator with appropriate quantum numbers
  Example: $V(t) = \sum_{\vec{x}} \overline{\psi}(\vec{x}, t) \gamma_\mu \psi(\vec{x}, t)$ for vector meson with zero momentum
  $\longrightarrow C(t) = \sum_{\vec{x}} \text{Tr}\left[\overline{\psi}(x) \gamma_\mu \psi(x) \overline{\psi}(0) \gamma_\mu \psi(0)\right] = \sum_{\vec{x}} \text{Tr}\left[D^{-1}(-x) \gamma_\mu D^{-1}(x) \gamma_\mu\right]$

- (Aside: Flavor-singlet scalar mixes with vacuum $\longrightarrow$ signal-to-noise problem,
  $C(t) \longrightarrow$ const. $+ Ae^{-Mt}$ with const. $= \langle \mathcal{O}_f \rangle \langle \mathcal{O}_i \rangle$)

- Amplitude A contains decay constant, for example $\langle 0 | V | \rho \rangle = -i Z_V \sqrt{2} F_V M_V$
  (lattice-to-continuum renormalization factors Z computable (non)-perturbatively)

Some results for composite spectrum beyond QCD

- QCD-like SU(2) gauge theory with $N_F = 2$ fundamental fermions
  ($\longrightarrow$ SU(4)/Sp(4) Next-to-MCHM without partial compositeness)
  Chiral and continuum extrapolations give large $M_V/f \approx 15.7 \longrightarrow M_V \approx \frac{3.9}{\sqrt{\xi}}$ TeV

- SU(3) QCD has ratio $M_V/f \simeq 8$
  Same $M_V/f \simeq 8$ found for SU(3) gauge theories with more fermion d.o.f.
  Examples: $N_F = 8$ fundamental fermions (Higgs could be either dilaton or PNGB)
  $N_S = 2$ fermions in two-index-symmetric (sextet) rep
  (Higgs can't be PNGB, must be dilaton $\longrightarrow f = v$ and $M_V \simeq 2$ TeV)

- Lattice calculations of $N_F = 8$ decay constants allow estimate
  of vector width-to-mass ratio $\frac{\Gamma_{V \to PP}}{M_V} \approx 0.24$, also similar to QCD value

- Multi-rep composite Higgs UV completions have begun to be explored
  For simplicity, SU(4) gauge theory with $N_F = 2$ fund. and $N_A = 2$ in AS2 rep
  rather than $N_F = 3$ and $N_A = 2.5$ (i.e., 5 Weyl) needed by composite Higgs model

- In large-N limit expect $f \propto \sqrt{N}$ while $M_V \sim$ const.
  Rescaling QCD $\longrightarrow M_V/f \simeq 8\sqrt{\frac{3}{4}} \approx 4.9\sqrt{2}$ in good agreement with lattice results
  which also show a narrower width $\Gamma_{V \to PP}/M_V$ as expected from the larger $N = 4$

Supplement: Near conformality and anomalous dimensions

- SU(3) gauge theories with many fermion d.o.f. generically exhibit a light scalar
  qualitatively different from the heavy ($M_S/f \approx 5$), broad σ meson of QCD
  Empirically related to near conformality (small β function) in the IR

- Near conformality $\longrightarrow$ large scale separation, complicating lattice calculations

- Lattice calculations can investigate a discrete β function
  First need to choose scheme for renormalized coupling g^2
  $\longrightarrow$ gradient flow schemes now used in most invetigations

- Coupling is tied to lattice size L and its evolution is studied
  as $L \to sL$ increases by discrete scale factor (typically $s = 2$ or $3/2$):
  $$\beta_s(g^2, L) = \frac{g^2(sL, a) - g^2(L, a)}{\log s^2}$$
  Note matched lattice spacings a for both L and sL
  $\longrightarrow$ results must be extrapolated to $a \to 0$ continuum limit

- Continuum-extrapolated results show dramatic decrease
  in magnitudes of discrete β functions as number of fermion d.o.f. increases

- For SU(3) gauge theory with $N_F = 12$, signs of conformal IR fixed point ($\beta_s = 0$)

- Slope of β_s at fixed point $\longrightarrow$ scheme-independent anomalous dimension $\gamma_g^\star$
  (full scaling dimension $y_g = \gamma_g^\star$ is irrelevant)

- A relevant scaling dimension $y_m = 1 + \gamma_m^\star$ can be computed
  from finite-size scaling of hadron spectrum or from Dirac eigenmodes

- Future work: Compute baryon anomalous dimension for partial compositeness
  (from RI/MOM renormalization of three-fermion operator)
  Using expensive domain wall fermions to avoid staggered mixing of spin and flavor

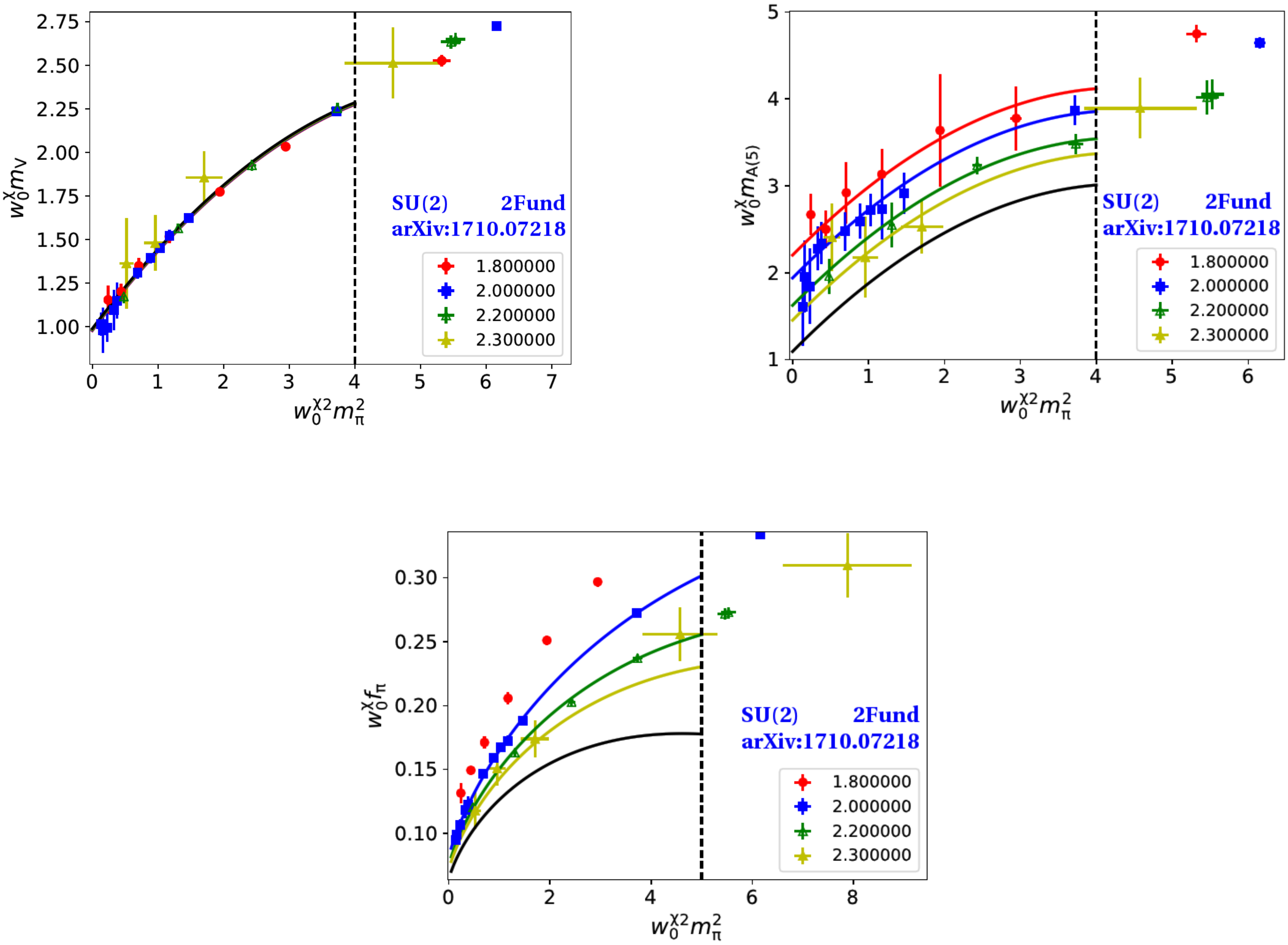

Figure 1: Selected results from lattice investigations of SU(2) gauge theory with $N_F = 2$ (Wilson) fermions in the fundamental representation, from arXiv:1710.07218. The top plots show the masses m_V (left) and m_A (right) of the lightest vector and axial-vector particles (respectively analogs of the $\rho(770)$ and $a_1(1260)$). The bottom plot shows the symmetry breaking scale f_π from the pion-to-vacuum matrix element in the amplitude of the pseudoscalar two-point function. For all three quantities the multiple different lattice spacings (parameterized by w_0^χ) and fermion masses (parameterized by the PNGB mass m_π) allow combined extrapolations to the continuum limit $a \to 0$ and the chiral limit $m_\pi \to 0$. In this combined limit, the physical $m_V/f_\pi \approx 15.7 \longrightarrow M_V \approx \frac{3.9}{\sqrt{\xi}}$ TeV and $m_A/f_\pi \approx 17.3 \longrightarrow M_A \approx \frac{4.2}{\sqrt{\xi}}$ TeV, not yet within reach of experimental searches even if $\xi \simeq 1$.

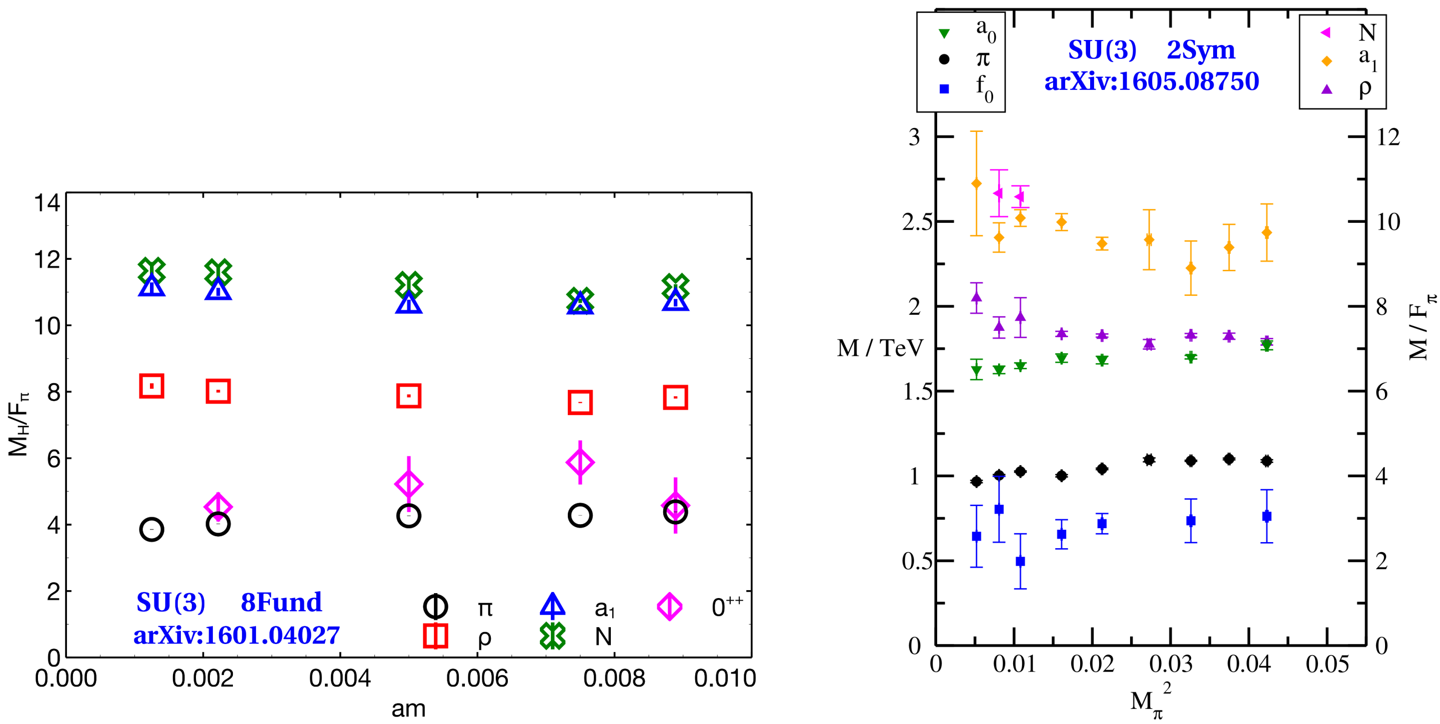

Figure 2: Ratios of composite particle masses to the pseudoscalar decay constant F_π for SU(3) gauge theories with either $N_F = 8$ fundamental fermions (left) or $N_F = 2$ two-index-symmetric 'sextet' fermions (right), both using staggered discretizations. In both cases the lightest vector has $M_\rho/F_\pi \simeq 8$, roughly the same as QCD. Unlike QCD, the lightest scalar is degenerate to or even lighter than the pions, making it possible to interpret the Higgs as a dilaton rather than a PNGB. (The $N_F = 8$ system allows both possibilities, but the sextet Higgs can't be a PNGB and must be a dilaton.) Note that the expected behavior $M_\pi/F_\pi \to 0$ in the chiral limit is not yet visible. In both cases only a single lattice spacing is shown and no extrapolations are attempted, but tests at other lattice spacings show little visible dependence on a compared to statistical uncertainties.

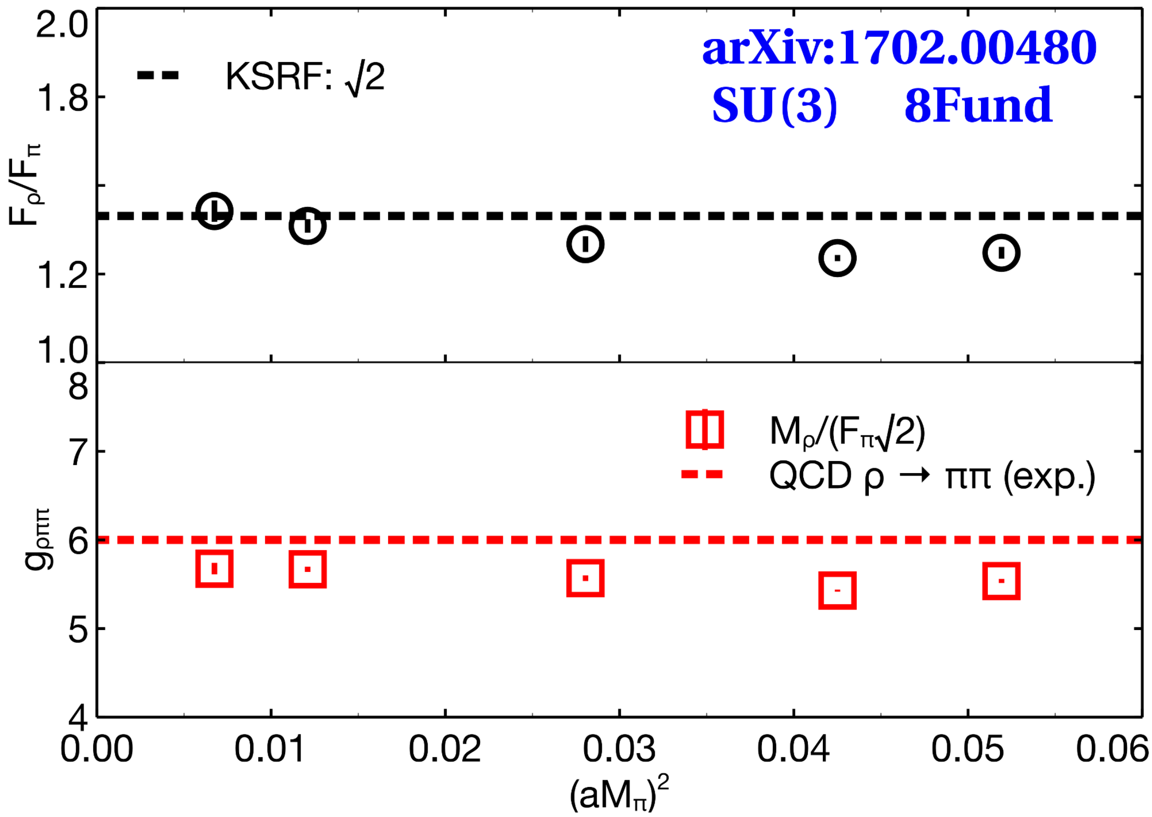

Figure 3: For the SU(3) staggered $N_F = 8$ fundamental system considered above, lattice calculations of the vector and pseudoscalar decay constants F_ρ and F_π are consistent with the first Kawarabayashi–Suzuki–Riazuddin–Fayyazuddin (KSRF) relation $F_\rho/F_\pi \approx \sqrt{2}$. This is another similarity between this theory and QCD. The second KSRF relation $g_{\rho\pi\pi} = M_\rho/(F_\pi\sqrt{2})$ enables an estimate of the vector width-to-mass ratio $\frac{\Gamma_{\rho\to\pi\pi}}{M_\rho} \approx \frac{g^2_{\rho\pi\pi}}{48\pi} \approx 0.24$, also similar to QCD and probably invalidating narrow width approximations.

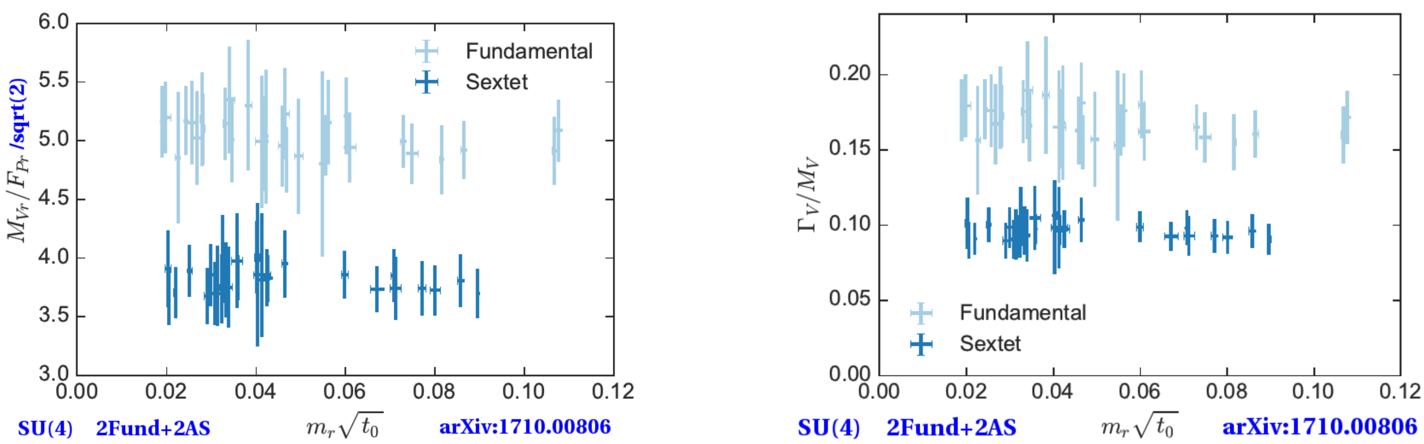

Figure 4: Lattice results for M_V/F_P and Γ_V/M_V (as in the plots above) for SU(4) gauge theory with $N_F = 2$ fundamental fermions and $N_A = 2$ fermions in the two-index-antisymmetric (AS2 or 'sextet') rep (a simplified step towards a multi-rep composite Higgs model with partial compositeness). The quantities are computed separately for the (Wilson) fermions in each rep, and compared to the previous plots the normalization of F_P has changed by a factor of $\sqrt{2}$. For SU(N) gauge groups in the limit where the number of 'colors' N is large, we expect $M_V \sim$ const. while $F_P \sim \sqrt{N}$. Naively rescaling QCD would suggest $M_V/F_P \simeq 8\sqrt{\frac{3}{4}\frac{1}{\sqrt{2}}} \approx 4.9$, in good agreement with the fundamental-rep results in the left plot. The right plot suggests that the width Γ_V decreases compared to QCD, which is also expected in the large-N limit.

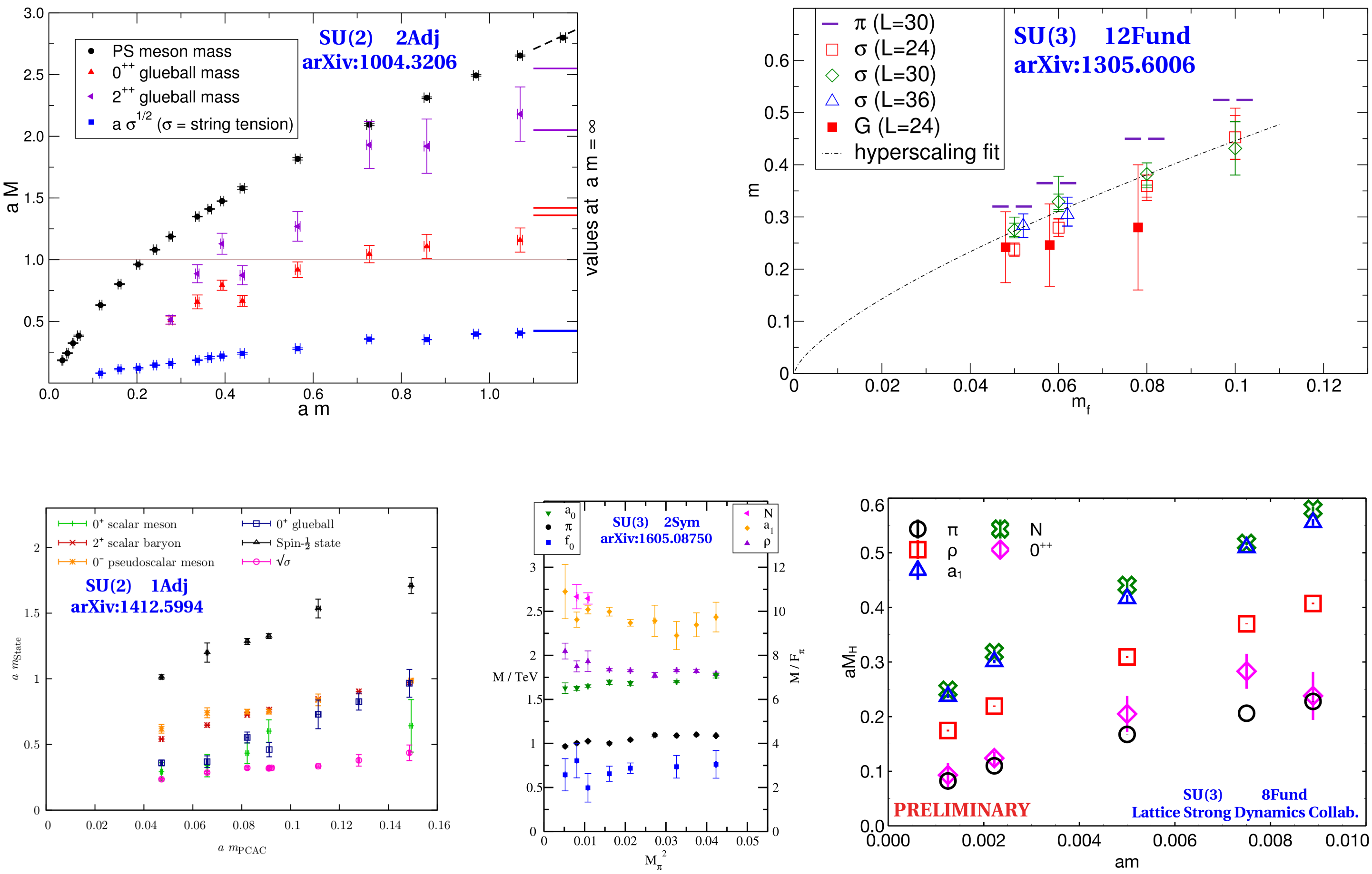

Figure 5: A collection of lattice results for the spectra of more SU(2) and SU(3) gauge theories, all of which feature a non-QCD-like light scalar that might (with careful model building) serve as a candidate for a non-PNGB (potentially dilatonic) composite Higgs boson. The top row includes systems believed to possess a conformal IR fixed point rather than exhibiting confinement and spontaneous symmetry breaking (i.e., $f \to 0$ in the chiral limit): SU(2) gauge theory with $N_{\text{Adj}} = 2$ (Wilson) fermions in the adjoint rep (left, arXiv:1004.3206) and SU(3) gauge theory with $N_F = 12$ (staggered) fermions in the fundamental rep (right, arXiv:1305.6006). The bottom row includes systems more likely to exhibit spontaneous symmetry breaking: SU(2) with $N_{\text{Adj}} = 1$ Wilson (left, arXiv:1412.5994), SU(3) with $N_S = 2$ (staggered) fermions in the two-index-symmetric (sextet) rep (center, arXiv:1605.08750) and SU(3) with $N_F = 8$ staggered (right, a preliminary update of arXiv:1601.04027).

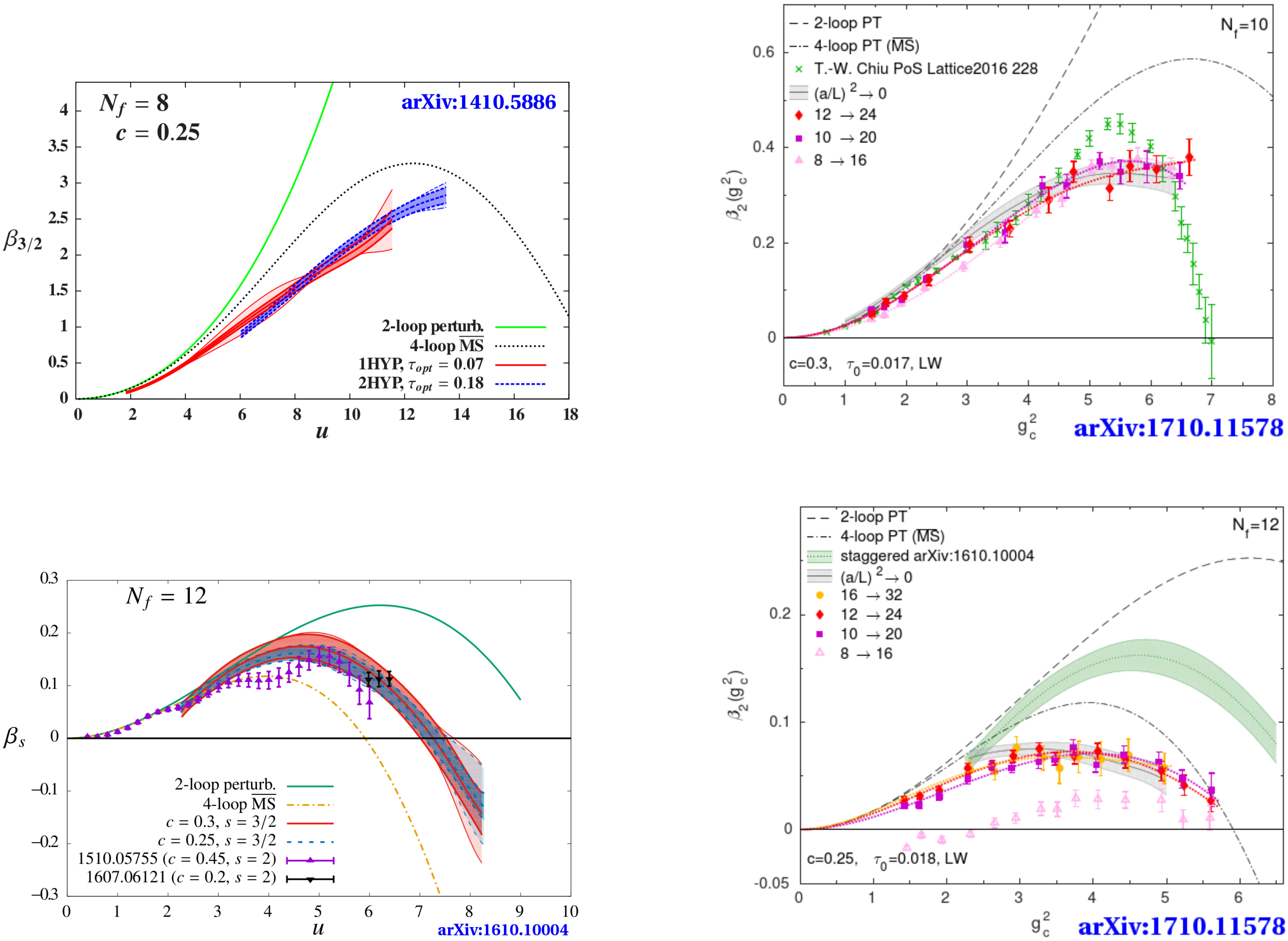

Figure 6: The magnitudes of continuum-extrapolated discrete β functions decrease dramatically as the number N_F of SU(3) fundamental fermions increases from $N_F = 8$ (top left, arXiv:1410.5886) to $N_F = 10$ (top right, arXiv:1710.11578) and $N_F = 12$ (bottom, arXiv:1610.10004 on left and arXiv:1710.11578 on right). All of these investigations consider some form of the gradient flow renormalized coupling scheme proposed in arXiv:1208.1051. For $N_F \geq 10$ there are (disputed) signs that the theories may approach a conformal fixed point in the IR, with no confinement or spontaneous symmetry breaking. The slope of the β function at that fixed point predicts the scheme-independent anomalous dimension $\gamma_g^\star$. One complication is the current disagreement between staggered (left) and domain wall (right) results for $N_F = 12$, which may indicate that the different symmetries of those lattice fermion formulations lead to IR fixed points in different universality classes.