Results from lattice studies of maximally supersymmetric Yang–Mills

David Schaich (Syracuse)

Lattice 2014, 25 June

arXiv:1405.0644 (submitted to PRD) and work in progress
with Simon Catterall, Poul Damgaard, Tom DeGrand and Joel Giedt

Practical lattice $\mathcal{N} = 4$ SYM

The previous talks reviewed the motivations for
and formulation of lattice $\mathcal{N} = 4$ SYM

$$S = \frac{N}{\lambda_{\text{lat}}} \sum_x \left[-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{C_2}{2}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a\right)^2 - \chi_{ab}\mathcal{D}_{[a}^{(+)}\psi_{b]} - \eta\overline{\mathcal{D}}_a^{(-)}\psi_a - \frac{1}{4}\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab} \right]$$
$$+ \mu^2 \sum_{x,\ a} \left(\frac{1}{N}\text{Tr}\left[\overline{\mathcal{U}}_a\mathcal{U}_a\right] - 1 \right)^2 + \kappa \sum_{\mathcal{P}} |\det \mathcal{P} - 1|^2$$

—First line exactly preserves one supersymmetry $\mathcal{Q}$, other 15 broken
—μ term regulates flat directions, acts like bosonic mass
—κ term reduces U(N) $\longrightarrow$ SU(N), suppressing U(1) lattice phase
(I focus on $N = 2$, larger-N studies underway)

How well does this work in our existing lattice calculations?

First issue: Both μ and κ deformations break the $\mathcal{Q}$ supersymmetry
in our numerical computations

Monitoring $\mathcal{Q}$ supersymmetry breaking

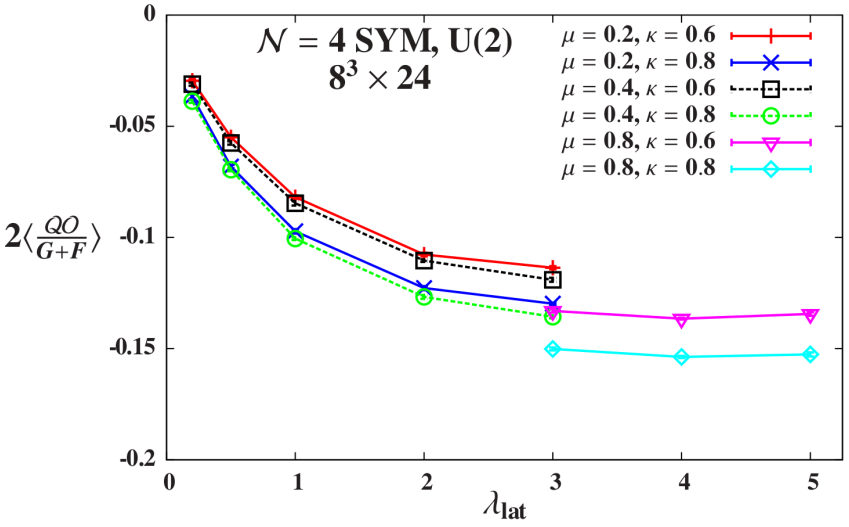
Exactly preserved $\mathcal{Q}$ supersymmetry $\Longrightarrow$ Ward identity $\langle \mathcal{Q}\mathcal{O} \rangle = 0$

Fermionic $\mathcal{O} = \mathrm{Tr}\left[\eta \sum_a \mathcal{U}_a \overline{\mathcal{U}}_a\right]$ (not already in action) gives bosonic

$$\mathcal{Q}\mathcal{O} = \mathrm{Tr}\left[C_2 \sum_b \left(\mathcal{U}_b \overline{\mathcal{U}}_b - \overline{\mathcal{U}}_b \mathcal{U}_b\right) \sum_a \mathcal{U}_a \overline{\mathcal{U}}_a\right] - \mathrm{Tr}\left[\eta \sum_a \psi_a \overline{\mathcal{U}}_a\right] = G - F$$

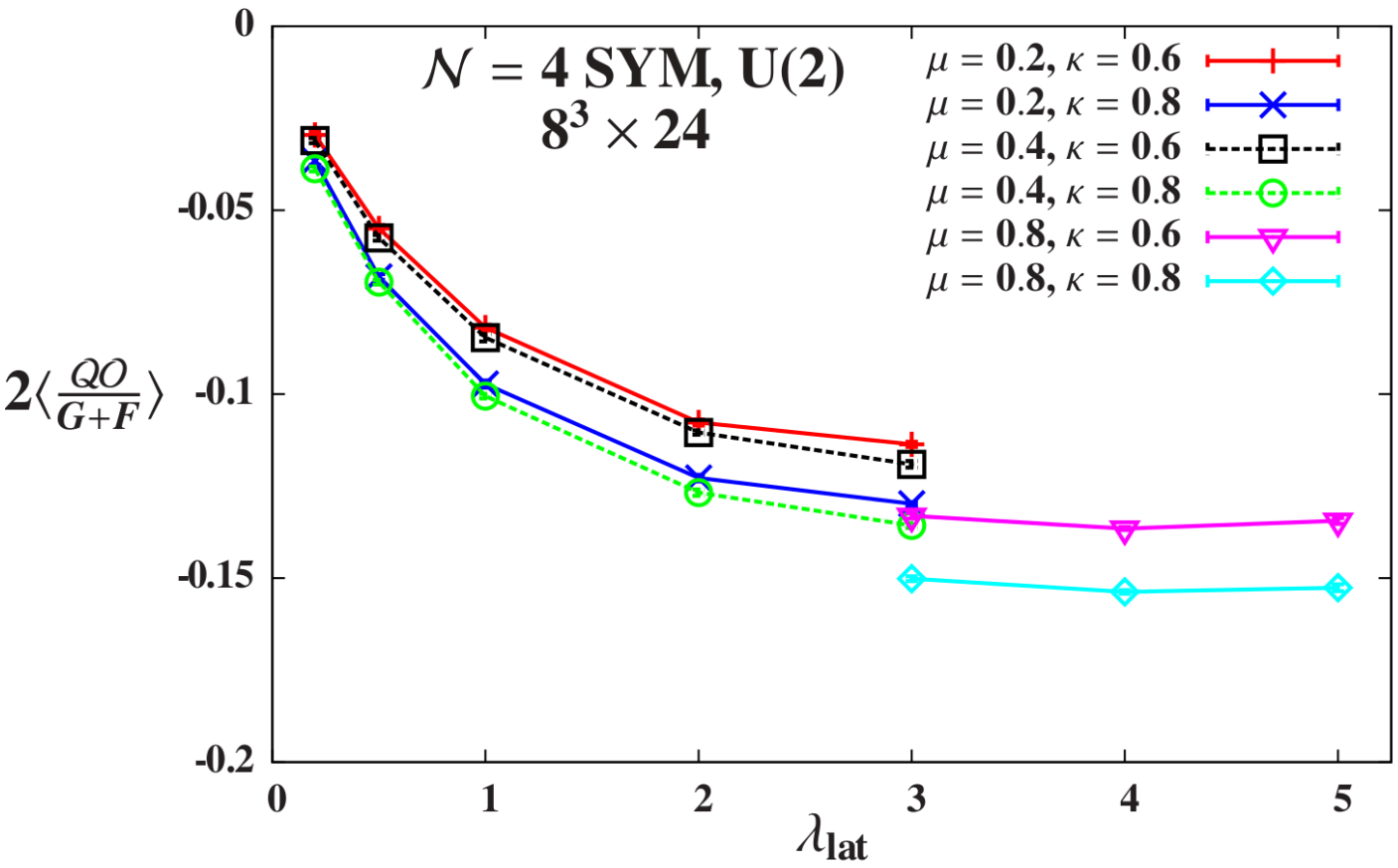
(difference of gauge term and fermion-bilinear term)

Normalized Ward identity violations $\left\langle \frac{G-F}{(G+F)/2} \right\rangle$ measure susy breaking



We observe mild $\mathcal{Q}$ supersymmetry breaking

Normalized Ward identity violations $\left\langle \frac{G-F}{(G+F)/2} \right\rangle$ measure susy breaking



Observations: $\sim$10% violations grow with each of $\lambda_{\rm lat}$, μ and κ
More sensitive to κ than to μ

The other 15 supersymmeties $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

Previous talk reviewed role of discrete R symmetries R_a & R_{ab}

Qualitatively, $\mathcal{Q}_a \sim R_a \times \mathcal{Q}$ and $\mathcal{Q}_{ab} \sim R_{ab} \times \mathcal{Q}$

where R_a and R_{ab} transform $\mathcal{U}_c \to \overline{\mathcal{U}}_c^{-1}$ for $c \neq a$ (or b)

Act on $m \times n$ Wilson loop: $\mathcal{W}_{ab} \longrightarrow \widetilde{\mathcal{W}}_{ab} \equiv R_a \mathcal{W}_{ab}$ where

$$\mathcal{W}_{ab} = \mathrm{Tr}\left[\prod_m \mathcal{U}_a(x) \prod_n \mathcal{U}_b(x + m\widehat{\boldsymbol{e}}_a) \prod_m \overline{\mathcal{U}}_a(x + n\widehat{\boldsymbol{e}}_b) \prod_n \overline{\mathcal{U}}_b(x)\right]$$

$$\widetilde{\mathcal{W}}_{ab} = \mathrm{Tr}\left[\prod_m \mathcal{U}_a(x) \prod_n \overline{\mathcal{U}}_b^{-1}(x + m\widehat{\boldsymbol{e}}_a) \prod_m \overline{\mathcal{U}}_a(x + n\widehat{\boldsymbol{e}}_b) \prod_n \mathcal{U}_b^{-1}(x)\right]$$

Loop still closes since $\mathcal{U}_b$ and $\overline{\mathcal{U}}_b^{-1}$ both go from $x + \widehat{\boldsymbol{e}}_b$ to x

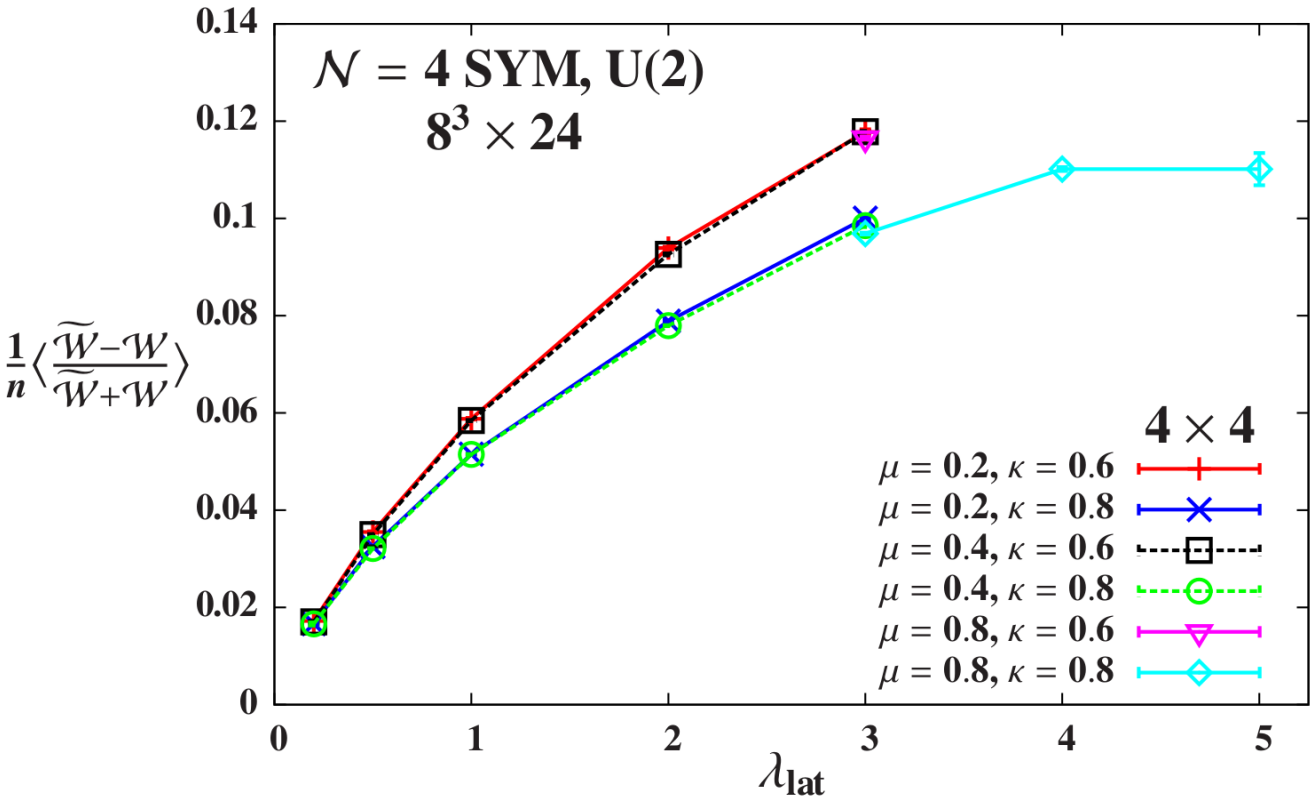
Relative difference $\left\langle \frac{(\widetilde{\mathcal{W}} - \mathcal{W})/2n}{(\widetilde{\mathcal{W}} + \mathcal{W})/2} \right\rangle$ measures R symmetry breaking

$\longrightarrow \mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ breaking

R symmetry breaking also appears mild

Relative difference $\left\langle \frac{(\widetilde{\mathcal{W}}-\mathcal{W})/2n}{(\widetilde{\mathcal{W}}+\mathcal{W})/2} \right\rangle$ measures R symmetry breaking

$\longrightarrow$ $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ breaking



Observations: $\sim$10% violations grow with $\lambda_{\rm lat}$ but shrink with κ

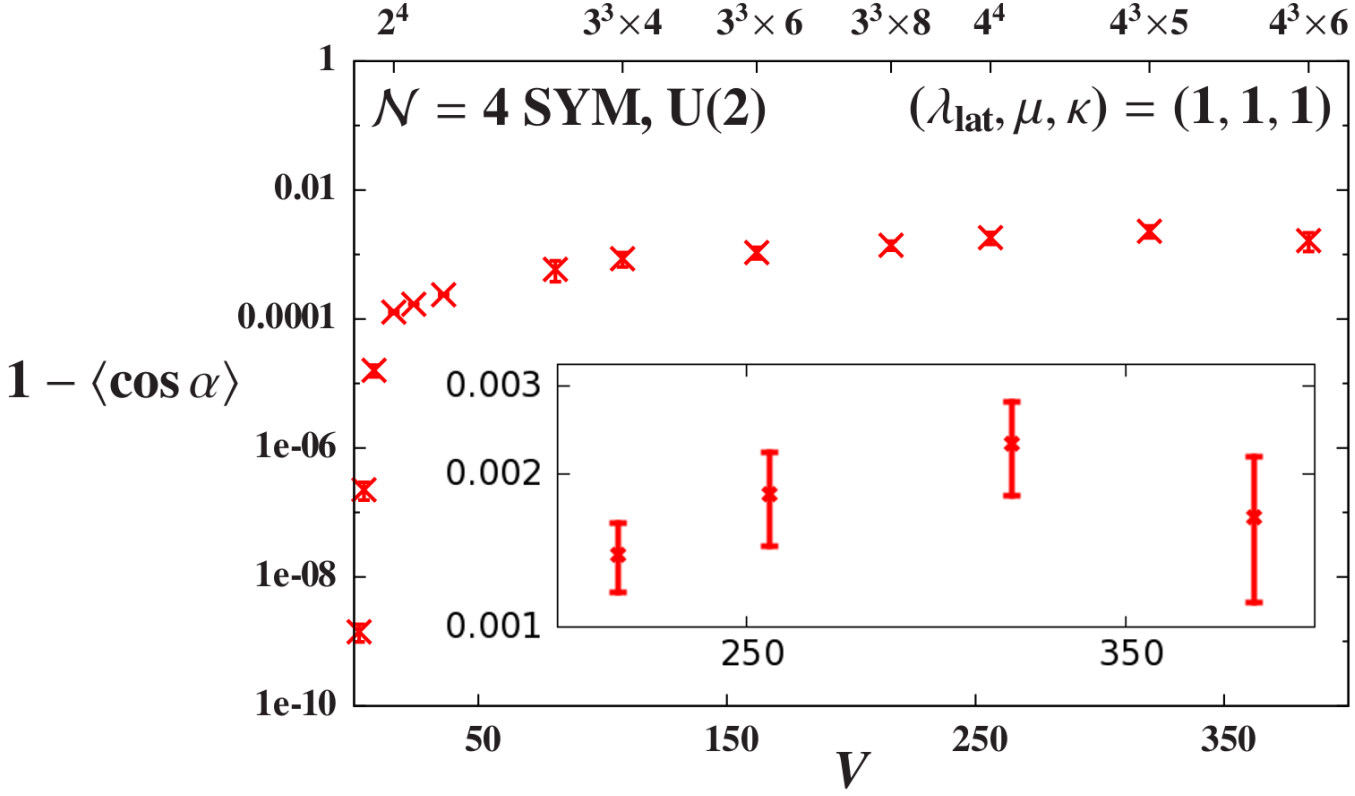
Connection to U(1) sector?

Complex pfaffian $P = |P|e^{i\alpha} \rightarrow$ potential sign problem

Our calculations are all phase-quenched:

Omit $e^{i\alpha}$ in RHMC, compute full pfaffian P on saved configurations

We measure P to be nearly real and positive: $\langle e^{i\alpha} \rangle \approx \langle \cos(\alpha) \rangle \approx 1$



Fluctuations don't grow with volume

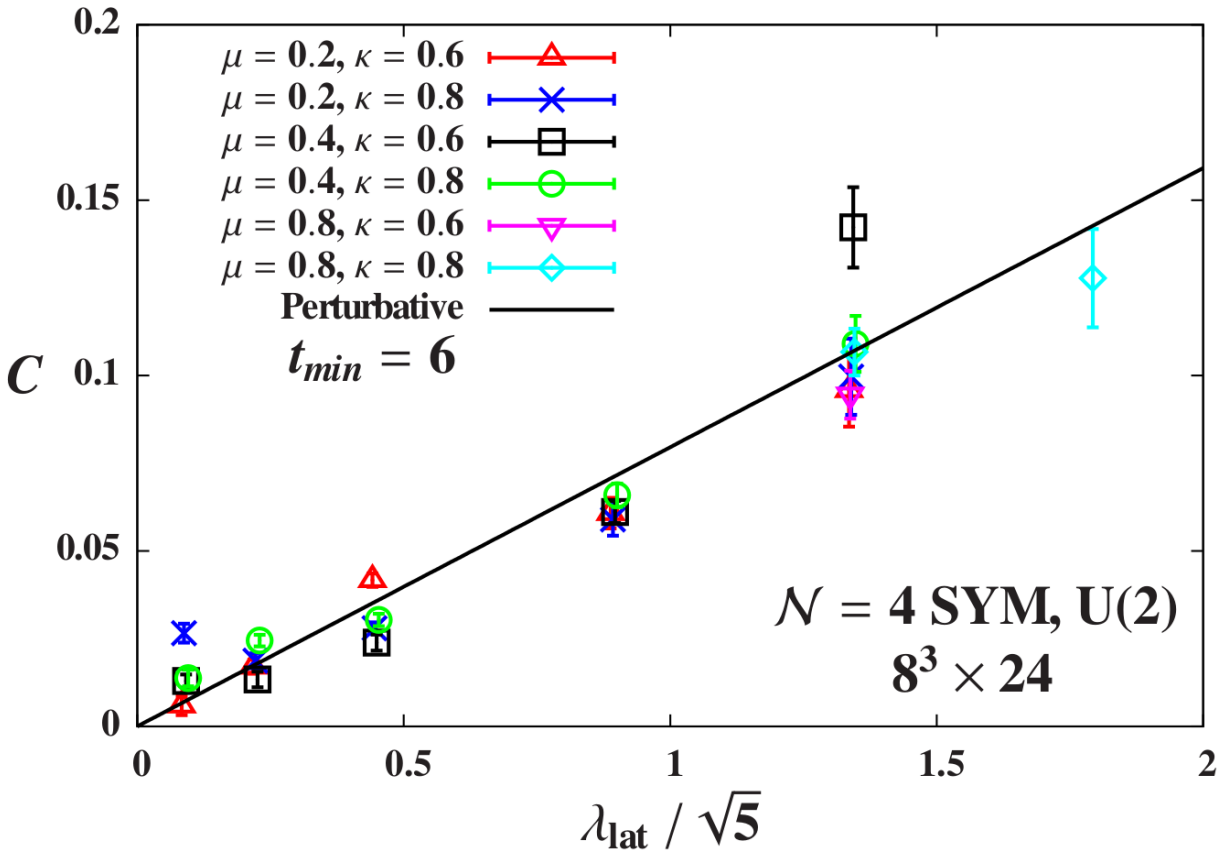
Also shrink with N for fixed $V = 32$:

	$\langle \cos\alpha \rangle$
U(2)	0.99978(4)
U(3)	0.99980(3)
U(4)	0.99989(4)

We have no good explanation for the absence of a sign problem

Static potential: Comparison with continuum theory

—Wilson loops $W(\vec{r}, t) = \exp\left[-V(\vec{r})t\right] \longrightarrow V(r) = A - C/r$
—Coulomb coefficients agree with perturbative $C = \lambda_{\rm lat}/(4\pi\sqrt{5})$

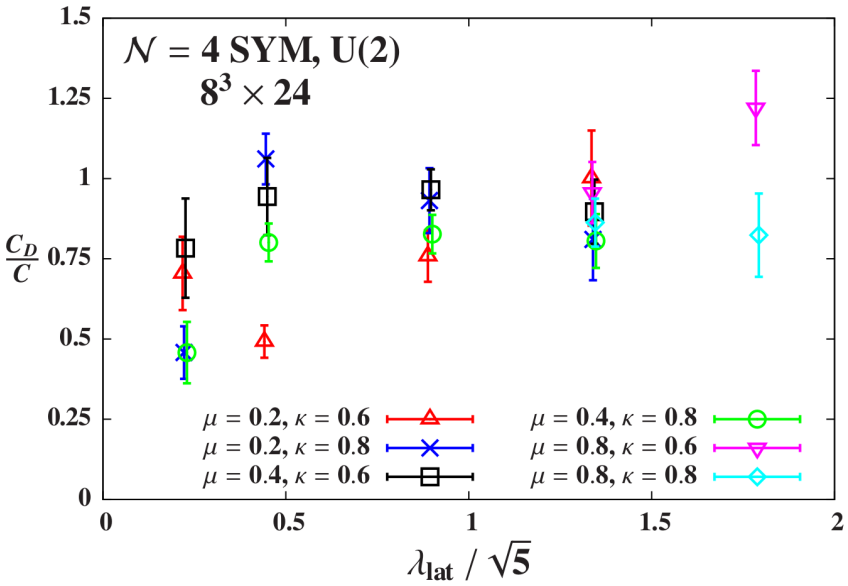


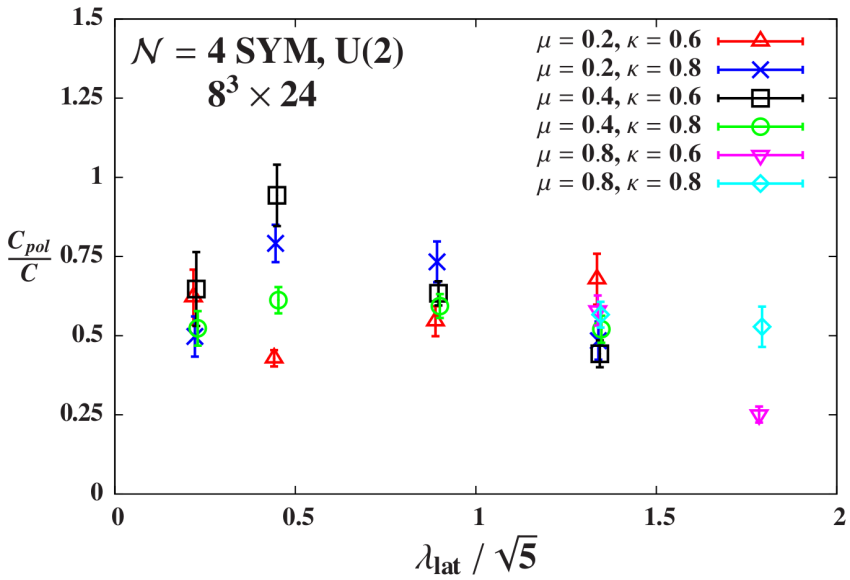
Smearing may help reduce noise in static potential results
We have implemented stout smearing, re-analysis underway

Coulomb coefficients for different Wilson loops

Left: Project Wilson loops from U(2) $\longrightarrow$ SU(2)
Expect C to decrease by factor of $\frac{N^2-1}{N^2} = 3/4$

Right: Build Wilson loops from unitarized links (removing scalars)
Expect C to decrease by factor of $1/2$





Both expected factors present, although again noisily

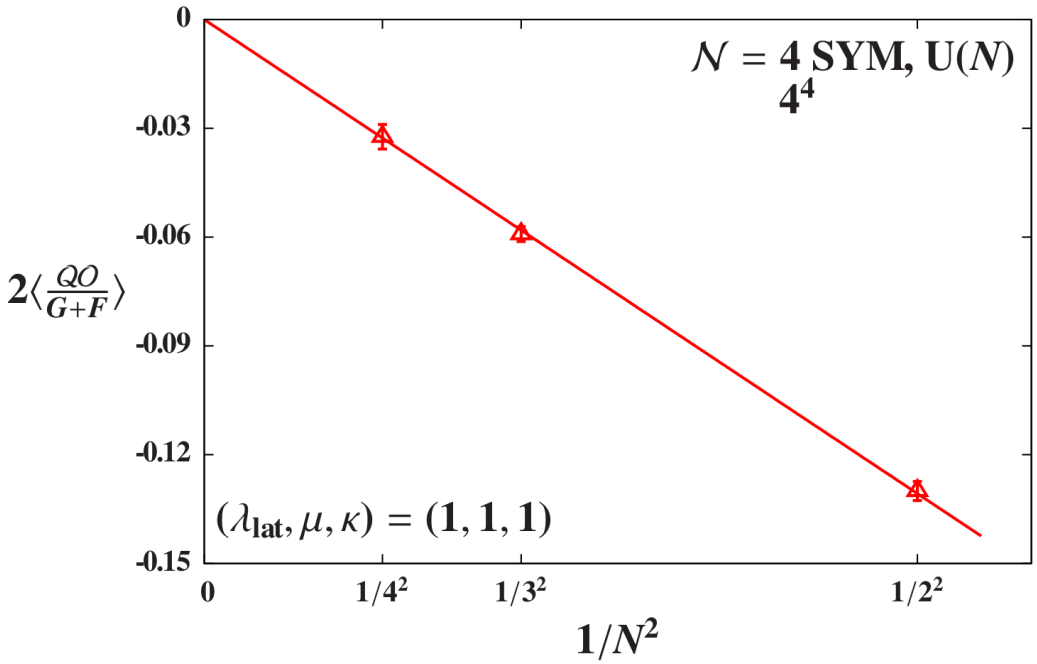
There are many open directions for further studies

Moving to larger N important for contact with continuum theory
Example: Maldacena prediction $C \propto \sqrt{\lambda}$ for strong coupling $\lambda \ll N$

Our code allows arbitrary N

We are running $N = 2$, 3 and 4

We see susy breaking $\propto 1/N^2$
costs increasing $\propto N^5$



Other projects underway include...

—Computation of Konishi & SUGRA correlators and their anom. dims.
—Stout smearing to improve signals $\longrightarrow$ gradient flow?
—Blocking, tuning to desired continuum limit (previous talk)

Recapitulation

Supersymmetric field theories very interesting to study on the lattice

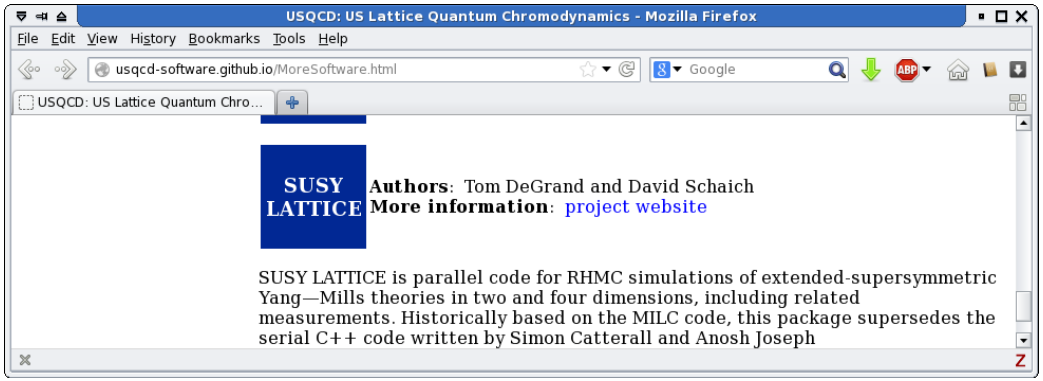
Lattice formulation of $\mathcal{N} = 4$ SYM preserves one supersymmetry,
only known example of such discretization in 4d

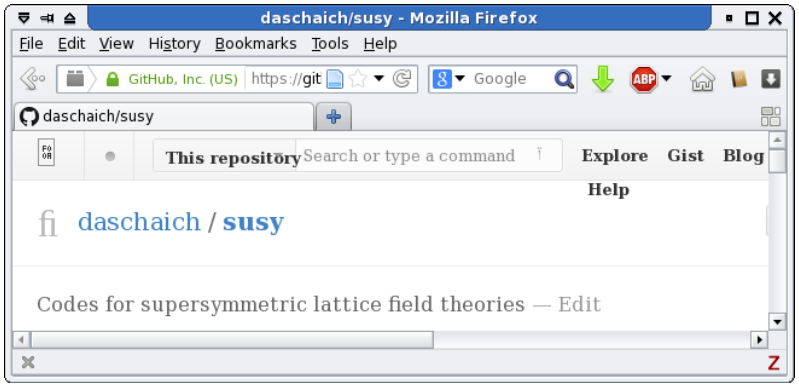
Current results from lattice $\mathcal{N} = 4$ SYM calculations

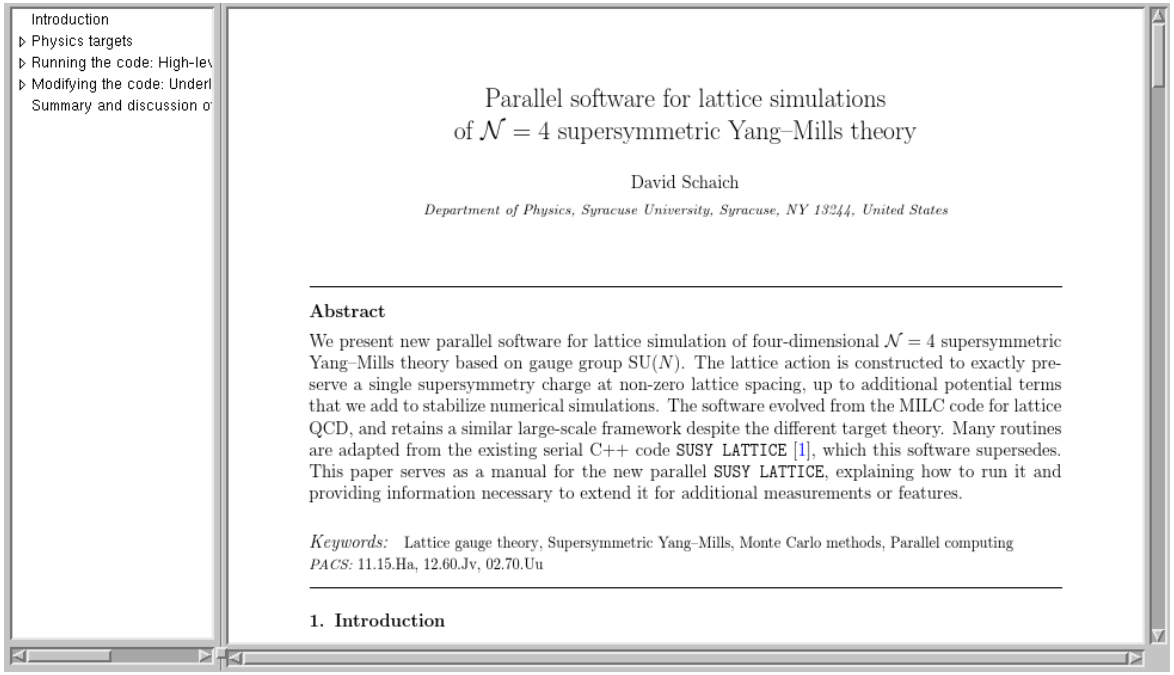
- $\mathcal{Q}$ supersymmetry breaking is under control
  R symmetry breaking for $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ also appears mild
- The pfaffian is nearly real and positive on all accessible volumes
  and fluctuations don't grow with volume or with N
- Static potentials are coulombic at all investigated couplings
- Coulomb coefficients agree with perturbation theory
  and scale as expected for different types of Wilson loops

It will be healthy to have more groups studying lattice susy
$\longrightarrow$ We publicly release our software to reduce barriers to entry

MILC-based code through USQCD (github.com/daschaich/susy)







Thank you!

Thank you!

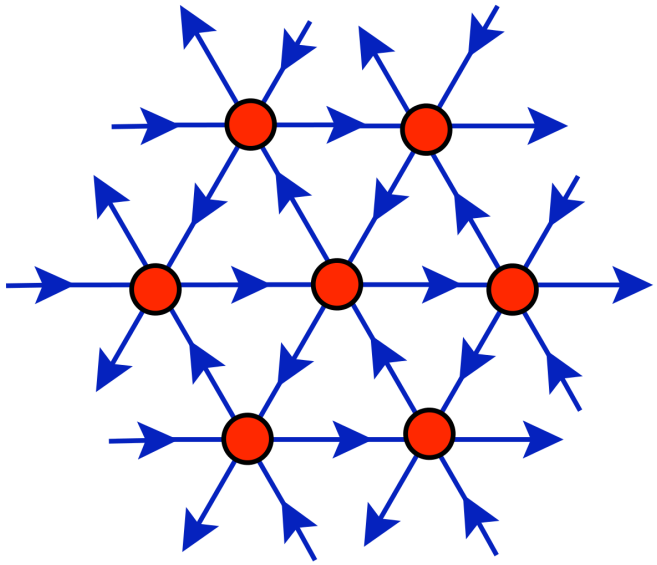
Collaborators

Simon Catterall, Poul Damgaard, Tom DeGrand, Joel Giedt, Aarti Veernala

Funding and computing resources

Backup: Discretization on A_4^* lattice

—Five links symmetrically spanning 4d
—Analog of 2d triangular lattice



Non-orthogonal links

$$\Longrightarrow \text{continuum } \lambda = \lambda_{\text{lat}} / \sqrt{5}$$

A_4^* lattice has S_5 point group symmetry
S_5 irreducible representations of lattice fields

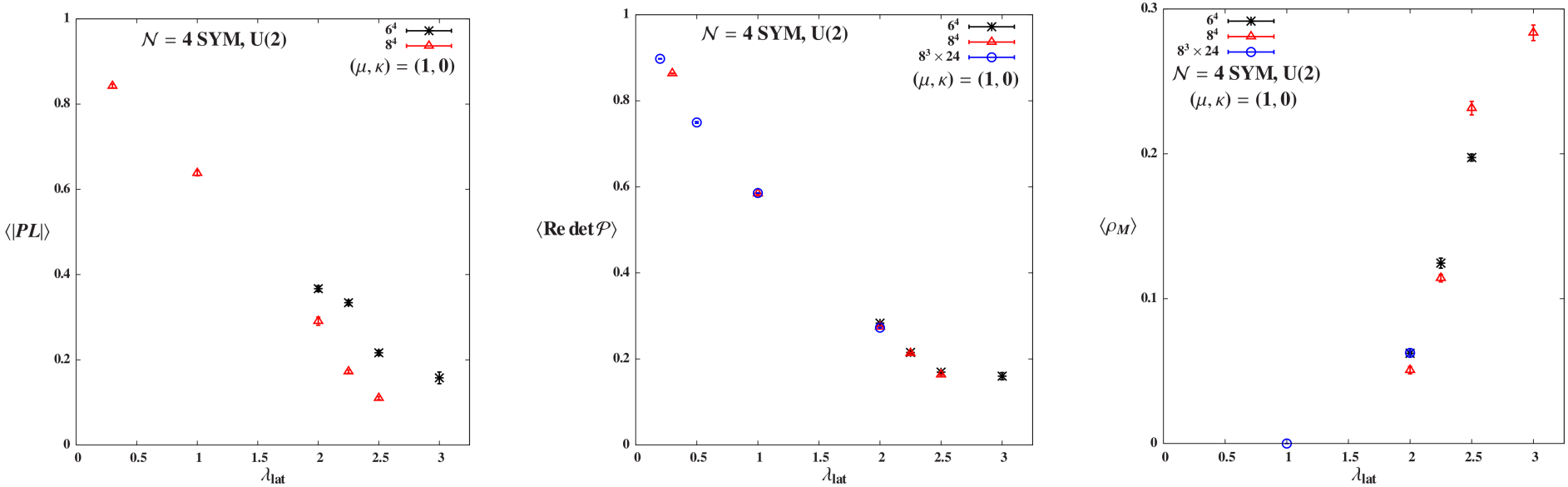
$\longrightarrow$ continuum SO(4) euclidean Lorentz irreps.

$$\mathcal{U}_a = \mathbf{4} \oplus \mathbf{1} \longrightarrow U_\mu \text{ and } \Phi$$

$$\psi_a = \mathbf{4} \oplus \mathbf{1} \longrightarrow \psi_\mu \text{ and } \overline{\eta}$$

$$\chi_{ab} = \mathbf{6} \oplus \mathbf{4} \longrightarrow \chi_{\mu\nu} \text{ and } \overline{\psi}_\mu$$

Backup: Lattice phase due to U(1) sector



1. Polyakov loop collapses $\Longrightarrow$ confining phase
   (**not** present in continuum $\mathcal{N} = 4$ SYM)

2. Plaquette determinant is associated with U(1) sector
   Drops around same coupling $\lambda_{\rm lat}$ as Polyakov loop

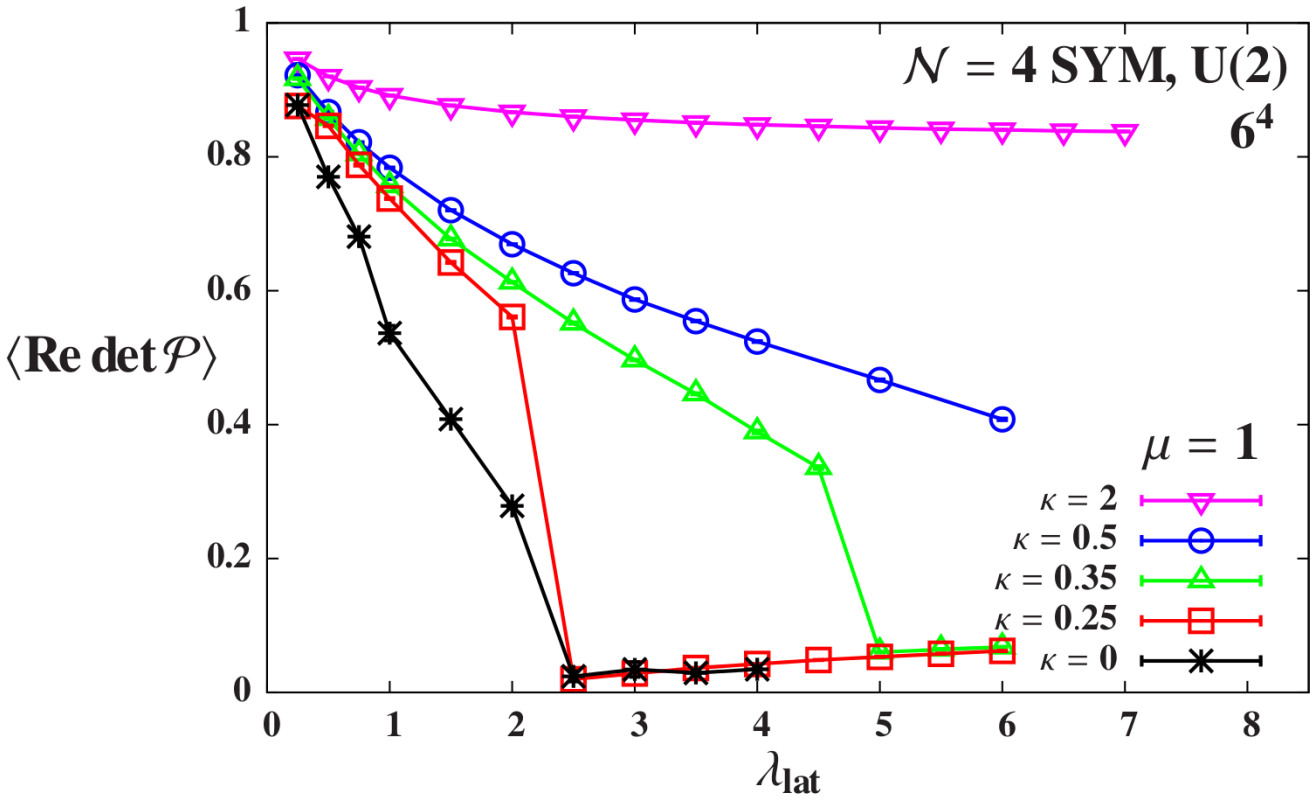
3. ρ_M is density of U(1) monopole world lines (DeGrand & Toussaint)
   Non-zero when Polyakov loop and plaq. determinant collapse

Backup: Removing the U(1) sector, U(N) $\longrightarrow$ SU(N)

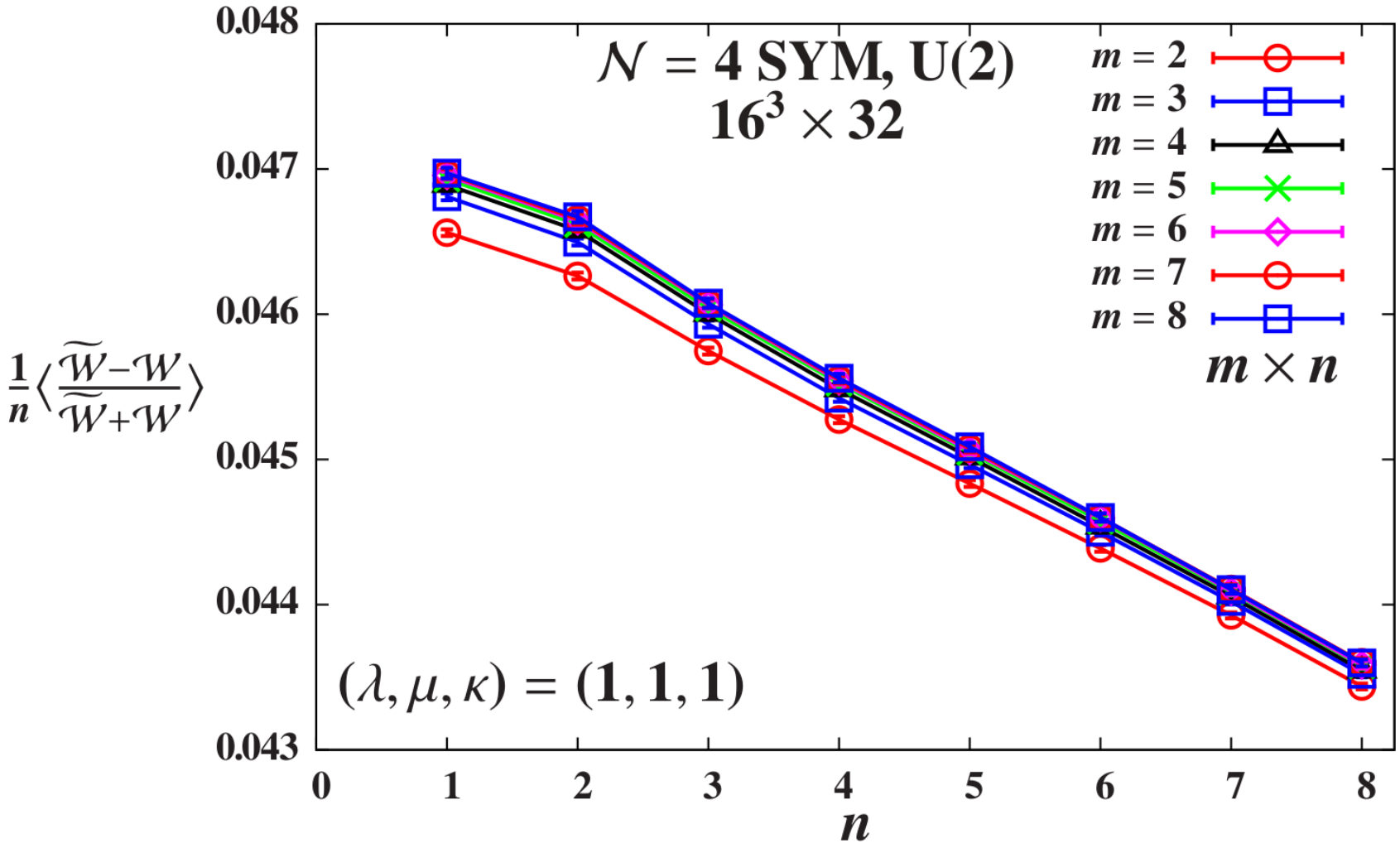
$\kappa \sum_{\mathcal{P}} |\det \mathcal{P} - 1|^2 \in S$ suppresses the strongly-coupled lattice phase

Produces $2\kappa F_{\mu\nu} F^{\mu\nu}$ term in U(1) sector

$\Longrightarrow$ QED critical $\beta_c = 0.99 \longrightarrow$ critical $\kappa_c \approx 0.5$

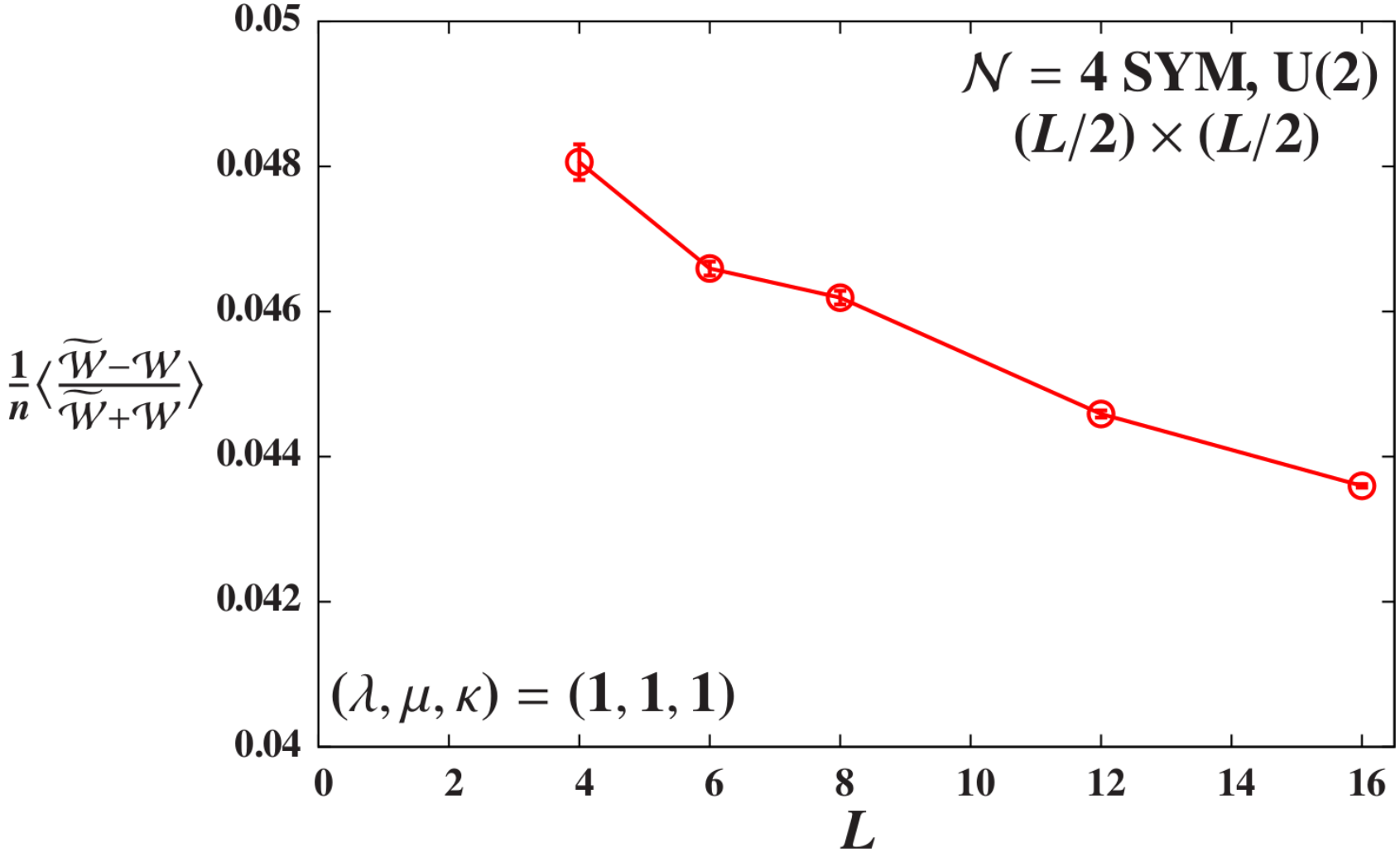


Backup: R symmetry breaking vs. size of Wilson loop



R symmetry breaking decreases slightly with $2n$ inverted links in $\widetilde{\mathcal{W}}$, largely insensitive to number ($2m$) of unaltered links

Backup: R symmetry breaking vs. lattice volume



R symmetry breaking from $(L/2)\times(L/2)$ Wilson loops decreases $\sim$10% for $16^3 \times 32$ volume compared to $4^3 \times 12$

Backup: New parallel pfaffian measurement

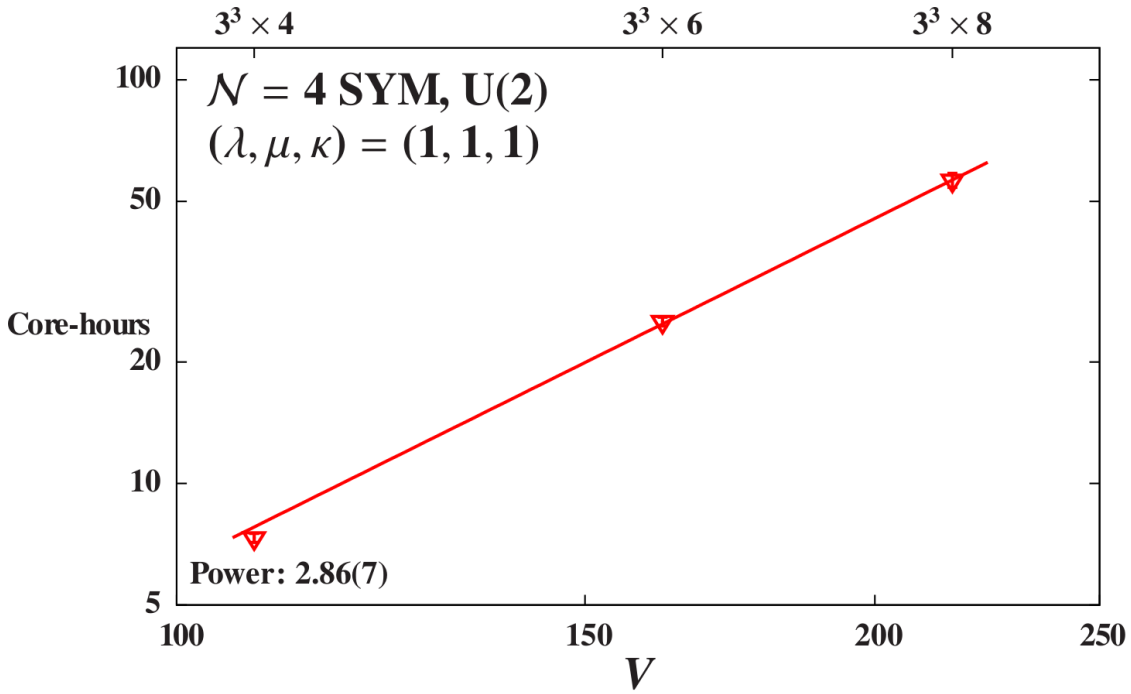
Pfaffian measurement is much harder than RHMC

Cost scales $\propto N_\psi^3$ where N_ψ is number of elements in fermionic fields

Good weak scaling
from new parallel software
(github.com/daschaich/susy)

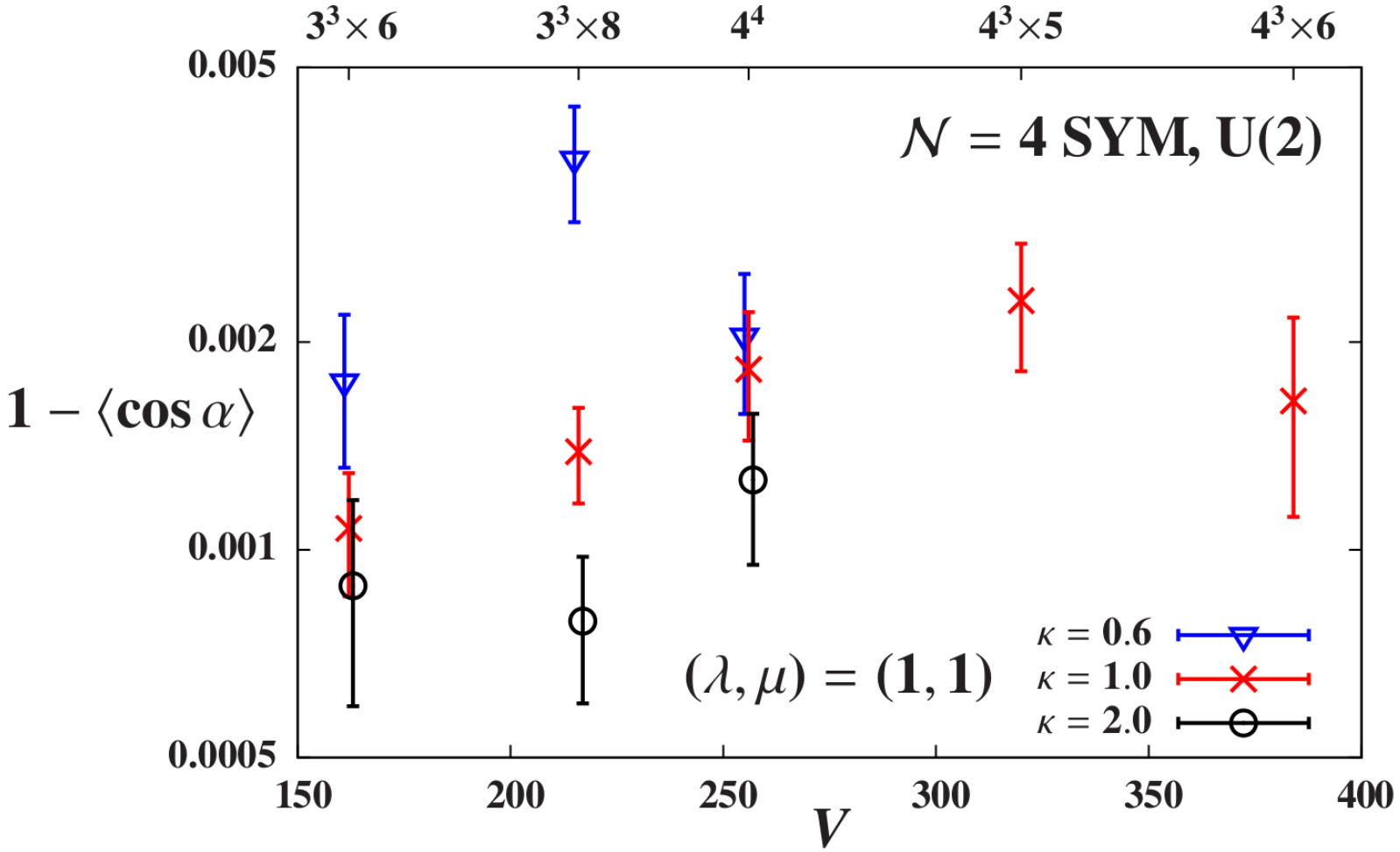
Local volume fixed to $3^3 \times 2$

Log–log axes with power fit



So far we haven't gone beyond $4^3 \times 6$ lattices for U(2) gauge group
This measurement takes ∼8 days (and ∼10GB memory) on 16 cores

Backup: Pfaffian phase for other values of $\lambda_{\rm lat}$ and κ



Fluctuations grow with $\lambda_{\rm lat}$ (not shown) but shrink with κ

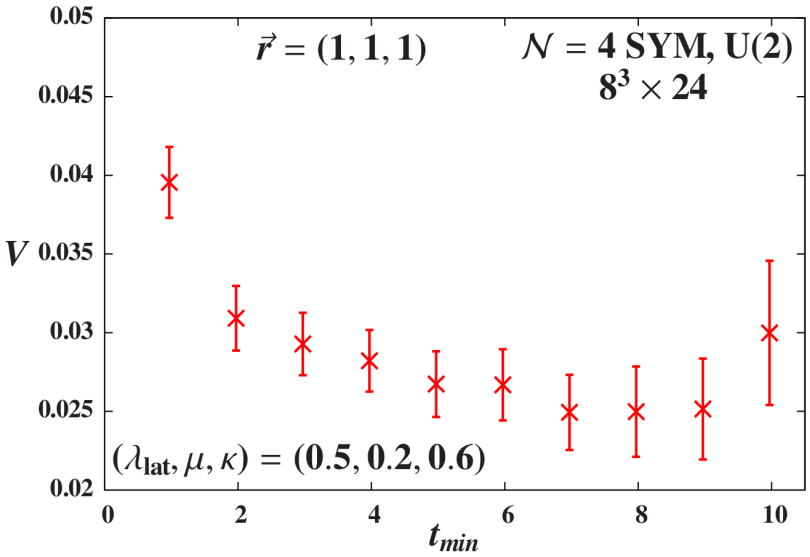
Connection to U(1) sector?

Backup: More details of static potential calculation

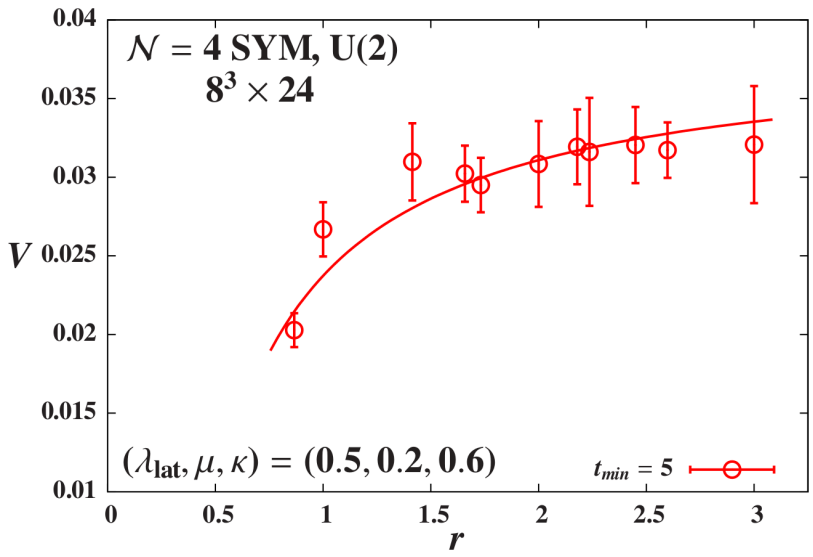
Wilson loops computed from temporal link products in Coulomb gauge

$$W(\vec{r}, T) = \frac{1}{V} \sum_{\vec{x},\ t_0} \mathrm{Tr} \left[\prod_T \mathcal{U}_t(\vec{x}, t_0 + T) \prod_T \overline{\mathcal{U}}_t(\vec{x} + \vec{r}, t_0) \right]$$

Checked against explictly-constructed on-axis loops

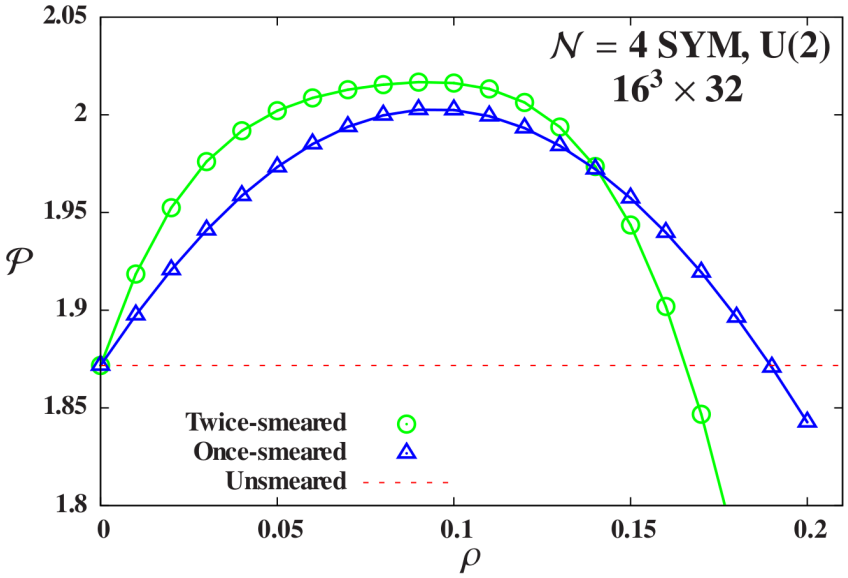


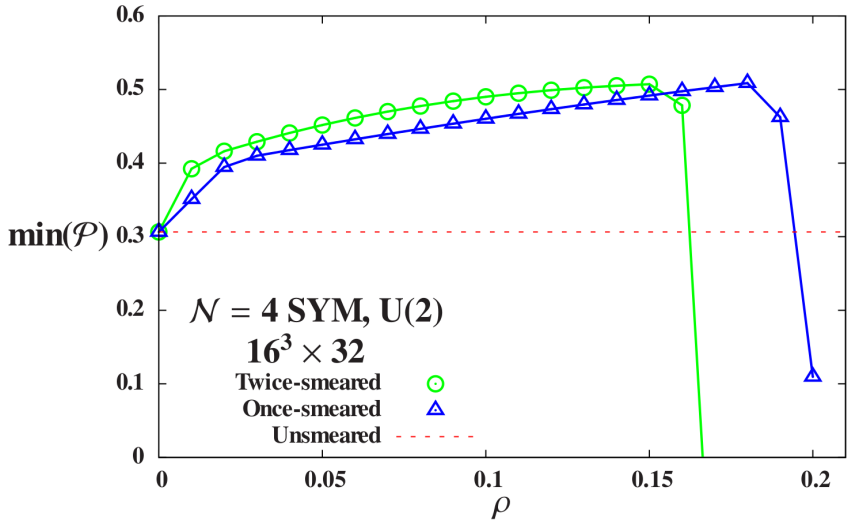
$\rightsquigarrow$



Left: Checking stability of fits to $W(\vec{r}, t) \propto \exp\left[-V(\vec{r})t\right]$

Backup: Stout smearing implemented on A_4^* lattice

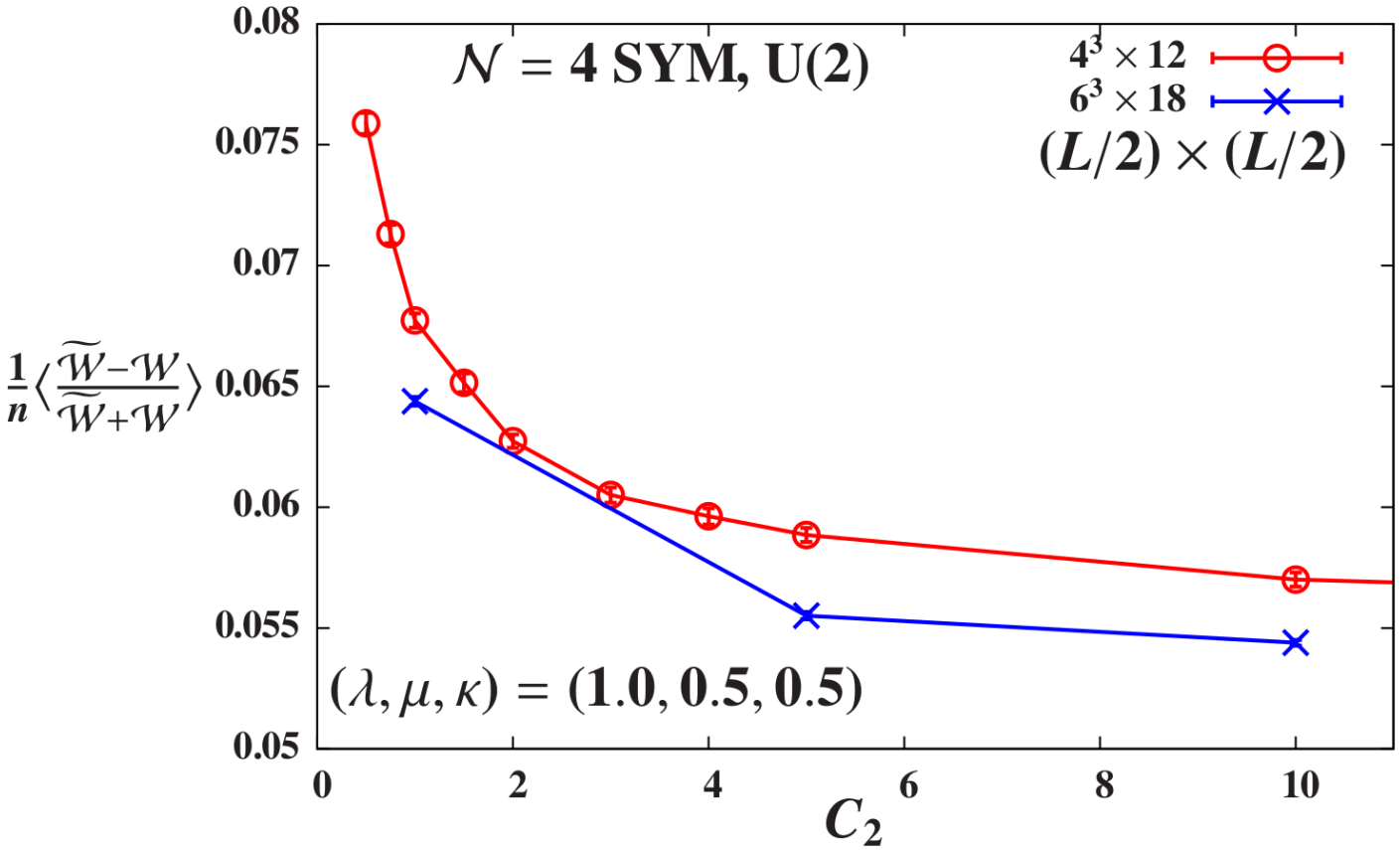




Average **(left)** and minimum **(right)** plaquette
as function of stout smearing parameter ρ on U(2) $16^3 \times 32$ lattice

Backup: R symmetry breaking vs. C_2 in gauge action

$$S = \frac{N}{\lambda_{\rm lat}} \sum_x \left[-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{C_2}{2}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a\right)^2 - \cdots \right]$$

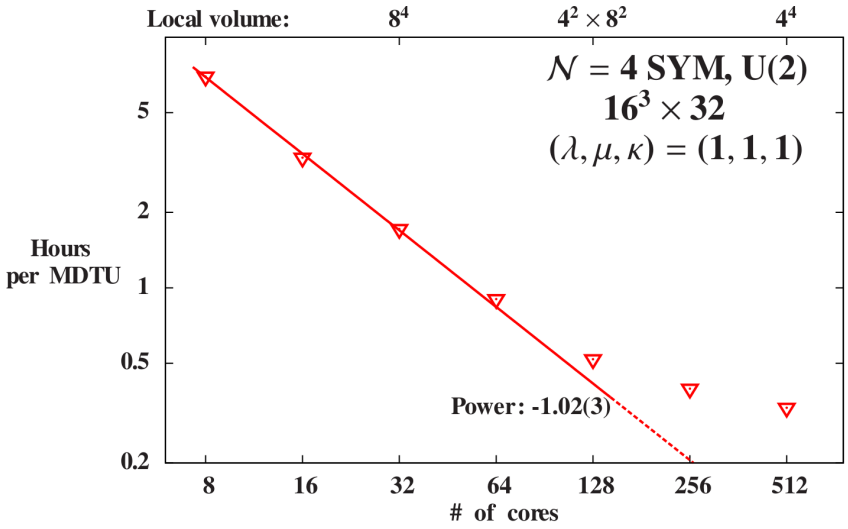


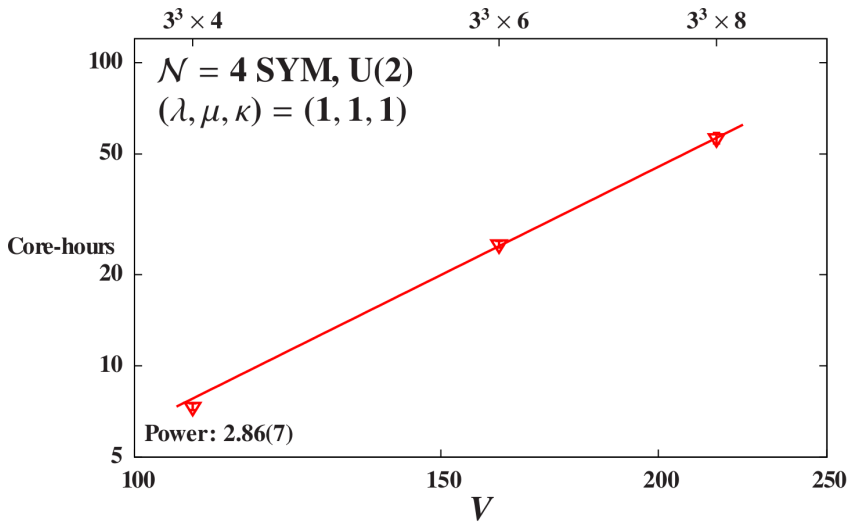
Want to tune C_2 to minimize breaking, then take continuum limit
Relatively large $C_2 \sim 10$ preferred on small volumes $L = 4$ and 6

Backup: $\mathcal{N} = 4$ SYM code performance at Fermilab

Left: Strong scaling for U(2) $16^3 \times 32$ RHMC gauge generation

Right: Weak scaling for $\mathcal{O}(N_\psi^3)$ pfaffian calculation
with local volume fixed to $3^3 \times 2$ sites per core





Both plots on log–log axes with power-law fits