Bulk and finite-temperature transitions in SU(3) gauge theories with many light fermions

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Motivation for studying finite-temperature transitions

Hope for contrast between confining vs. IR-conformal systems

Previous work: Groningen–INFN; Lattice Higgs Collaboration

→ Explore with action that showed $N_F = 12$ IR fixed point via MCRG

(For more on MCRG: Greg Petropoulos, Thursday 14:30)
Motivation for studying finite-temperature transitions

- nHYP smeared action with adjoint plaquette term $\beta_A = -0.25 \beta_F$
- $N_F = 8$ and 12 staggered fermions in fundamental rep.
- $T > 0$ volumes up to $40^3 \times 20$, as well $T = 0$ up to $32^3 \times 64$
  (large-volume runs still in progress $\rightarrow$ results preliminary)
For $N_F = 12$ we observe two bulk transitions

 Strange behavior in $\langle \bar{\psi} \psi \rangle$ observed using three staggered actions

Our observations (may not yet be consensus)

- Large jump in $\langle \bar{\psi} \psi \rangle$ at stronger coupling
- Large jump in RG-blocked Polyakov loop at weaker coupling

Suggest a confined but chirally symmetric phase

Study through low-lying eigenvalues, and new order parameters... (and spectrum and static potential, omitted from this talk)
Eigenvalue density $\rho(\lambda)$ in intermediate phase

Good observable for exploring chiral properties

$$\langle \bar{\psi}\psi \rangle \propto m \int \frac{\rho(\lambda)d\lambda}{\lambda^2 + m^2}$$

$$\omega \equiv \chi_P - \chi_S = 4m^2 \int \frac{\rho(\lambda)d\lambda}{(\lambda^2 + m^2)^2}$$

“Soft edge”: $\lim_{V \to \infty} \rho(\lambda) \propto (\lambda - \lambda_0)^\alpha$ with $\lambda_0 > 0$ ($\alpha \approx 0.5$)

Gap in infinite-volume extrapolation of eigenvalue density

$$\implies \langle \bar{\psi}\psi \rangle = 0 \text{ and } \chi_S = \chi_P \text{ (axial } U(1)_A \text{ restored) in chiral limit}$$

(For more on eigenvalues at weak coupling: A. Hasenfratz, Tuesday 14:50)
Novel behavior: single-site shift symmetry breaking

Order parameters: differences of plaquettes $\Box$ or links $\overline{\chi} U \chi$

$$\Delta P_\mu = \langle \text{ReTr} \, \Box_{n,\mu} - \text{ReTr} \, \Box_{n+\mu,\mu} \rangle_{n\mu} \text{ even}$$

$$\Delta U_\mu = \langle \alpha_{\mu,n} \overline{\chi}_n U_{\mu,n} \chi_{n+\mu}$$

$$- \alpha_{\mu,n+\mu} \overline{\chi}_{n+\mu} U_{\mu,n+\mu} \chi_{n+2\mu} \rangle_{n\mu} \text{ even}$$

Single-site shift symmetry of staggered action spontaneously broken in intermediate phase (“$S^4$”)

![Graphs showing $\Delta P_t$ and $\Delta U_t$](image)
Consequences of shift symmetry breaking

Observables alternate between slices

Breaking can develop in one or more directions
Breaking can change direction(s) during HMC evolution

Present in plaquette \(\rightarrow\) feature of gauge configurations themselves
Implications of $S^4$ lattice phase

$S^4$ phase seems to have no continuum limit

- Confining but chirally symmetric (forbidden by anomaly matching)
- Bounded by first-order bulk transitions (merge as $m$ increases)
- Observed for both $N_F = 8$ and $12$
  (Potential Aoki-like phase? Relation to staggered taste breaking?)

How do finite-temperature transitions behave around bulk transitions?
Finite-temperature transitions around the $S^4$ phase

$N_F = 12$, $m = 0.01$, $24^3 \times 12$ and $32^3 \times 16$

$S^4$ order parameters fall to zero at the same time as the RG-blocked Polyakov loop becomes large (compared to $N_F = 2+1$ deconfinement transition)

$\implies$ At $m = 0.01$, move from $S^4$ phase into deconfined phase
Finite-temperature transitions around the $S^4$ phase

$N_F = 8, \quad m = 0.01, \quad 24^3 \times 12$ and $32^3 \times 16$

Rise in RG-blocked Polyakov loop depends on $N_T$, not simultaneous with fall in $S^4$ order parameters

$\implies$ At $m = 0.01$, move from $S^4$ phase into confined phase;
Deconfinement transition moves with $N_T$

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Graphs showing $N_F = 8$ and $N_F = 12$ with various parameters.
Eigenvalue density $\rho(\lambda)$ around the $S^4$ phase

$N_F = 8$ and $12, \quad m = 0.01, \quad 32^3 \times 16$

- Soft edge appears both in $S^4$ phase and at high temperature
- For $N_F = 12$, we move straight from $S^4$ phase to chiral symmetry
- For $N_F = 8$, we observe a chirally broken phase in between the $S^4$ phase and chiral symmetry restoration

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$N_F = 8$

$N_F = 12$
Resulting $N_F = 8$ and $12$ phase diagrams

$N_F = 8$

$N_F = 12$

$T > 0$ transitions pass through $S^4$ bulk transition
$N_T = 12$, $16$ move to weaker $\beta$

$T > 0$ transitions congregate at $S^4$ bulk transition
$N_T = 12$, $16$ indistinguishable

Finite-temperature analysis appears feasible despite novel $S^4$ phase
Outlook

Complementing finite-temperature studies with MCRG, Dirac eigenvalues and meson spectrum analyses

Eigenvalues: Anna Hasenfratz, Tuesday 14:50
MCRG: Greg Petropoulos, Thursday 14:30
Thank you!
Thank you!

Collaborators
Anqi Cheng, Anna Hasenfratz, Greg Petropolous

Funding and computing resources

[Logos of the collaborating institutions]
We add a negative adjoint plaquette term to the gauge action \((\beta_A = -0.25\beta_F)\) to avoid a well-known spurious UV fixed point.
RG-blocked observables enhance signals over noise

Simply the usual observables measured on RG-blocked configurations
Can be thought of as extended observables on original lattices,
improved to remove UV fluctuations

**Example below:** Polyakov loop for $N_F = 12$, $\beta_F = 2.7$, $m = 0.01$

Note different volumes permit different numbers of blocking steps
Backup: Blocked Polyakov loop

**RG-blocked observables preserve existing signals**

Simply the usual observables measured on RG-blocked configurations
Can be thought of as extended observables on original lattices,
improved to remove UV fluctuations

**RG-blocked Polyakov loop can still indicate transition**

\[ N_F = 8 \]

\[ N_F = 12 \]

As for other observables, qualitative difference between \( N_F = 8 \) and 12
Backup: Blocked Polyakov loop

RG-blocked observables checked for $N_F = 2+1$

Simply the usual observables measured on RG-blocked configurations
Can be thought of as extended observables on original lattices, improved to remove UV fluctuations

Behave as expected for finite-temperature lattice QCD

Thanks!

Tested on $48^3 \times 12$ configurations provided by HotQCD Collaboration
Backup: $S^4$ phase is confining but chirally symmetric

**Confinement:**
- RG-blocked Polyakov loop is small
- Potential has clear linear term, small Sommer parameter $r_0 \approx 3$

**Chiral symmetry:**
- Meson spectrum is parity-doubled and volume-independent
- Dirac eigenvalue distribution has “soft edge” $\lambda_0 = 0.0175(5)$
Backup: parity doubling in the $S^4$ meson spectrum

In the $S^4$ phase, meson spectrum is parity-doubled and volume-independent. Goldstone pion possesses a scalar parity partner “$a_5$” (forbidden in QCD-like systems).

**Scalars and pseudoscalars**

**Vector and axial**
Backup: Volume scaling of Dirac eigenvalues

$S^4$ phase soft edge visible in eigenvalues themselves (left)
Contrast with weak-coupling phase (right)

\[
\lim_{V \to \infty} \rho(\lambda) \propto (\lambda - \lambda_0)^\alpha
\]

\[
\lambda_0 = 0.0175(5) > 0 \text{ is soft edge}
\]
Backup: Cartoon of phase diagram including $S^4$ phase

Staggered single-site shift symmetry:

\[
\begin{align*}
\chi(n) &\rightarrow \xi_\mu(n)\chi(n + \mu) \\
\bar{\chi}(n) &\rightarrow \xi_\mu(n)\bar{\chi}(n + \mu)
\end{align*}
\]

\[
\xi_\mu(n) \equiv (-1)^{\sum_{\nu > \mu} n_\nu}
\]

\[
U_\mu(n) \rightarrow U_\mu(n + \mu)
\]