Extremely supersymmetric lattice gauge theory

David Schaich (Syracuse University)

arXiv:1405.0644 with Simon Catterall, Poul Damgaard, Tom DeGrand, Joel Giedt

Maximally ($\mathcal{N}=4$) supersymmetric Yang–Mills theory on the lattice

Superconformal $\mathcal{N}=4$ SYM is cornerstone of AdS/CFT duality, & admits a natural lattice formulation

Field content: Gauge field A_μ, four Majorana fermions Ψ and six scalars Φ all in adjoint rep.

Lattice formulation: Gauge & scalar fields in **five** complex links $\mathcal{U}_a \in \mathfrak{gl}(N,\mathbb{C})$ with field strength $\mathcal{F}_{ab}$

Fermion field components grouped into singlet η, vector ψ_a and anti-symmetric tensor χ_{ab}

$$S = \frac{N}{\lambda_{\text{lat}}}\sum_x \left[-\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} + \frac{1}{2}\left(\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a\right)^2 - \chi_{ab}\mathcal{D}_{[a}^{(+)}\psi_{b]} - \eta\overline{\mathcal{D}}_a^{(-)}\psi_a - \frac{1}{4}\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}\right]$$

$$+\mu^2\sum_{x,\ a}\left(\frac{1}{N}\text{Tr}\left[\overline{\mathcal{U}}_a\mathcal{U}_a\right]-1\right)^2 + \kappa\sum_{\mathcal{P}}\left|\det\mathcal{P}-1\right|^2 \qquad (\mathcal{P}\text{ is plaquette})$$

—First line exactly preserves a single supersymmetry $\mathcal{Q}$ (other 15 broken) $\longrightarrow$ practical lattice susy

—μ term regulates flat directions, stabilizes continuum limit, acts like bosonic mass

—κ term approximately reduces U(N) $\longrightarrow$ SU(N), suppressing U(1) confinement lattice phase

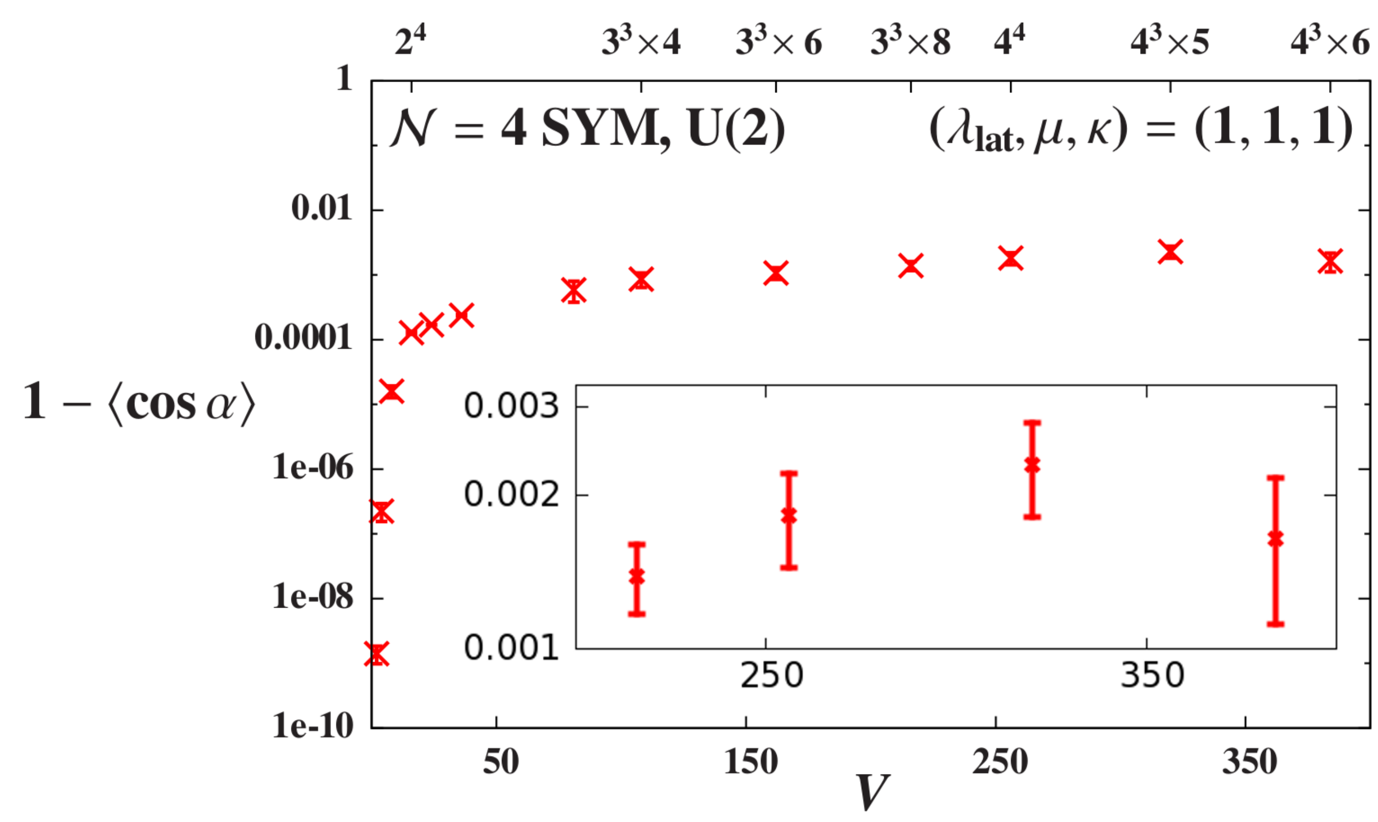
Complex pfaffian $P = |P|e^{i\alpha} \longrightarrow$ potential sign problem in numerical simulations

Our calculations are all "phase-quenched":
Omit $e^{i\alpha}$ in RHMC, measure P on saved configs

With new parallel software (github.com/daschaich/susy)
$4^3\times6$ measurement takes ~8 days, ~10GB memory

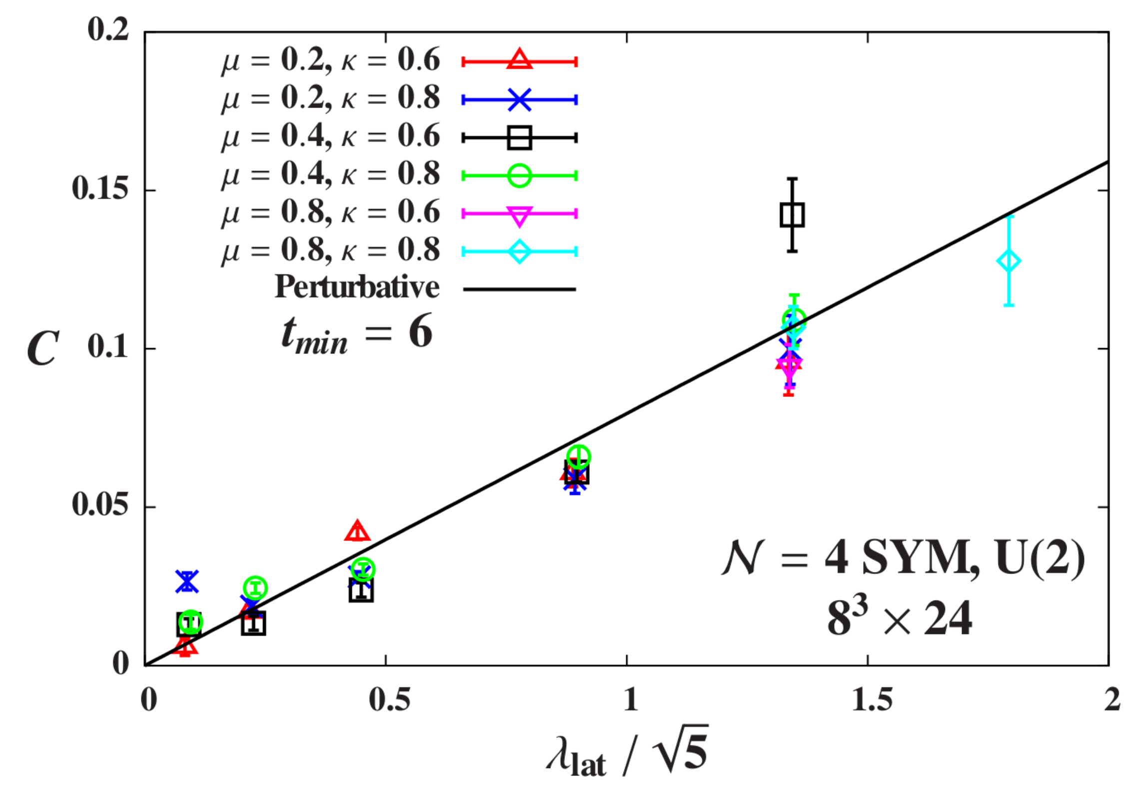
P is nearly real and positive $\longrightarrow$ no sign problem?
Fluctuations don't grow with lattice volume or N

$V=32$	U(2)	U(3)	U(4)
$\langle\cos\alpha\rangle$	0.99978(4)	−0.99980(3)	0.99989(4)



Coulombic static potential $V(r) = A - C/r$

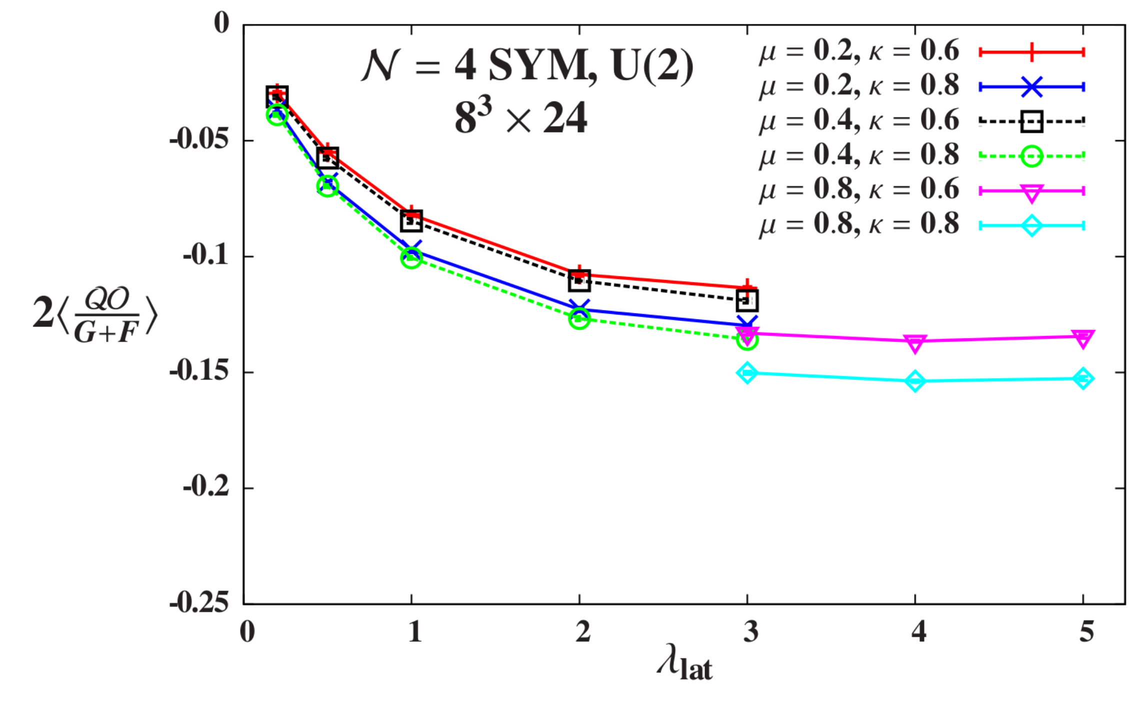
Agreement with perturbative $C = \lambda_{\text{lat}}/(4\pi\sqrt{5})$



Supersymmetry breaking from μ and κ

—Exact $\mathcal{Q} \Longrightarrow$ Ward identity $\langle\mathcal{Q}\mathcal{O}\rangle = 0$

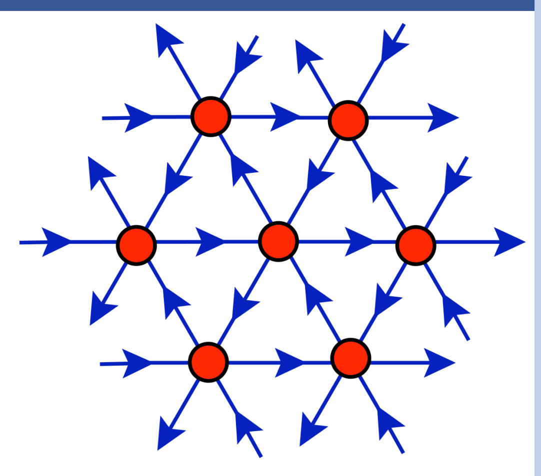
—Ward identity violations from non-zero μ, κ suggest $\mathcal{O}(10\%)$ supersymmetry breaking



Details of discretization on A_4^* lattice

5 links symmetrically span 4d
Analog of 2d triangular lattice

Non-orthogonal links
$\Longrightarrow$ continuum $\lambda = \lambda_{\text{lat}}/\sqrt{5}$



A_4^* lattice has S_5 point group symmetry
S_5 irreducible representations of lattice fields
$\longrightarrow$ continuum SO(4) euclidean Lorentz irreps.

$$\mathcal{U}_a = \mathbf{4}\oplus\mathbf{1} \longrightarrow U_\mu, \Phi$$
$$\psi_a = \mathbf{4}\oplus\mathbf{1} \longrightarrow \psi_\mu, \overline{\eta}$$
$$\chi_{ab} = \mathbf{6}\oplus\mathbf{4} \longrightarrow \chi_{\mu\nu}, \overline{\psi}_\mu$$

Towards the large-N limit

—Important for contact with continuum theory

—Challenge: computational costs grow $\propto N^5$

—Benefit: supersymmetry breaking $\propto 1/N^2$