Lattice fields and lattice action: exact supersymmetry vs. stable numerical calculations

— All fields transform in the adjoint representation of gauge group U(N)
— Gauge & scalar fields combined into five complexified links $U_a$ with field strength $F_{ab}$
— Fermion field components grouped into singlet $\eta$, vector $\psi_a$ and anti-symmetric tensor $\chi_{ab}$

\[
S = \frac{N}{\lambda_{\text{lat}}} \sum_x \left[ -F_{ab}F_{ab} + \frac{1}{2} (\mathcal{D}_a \eta)_a \right]^2 - \chi_{ab} \mathcal{D}_a \left[ \psi_a b \right] - \eta \mathcal{D}_a \psi_a - \frac{1}{4} \varepsilon_{abcde} \chi_{de} \mathcal{D}_c \chi_{ab} \right] \\
+ \mu^2 \sum_{x, a} \left( \frac{1}{N} \text{Tr} \left[ U_{a \dagger} U_{a - 1} \right] - 1 \right)^2 + \kappa \sum_P |\text{det} \mathcal{P} - 1|^2 \quad (\mathcal{P} \text{ is plaquette})
\]

— First line exactly preserves a single supersymmetry $Q$, other 15 broken
— $\mu$ term regulates flat directions, stabilizes continuum limit, acts like bosonic mass
— $\kappa$ term approximately reduces U(N) $\rightarrow$ SU(N), suppressing U(1) confinement lattice phase

Supersymmetry breaking from $\mu$ and $\kappa$:

— Exact $Q \implies$ Ward identity $\langle QO \rangle = 0$
— Ward identity violations from non-zero $\mu, \kappa$ suggest $O(10\%)$ supersymmetry breaking

Discretization on $A_4^*$ lattice

5 links symmetrically span 4d
Analog of 2d triangular lattice

Non-orthogonal links

$\implies$ continuum $\lambda = \lambda_{\text{lat}} / \sqrt{5}$

$A_4^*$ lattice has $S_5$ point group symmetry
$S_5$ irreducible representations of lattice fields

$\implies$ continuum $SO(4)$ euclidean Lorentz irreps.

$U_a = 4 \oplus 1 \implies U_\mu, \Phi$
$\psi_a = 4 \oplus 1 \implies \psi_\mu, \bar{\eta}$
$\chi_{ab} = 6 \oplus 4 \implies \chi_{\mu \nu}, \psi_\mu$

Towards the large-$N$ limit

— Important for contact with continuum theory
— Challenge: computational costs grow $\propto N^5$
— Benefit: supersymmetry breaking $\propto 1/N^2$

Is there a sign problem?

— Complex pfaffian $P = |P| e^{i\alpha}$ from fermions
— Our “phase-quenched” calculations ignore $e^{i\alpha}$
— We measure $P$ to be nearly real and positive $\implies 1 - \langle \cos \alpha \rangle \ll 1$
— Fluctuations aren’t growing with volume

Static potential is coulombic at both weak and strong coupling

$V(r) = A - \frac{C}{r} \quad \rightarrow$ Coulomb coefficients in agreement with perturbation theory, $C = \lambda_{\text{lat}}/(4\pi \sqrt{5})$