# *S* parameter and parity doubling below the conformal window

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for the Lattice Strong Dynamics Collaboration

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# Lattice Strong Dynamics Collaboration

| | |
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| Brookhaven | Oliver Witzel |
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| Harvard | Mike Clark |
| Livermore | Mike Buchoff, Michael Cheng, Pavlos Vranas, Joe Wasem |
| UC Davis | Joseph Kiskis |
| U Wash. | Saul Cohen |
| Yale | Thomas Appelquist, George Fleming, Meifeng Lin, Gennady Voronov |

Performing non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider

# The $S$ parameter: Motivation and Definition

Constrain the physics of electroweak symmetry breaking (EWSB)
from its effects on vacuum polarizations $\Pi(Q)$ of EW gauge bosons


(Peskin and Takeuchi, 1991)

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

1. $\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$

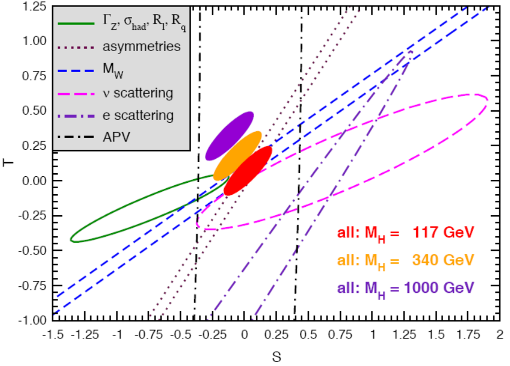
   (more details later)

2. $N_D \geq 1$ is the number of doublets with chiral electroweak couplings
3. $\Delta S_{SM}$ is subtracted so that $S = 0$ in the standard model

# The $S$ parameter: Experiment and QCD

Experimentally $S \lesssim 0$

For a reference Higgs mass scale of 1 TeV
$S \approx -0.15(10)$




(PDG)

Recall technicolor: electroweak symmetry breaking results from chiral symmetry breaking in a new strongly-interacting sector

QCD-based scaling: $S \simeq 0.3 \frac{N_f}{2} \frac{N_c}{3} + \frac{1}{12\pi} \left( \frac{N_f^2}{4} - 1 \right) \log \left( \frac{M_{\rho_T}^2}{M_{\pi_T}^2} \right)$

Recall "walking": realistic models must have non-QCD dynamics

Need lattice calculations!

# LSD Philosophy and Simulation Details

- Use QCD as a baseline, explore changes as $N_f$ increases
  $\longrightarrow SU(3)$ gauge theory with $N_f =$2, 6, 10
- Use domain wall fermions with Iwasaki gauge action
  for good chiral and flavor symmetries
  $L_s = 16 \longrightarrow m_{res} \approx 3\times10^{-5}$ (2f); $8\times10^{-4}$ (6f); $2\times10^{-3}$ (10f)
- Tune $\beta$ to hold IR scale(s) fixed in the chiral limit
  $M_\rho \approx 0.2 \longrightarrow \beta =$2.7 (2f); 2.1 (6f); 1.95 (10f)
- $M_P L > 4 \longrightarrow 32^3\times64$ lattices; $0.005 \leq m_f \leq 0.03$
- Exploratory calculations $\longrightarrow\sim$1000 trajectories per run

Unpublished results in this talk are PRELIMINARY

Other LSD talks: M. Lin, Mon.; G. Voronov and M. Buchoff, this session

# Currents and correlators

$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

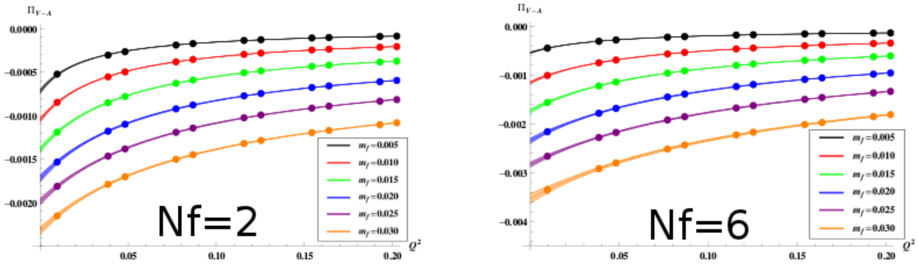

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

$$\Pi^{\mu\nu}(Q) = \left(\delta^{\mu\nu} - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\right)\Pi(Q^2) - \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\Pi^L(Q^2) \qquad \widehat{Q} = 2\sin(Q/2)$$

- Conserved currents $\mathcal{V}$ and $\mathcal{A}$ ensure that lattice artifacts cancel (only need one conserved current in each correlator)
- Renormalization constant $Z$ computed non-perturbatively
  $Z =$ 0.85 (2f); 0.73 (6f); 0.71 (10f)

# Polarization function data and fits, $N_f = 2$ and $N_f = 6$

$$S = 4\pi N_D \lim_{Q^2\to 0} \frac{d}{dQ^2}\Pi_{V-A}(Q^2) - \Delta S_{SM}$$




Very smooth data ⇒ extract slope at $Q^2 = 0$

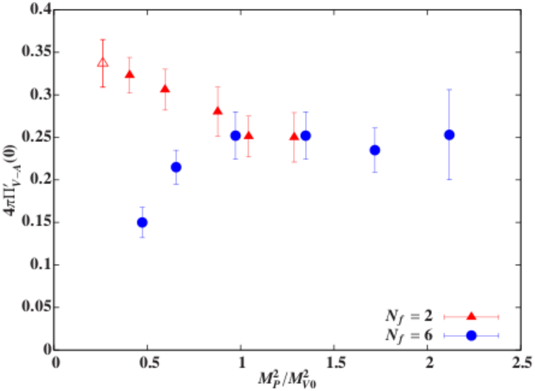
from fit to Padé-(1, 2) functional form

$$\Pi_{V-A}(Q^2) = \frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \frac{\sum_{m=0}^{1} a_m Q^{2m}}{\sum_{n=0}^{2} b_n Q^{2n}}$$

(similar to single-pole-dominance approximation)

Results stable and $\chi^2/dof \ll 1$ as $Q^2$ fit range varies

# Fit results for $\Pi'_{V-A}(0)$, $N_f = 2$ and $N_f = 6$




Vertical axis: $4\pi\Pi'_{V-A}(0)$

where

$$\Pi'(0) = \lim_{Q^2\to 0} \frac{d}{dQ^2}\Pi(Q^2)$$

$$S = 4\pi N_D \Pi'_{V-A}(0) - \Delta S_{SM}$$

Horizontal axis: $M_P^2/M_{V0}^2$ gives a more physical comparison than $m_f$
$M_{V0} \equiv \lim_{m_f\to 0} M_V$ is matched between $N_f = 2$ and $N_f = 6$

Expect agreement in the quenched limit $M_P^2 \to \infty$

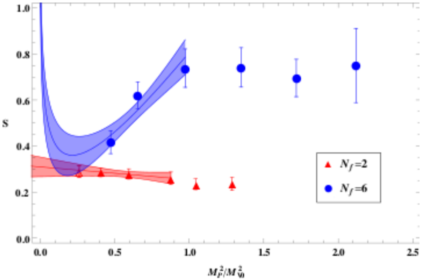
# The last steps from slope to $S$

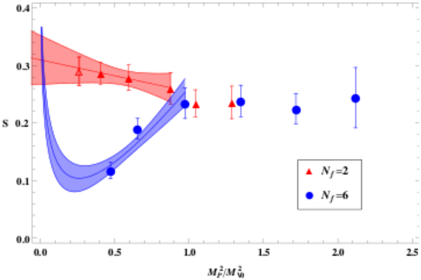
$$S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}$$

1. $N_D$ doublets with chiral electroweak couplings contribute to $S$
   Usually consider worst-case scenario $N_D = N_f/2$
   but only $N_D \geq 1$ is required for electroweak symmetry breaking

2. $$\Delta S_{SM} = \frac{1}{4} \int_{4M_P^2}^{\infty} \frac{ds}{s} \left[ 1 - \left( 1 - \frac{M_{V0}^2}{s} \right)^3 \Theta(s - M_{V0}^2) \right]$$
   No direct dependence on $N_f$ or $N_D$
   Diverges logarithmically in the limit $M_P^2 \to 0$
   to cancel contribution of three “eaten” modes
   Small ($\lesssim 10\%$) reduction in this work ($M_P^2 > 0$)

# $S$ parameter, $N_f = 2$ and $N_f = 6$




Maximum $N_D = N_f/2$
$C = N_f^2 - 1 - 3$

Minimum $N_D = 1$
$C = 4N_f - 5 - 3$

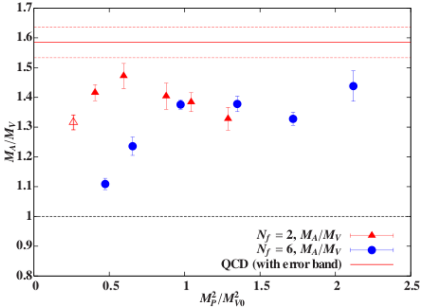
Linear fit to light points guides the eye,
accounts for known $M_P^2 \to 0$ log divergence remaining after $\Delta S_{SM}$

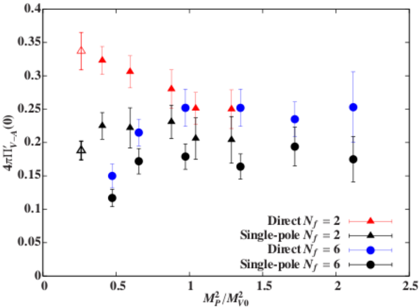
$$S = A + BM_P^2 + \frac{C}{48\pi}\log\left(\frac{M_{V0}^2}{M_P^2}\right) \longrightarrow \lim_{M_P^2 \to 0} S = 0.31(4) \text{ for } N_f = 2$$

# Connection to parity-doubling of lightest states

$$4\pi\Pi'_{V-A}(0) = \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}[R_V(s) - R_A(s)] \longrightarrow 4\pi\left[\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right]$$

$$\text{for } R(s) \longrightarrow 12\pi F^2\delta(s - M^2)$$



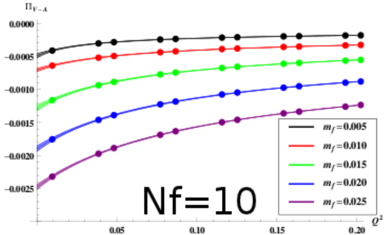




Signs of $N_f = 6$ parity-doubling consistent with reduced $S$
Direct checks of finite-volume effects underway

# Preliminary results for $N_f = 10$

Following the same procedure described above

(but not claiming $N_f = 10$ is below the conformal window)

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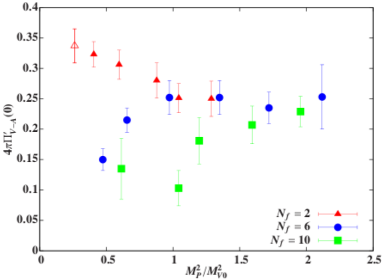
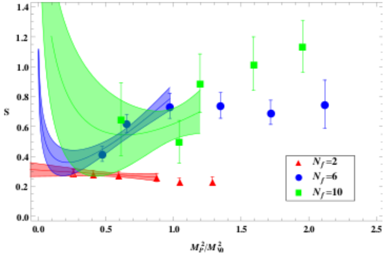
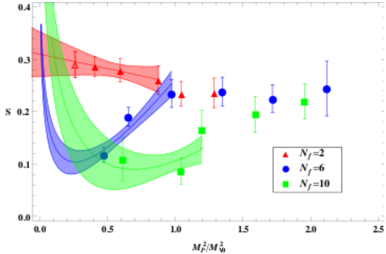


# Conclusion and next steps

## Conclusions

- Reduction in $S$ parameter for $N_f = 6$ and $N_f = 10$
  compared to QCD-based expectations
- Discrepancy with experiment as small as $\sim 2\sigma$

## Ongoing work

- $N_f = 10$ runs still in progress,
  as are $m_f = 0.0075$ runs for $N_f = 2$ and $N_f = 6$
- Directly checking finite-volume effects
- Testing twisted boundary conditions for better low-$Q^2$ resolution

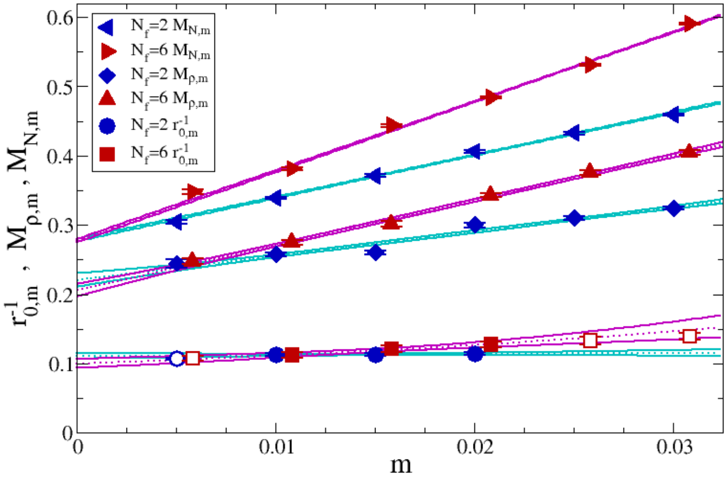
The $S$ parameter is a crucial observable to measure
in lattice studies of new strong dynamics

# Thank you!

## Funding and computing resources

USQCD

# Backup: matching IR scales in the chiral limit




Vector mass, nucleon mass, and inverse Sommer scale

all match at 10% level between $N_f = 2$ and $N_f = 6$

$M_{V0} = 0.215(3)$ [2f]; 0.209(3) [6f]; 0.192(10) [10f, not shown]

# Backup: Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^{\mu a}(x) = \sum_{s=0}^{L_s-1} j^{\mu a}(x,s) \qquad \mathcal{A}^{\mu a}(x) = \sum_{s=0}^{L_s-1} \text{sign}\left(s - \frac{L_s-1}{2}\right) j^{\mu a}(x,s)$$

$$\begin{aligned} j^{\mu a}(x,s) = {} & \overline{\Psi}(x+\widehat{\mu},s)\frac{1+\gamma^\mu}{2}\tau^a U^\dagger_{x,\mu}\Psi(x,s) \\ & -\overline{\Psi}(x,s)\frac{1-\gamma^\mu}{2}\tau^a U_{x,\mu}\Psi(x+\widehat{\mu},s) \end{aligned}$$

Local currents:

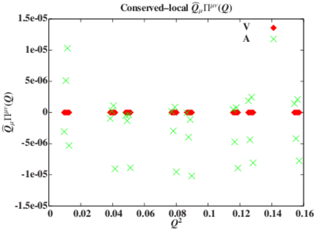
$$V^\mu(x) = \overline{q}(x)\gamma^\mu\tau^a q(x) \qquad A^\mu(x) = \overline{q}(x)\gamma^\mu\gamma^5\tau^a q(x)$$

$$q(x) = P_L\Psi(x,0) + P_R\Psi(x,L_s-1)$$

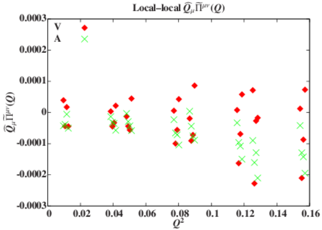
Normalization: $\text{Tr}\left[T^a T^b\right] = \frac{\delta^{ab}}{2}$

# Backup: Ward identities and violations

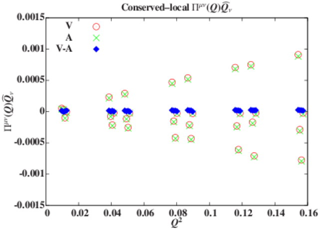
$$\widehat{Q}_\mu \left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x) V^a_\nu(0) \right\rangle\right] = 0$$



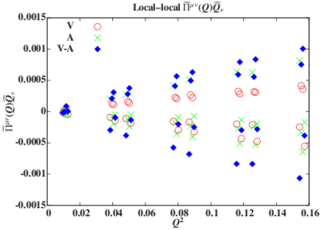

$$\widehat{Q}_\mu \left[\sum_x e^{iQ\cdot x} \left\langle V^a_\mu(x) V^a_\nu(0) \right\rangle\right] \neq 0$$



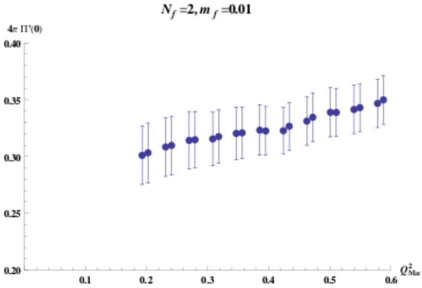

$$\left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left(\left\langle \mathcal{V}^a_\mu V^a_\nu \right\rangle - \left\langle \mathcal{A}^a_\mu A^a_\nu \right\rangle\right)\right] \widehat{Q}_\nu \approx 0$$

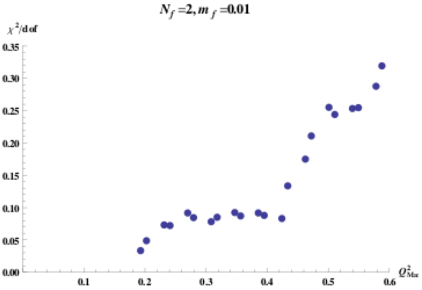



$$\left[\sum_x e^{iQ\cdot x} \left(\left\langle V^a_\mu V^a_\nu \right\rangle - \left\langle A^a_\mu A^a_\nu \right\rangle\right)\right] \widehat{Q}_\nu \neq 0$$



# Backup: Padé fit $Q^2$-range dependence




Results reported here used $Q_{Max} = 0.4$

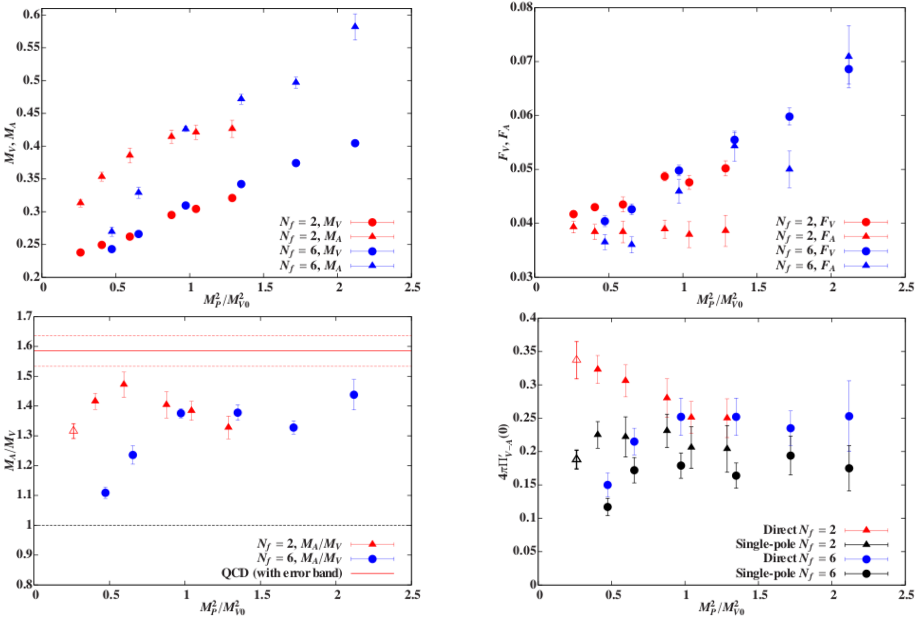
# Backup: LSD ensembles and measurements of $S$

Number of measurements after applying thermalization cuts:

| | $N_f = 2$ | | $N_f = 6$ | | $N_f = 10$ | |
|---|---|---|---|---|---|---|
| $m_f$ | $M_P L$ | $N_{meas}$ | $M_P L$ | $N_{meas}$ | $M_P L$ | $N_{meas}$ |
| 0.0050 | (3.5) | (900) | 4.6 | 538 | 4.8 | 130 |
| 0.0075 | (4.4) | (126) | (5.3) | (172) | – | – |
| 0.0100 | 4.4 | 920 | 5.4 | 482 | 6.3 | 104 |
| 0.0150 | 5.3 | 410 | 6.6 | 218 | 6.7 | 116 |
| 0.0200 | 6.4 | 286 | 7.8 | 158 | 7.8 | 180 |
| 0.0250 | 7.0 | 250 | 8.8 | 170 | 8.6 | 184 |
| 0.0300 | 7.8 | 158 | 9.7 | 104 | (8.9) | (114) |

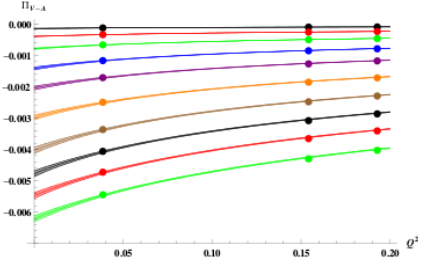
Ensembles in red still evolving; thermalization cuts are preliminary
Ensembles in (parentheses) not included in analyses reported here

# Backup: More parity-doubling plots

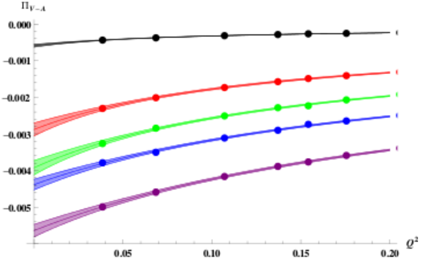


# Backup: $\Pi_{V-A}(Q^2)$ on smaller volumes, $N_f = 6$

PRELIMINARY




$16^3 \times 32$




$24^3 \times 32$

- Slope at $Q^2 = 0$ decreases for smaller $m_f$, just as on $32^3 \times 64$
- Initial indication that finite-volume effects are not significant
- Smaller volumes will be first application of twisted BCs