Exploring a new lattice phase

- **Review** lattice parameter space; "spurious " transitions from lattice artifacts
- **Origin:** Two "jumps" observed in $N_f = 12$ $\langle\overline{\psi}\psi\rangle$ when only one expected

  Deuzeman, Lombardo & Pallante in 2010

  Lattice Higgs Collaboration; Cheng, Hasenfratz & Schaich in 2011
- **Outline:** Observations vs. interpretations
  - Properties and identification
  - Order of transitions
  - N_f dependence
  - Connection to new relevant operator?
  - Relation to improved lattice action?
- **Question:** Do we care?
  - May be strange lattice artifact with no information about continuum
  - May be present only for theories that are IR conformal in the continuum
  - More modest approach is to investigate transitions (e.g., scaling with N_t) without worrying about what they transition between

  $\longrightarrow$ Discussion of "methods" next week?

Properties and identification

- **Chiral symmetry:** $\langle\overline{\psi}\psi\rangle = 0$ and $\Sigma = m\int_0^\infty \frac{\rho(\lambda)d\lambda}{\lambda^2+m^2} = 0$

  since $\rho(\lambda)$ has a "soft edge" with $\rho(\lambda) \propto \sqrt{\lambda - \lambda_0}$

  $\Longrightarrow$ $\mathrm{U}(1)_A$ is also restored since $\chi_P - \chi_S = 4m^2\int_0^\infty \frac{\rho(\lambda)d\lambda}{(\lambda^2+m^2)^2} = 0$
- **Meson spectrum:** parity partners degenerate; forbidden partners appear
- **Single-site shift symmetry** is exact in staggered action:

  $$\chi(n) \to \xi_\mu(n)\chi(n+\mu), \quad \overline{\chi}(n) \to \xi_\mu(n)\overline{\chi}(n+\mu), \quad U_\mu(n) \to U_\mu(n+\mu),$$

  with $\xi_\mu \equiv (-1)^{\sum_{\nu>\mu} n_\nu}$. Spontaneous breaking shown by order parameters

  $\Delta\Box_\mu = \langle \mathrm{ReTr}\,\Box_n - \mathrm{ReTr}\,\Box_{n+\mu}\rangle_{n_\mu\ \mathrm{even}}$ (note planar plaquettes)

  $\Delta L_\mu = \left\langle \alpha_{\mu,n}\overline{\chi}_n U_{\mu,n}\chi_{n+\mu} - \alpha_{\mu,n+\mu}\overline{\chi}_{n+\mu}U_{\mu,n+\mu}\chi_{n+2\mu}\right\rangle_{n_\mu\ \mathrm{even}}$ $(\alpha_\mu \equiv (-1)^{\sum_{\nu<\mu} n_\nu})$

  Complication is that only some may be broken
- **Polyakov loop** is zero $\Longrightarrow$ confined? Non-zero string tension as well...

Order of transitions

- Order parameters for spontaneous single-site shift symmetry breaking implies phase separation
- Both $\cancel{S^4}$ transition and chiral transition must therefore be first order at least until they merge for large enough mass
- Possibility of second-order critical point at or after that merger? (Same as Jin–Mawhinney point where $m_\sigma \to 0$?)

N_f dependence

- $\beta_{\cancel{S^4}} \approx 4.7$ for $N_f = 8$, $\beta_{\cancel{S^4}} \approx 2.7$ for $N_f = 12$, $\beta_{\cancel{S^4}} \approx 0.5$ for $N_f = 16$
- Seems to correspond to the same physics (e.g., spectrum, γ_m) in each system; constant shift results from effect of fermions on bare coupling
- No longer present for systems without asymptotic freedom?

Connection to new relevant operator?

- Mass anomalous dimension $\gamma_m\left(\frac{1}{a}\right) \gtrsim 1$ around the scale of the lattice spacing
- Relevant four-fermion interaction? Required to obtain fixed point?

Relation to improved lattice action?

- Only observed when using **sufficiently** improved staggered lattice action
- **Two possible interpretations:**
  - Improvement introduces new operators required for the phase to appear
  - Improvement pushes chiral transition to strong enough coupling that this phase becomes accessible