Lattice strong dynamics for composite Higgs sectors

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and more to come

with the Lattice Strong Dynamics Collaboration
Overview and plan

Lattice field theory is a broadly applicable tool to study strongly coupled near-conformal theories

Composite Higgs motivation for near-conformal lattice studies

Light scalar and low-energy effective theory connection

Mass-split systems for improved control

More: S parameter, partial compositeness, grav. waves, . . .

These slides: davidschaich.net/talks/2204Liverpool.pdf

Interaction encouraged — complete coverage unnecessary
Motivation from composite Higgs sectors

**Large Hadron Collider priority**
Study fundamental nature of the Higgs

**Composite Higgs sector**
can stabilize electroweak scale

**New strong dynamics must differ from QCD**
—Flavour-changing neutral currents
—Electroweak precision observables
—SM-like Higgs boson with $M \approx 0.5 v_{EW}$
Near-conformality for composite Higgs

New strong dynamics must differ from QCD
— Flavour-changing neutral currents
— Electroweak precision observables
— SM-like Higgs boson with $M \approx 0.5 v_{EW}$

Near-conformal dynamics can help with all three issues

Near-conformality $\rightarrow$ natural scale separation, novel IR dynamics

Can’t rely on intuition from QCD or $\mathcal{N} = 4$ SYM $\rightarrow$ lattice calculations
Lattice field theory in a nutshell

Formally

\[ \langle O \rangle = \frac{1}{Z} \int D\Phi \, O(\Phi) \, e^{-S[\Phi]} \]

Regularize by formulating theory in finite, discrete, euclidean space-time

\[ \xleftarrow{\text{Gauge invariant, non-perturbative, 4-dimensional}} \]

Spacing between lattice sites ("a")

\[ \rightarrow \text{UV cutoff scale } \frac{1}{a} \]

Remove cutoff: \[ a \rightarrow 0 \quad (L/a \rightarrow \infty) \]

Hypercubic \[ \rightarrow \text{Poincaré symmetries } \checkmark \]
Numerical lattice field theory calculations

High-performance computing \(\rightarrow\) evaluate up to \(\sim\) billion-dimensional integrals
(Dirac operator as \(\sim 10^9 \times 10^9\) matrix)

Results to be shown, and work in progress, require state-of-the-art resources

Many thanks to USQCD–DOE, DiRAC–STFC–UKRI, and computing centres!

Lassen @Livermore
USQCD @JLab
DiRAC @Cambridge
**Numerical lattice field theory algorithms**

**Importance sampling Monte Carlo**

Algorithms sample field configurations with probability

\[
\langle O \rangle = \frac{1}{Z} \int D\Phi \ O(\Phi) \ e^{-S[\Phi]} \quad \rightarrow \quad \frac{1}{N} \sum_{i=1}^{N} O(\Phi_i) \quad \text{with stat. uncertainty} \quad \propto \quad \frac{1}{\sqrt{N}}
\]

David Schaich (Liverpool)
Numerical lattice field theory algorithms

Importance sampling Monte Carlo

Algorithms sample field configurations with probability $\frac{1}{Z} e^{-S[\Phi]}$

$$
\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\Phi \; \mathcal{O}(\Phi) \; e^{-S[\Phi]} \rightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i) \text{ with stat. uncertainty } \propto \frac{1}{\sqrt{N}}
$$

Lattice calculation requires specific theory $\leftrightarrow$ lattice action $S[\Phi]$

Our strategy aims to gain generic insights into near-conformal strong dynamics
Exploring the range of possible phenomena in strongly coupled field theories
Choosing near-conformal theories to analyze for generic insights

For SU(3) gauge group, observe near-conformal strong dynamics for $8 \lesssim N_F \lesssim 10$ light fundamental fermions

Intermediate between $N_F = 2$ QCD and weakly coupled Banks–Zaks IR fixed point for $N_F \simeq 16$ (massless)
From new strong dynamics to electroweak symmetry breaking

For SU(3) with $N_F$ light fundamental fermions, chiral symmetry breaking is

$$\text{SU}(N_F)_L \times \text{SU}(N_F)_R \rightarrow \text{SU}(N_F)_V$$

Lattice studies of strong sector apply to two distinct model interpretations

1) Electroweak symmetry breaks directly

$$\text{SU}(2)_L \times U(1)_Y \subset \text{SU}(N_F)_L \times \text{SU}(N_F)_R \rightarrow U(1)_{\text{em}} \subset \text{SU}(N_F)_V$$

2) Electroweak symmetry breaks radiatively via vacuum misalignment

$$\text{SU}(N_F)_V \supset \text{SU}(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$$
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Lattice studies of strong sector apply to two distinct model interpretations

1) Electroweak symmetry breaks directly

$$\Rightarrow \text{Symmetry breaking scale } F^2 = v^2_{\text{EW}} = (246 \text{ GeV})^2$$

$$\Rightarrow \text{Higgs boson as } 0^{++} \text{ isosinglet scalar (`}\sigma`')$$

2) Electroweak symmetry breaks radiatively via vacuum misalignment

$$\Rightarrow \text{Symmetry breaking scale } F^2 = v^2_{\text{EW}}/\xi \text{ with } \xi \lesssim 0.1$$

$$\Rightarrow \text{Higgs boson as pseudo-Goldstone (`}\pi`')$$

[affected by $0^{++}$]
\(N_F = 8\) light composite spectrum

Light \(0^{++}\) scalar observed, \(M_\sigma \approx M_\pi \lesssim M_\rho/2\) qualitatively different than QCD

Improved staggered fermions

Masses in units of lattice scale \(\sqrt{8t_0}\)

\[L \geq 5.3/(a \cdot M_\pi) \rightarrow \text{up to } 64^3 \times 128\]

\([96^3 \times 192 \text{ with } \sqrt{8t_0} m_f \sim 0.003 \text{ in progress}]\)

Large \(0^{++}\) uncertainties from mixing with vacuum

[novel re-analyses also in progress]
Light scalar is generic feature of near-conformal dynamics

Consistently observed by many groups considering various theories

✓ SU(3) with $N_F = 8$ fundamental

✓ SU(3) with $N_F = 12$ fundamental

arXiv:1710.08970
Light scalar is generic feature of near-conformal dynamics

Consistently observed by many groups considering various theories

✓ SU(3) with $N_F = 8$ fundamental
✓ SU(3) with $N_F = 12$ fundamental
✓ SU(3) with $N_F = 2$ sextet
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✓ SU(2) with \( N_F = 2 \) adjoint
Light scalar is generic feature of near-conformal dynamics

Consistently observed by many groups considering various theories

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✓ SU(2) with $N_F = 2$ adjoint
✓ SU(2) with $N_F = 1$ adjoint

\[ \beta = 2.05 \quad \beta = 2.1 \quad \beta = 2.15 \quad \beta = 2.2 \]

\[ 2^+ \text{ scalar baryon} \quad 2^- \text{ pseudoscalar baryon} \quad 2^- \text{ vector baryon} \quad 0^{++} \text{ scalar glueball} \]
$M_\rho/F_\pi \approx 8$ naively implies resonance $\sim 2\,\text{TeV}/\sqrt{\xi}$ — with broad $\Gamma_\rho/M_\rho \approx 0.2$

This may be too naive

1) Need $M_\pi/F_\pi \to 0$ as $m_f \to 0$

2) Need exactly 3 massless pions, other $N_F^2 - 4 = 60$ stay massive

Chiral extrapolation relies on low-energy effective field theory (EFT)
Low-energy chiral effective theories

Chiral perturbation theory ($\chi$PT) is EFT of pions familiar from QCD

Need to extend $\chi$PT to incorporate light scalar, with power-counting rules to organize interactions

Several candidate EFTs currently being explored

—Completely generic (no assumptions, many parameters)
  [Soto–Talavera–Tarrus; Hansen–Langaebel–Sannino; Catà–Müller]

—Based on linear sigma model
  [LSD, arXiv:1809.02624]

—Treating scalar as ‘dilaton’, pseudo-Goldstone of broken scale invariance
  [Matsuzaki–Yamawaki; Golterman–Shamir; Appelquist–Ingoldby–Piai]
Use lattice results to test candidate EFTs

— Completely generic (no assumptions, many parameters)
— Based on linear sigma model
— Treating scalar as ‘dilaton’, pseudo-Goldstone of broken scale invariance

Work in progress to test and compare EFTs — limited lattice results available
Focus on well-developed dilaton-EFT with relatively few parameters


Compute more results to input — two-pion elastic scattering
\( I = 2 \) s-wave pion scattering

Same-sign \( \pi^\pm\pi^\pm \) scattering avoids challenging ‘disconnected’ contributions

[Work in progress to relate to \( W^\pm W^\pm \) scattering via EFTs — cf. arXiv:1201.3977]
\( I = 2 \) s-wave pion scattering

Same-sign \( \pi^+\pi^- \) scattering avoids challenging ‘disconnected’ contributions

Well-established path from finite-volume interaction energy
to low-momentum scattering length \( a_{\pi\pi} \) and effective range \( r_{\pi\pi} \)

\[
k^2 = \frac{E_{\pi\pi}^2}{4} - M_{\pi}^2
\]

\[
\frac{1}{\pi L} \left[ \sum_{\vec{j} \neq 0}^{\Lambda} \frac{4\pi^2}{4\pi^2 |\vec{j}|^2 - k^2 L^2} \right] - 4\pi\Lambda = \frac{1}{a_{\pi\pi}} + \frac{1}{2} M_{\pi}^2 r_{\pi\pi} \left( \frac{k^2}{M_{\pi}^2} \right) + \mathcal{O} \left( \frac{k^4}{M_{\pi}^4} \right)
\]
N_F = 8 scattering results

Six-parameter fit to tree-level dilaton-EFT
\[ \chi^2/\text{dof} \approx 55/18 \] suggests NLO will matter
(still far better than QCD-like \( \chi \)PT)

Parameters suppress dilaton effects:
\[ M_\pi a_{\pi\pi} = -\frac{M_\pi^2}{16\pi^2 F_\pi^2} \left[ 1 - O(10^{-3}) \frac{M_\pi^2}{M_\sigma^2} \right] \]

Grey band estimates size of NLO effects
to be considered in future work
$N_F = 8$ scattering results

Six-parameter fit to tree-level dilaton-EFT

Parameters suppress dilaton effects:

$$M_\pi a_{\pi \pi} = -\frac{M_\pi^2}{16\pi^2 F_\pi^2} \left[ 1 - \mathcal{O} \left( 10^{-3} \right) \frac{M_\pi^2}{M_\sigma^2} \right]$$

Grey band estimates size of NLO effects to be considered in future work

$I = 0$ scattering another future possibility
Splitting $N_F = 10 \rightarrow 4 + 6$ for improved control

Consider four light flavours (mass $\tilde{m}_\ell$) and six heavy flavours (mass $\tilde{m}_h$)

[switch to domain-wall fermions — better symmetries, larger costs]

In UV, effectively massless $N_F = 10 \rightarrow$ flow toward conformal IR fixed point

Around $\Lambda_{IR} \sim \tilde{m}_h$ heavy flavours decouple

Subsequent 4-flavour symmetry breaking $\rightarrow$ composite Higgs sensitive to 10-flavour IR fixed point
Splitting $N_F = 10 \rightarrow 4 + 6$ for improved control

Consider four light flavours (mass $\tilde{m}_\ell$) and six heavy flavours (mass $\tilde{m}_h$)

Flow toward IR fixed point $\longrightarrow$ bare gauge coupling $\beta = 6/g_0^2$ irrelevant

$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \frac{1}{a \cdot \tilde{m}_h} \rightarrow a \rightarrow 0 \text{ continuum limit is } a \cdot \tilde{m}_h \rightarrow 0 \text{ (with $\tilde{m}_\ell / \tilde{m}_h$ fixed)}$$

Chiral limit is $\tilde{m}_\ell / \tilde{m}_h \rightarrow 0$ and then no free parameters remain
Continuum limit is \( a \cdot \tilde{m}_h \to 0 \) (with \( \tilde{m}_\ell / \tilde{m}_h \) fixed)

Lattice spacing decreases as heavy mass decreases.

Obeys conformal hyperscaling relation

\[
\frac{a}{\sqrt{8 t_0}} = \tilde{m}_h^{1/y_m} \Phi_a(\tilde{m}_\ell / \tilde{m}_h)
\]

Fit quadratic ansatz for \( \Phi_a(\tilde{m}_\ell / \tilde{m}_h) \)

\( \rightarrow \) mass anomalous dimension

\( \gamma_m = y_m - 1 = 0.47(2) \)

\( y_m = 1.469(23) \)
$N_F = 4 + 6$ hyperscaling and mass anomalous dimension

Spectrum also exhibits hyperscaling

Here pseudoscalar decay constants

\[ a \cdot F = \tilde{m}_h^{1/y_m} \Phi_F(\tilde{m}_\ell/\tilde{m}_h) \]

Fit polynomial ansatz for $\Phi_F(\tilde{m}_\ell/\tilde{m}_h)$

\[ \longrightarrow \text{mass anomalous dimension} \]

\[ \gamma_m = y_m - 1 = 0.47(5) \]

Large anomalous dim’s from strong dynamics phenomenologically desirable
Testing dilaton-EFT with $N_F = 4 + 6$

Dilaton-EFT relation:

$$\frac{M^2}{F^2} = \frac{1}{y_m d_1} W_0 \left( \frac{y_m d_1}{d_2} \frac{\tilde{m}_\ell/\tilde{m}_h}{\Phi_a(0) \sqrt{8 t_0}} \right)$$

in terms of Lambert W-function

Fix $y_m$ from hyperscaling

Good fit suggests light scalar

(still to be analyzed directly)

$I = 2$ scattering calculations done via DiRAC, analyses ongoing [C. Culver]

Domain-wall fermions also assist $S$ param. and partial compositeness analyses
Electroweak precision observable — the $S$ parameter

Constrain Higgs sector from vector-minus-axial vacuum polarization $\Pi_{V-A}(Q)$

$$\gamma, \ Z \xrightarrow{\text{new}} Q \rightarrow \gamma, \ Z$$

Experimental $S = -0.01 \pm 0.10$

vs. QCD-like $S \approx 0.43 \sqrt{\xi}$

Related to $\chi$PT low-energy constant $L_{10}$


Domain-wall fermion symmetries important
S parameter on the lattice

$$\mathcal{L}_\chi \supset -\frac{S}{32\pi^2} g_1 g_2 B_{\mu\nu} \text{Tr} \left[ U_\tau U_\tau^\dagger W^{\mu\nu} \right] \rightarrow \gamma, Z$$

Prior LSD study of $N_F = 2, 6, 8$ \[arXiv:1405.4752\]

$N_F = 4 + 6$ planned for the near future

$$S/\sqrt{\xi} = 0.42(2) \text{ for } N_F = 2 \text{ matches QCD } \checkmark$$

Significant reduction from larger $N_F$,
chiral extrapolation again challenging

$V-A$ vacuum polarization also contributes to Higgs potential \[arXiv:1903.02535\]
**Anomalous dimensions for partial compositeness**

**Old challenge for new strong dynamics**

Quark & lepton masses \( \sim \frac{\bar{q}q\bar{\psi}\psi}{\Lambda_{UV}^2} \) vs. flavour-changing NCs \( \sim \frac{\bar{q}q\bar{q}q}{\Lambda_{UV}^2} \)

**Partial compositeness alternative**

Linear mixing with composite partners

\[ \mathcal{L} \supset \lambda \bar{q}O_q + \text{h.c.} \]

\[ \longrightarrow m_q \sim v_{EW} \left( \frac{\text{TeV}}{\Lambda_{UV}} \right)^{4-2\gamma_q} \]

Large mass hierarchy \( \longleftrightarrow \mathcal{O}(1) \) anomalous dimensions
Anomalous dimensions for partial compositeness

Partial compositeness alternative
Linear mixing with composite partners

\[
\mathcal{L} \supset \lambda \bar{q}O_q + \text{h.c.} \\
\rightarrow m_q \sim v_{\text{EW}} \left( \frac{\text{TeV}}{\Lambda_{UV}} \right)^{4-2\gamma_q}
\]

Large mass hierarchy $\leftrightarrow \mathcal{O}(1)$ anomalous dimensions

Example: $\Lambda_{UV} = 10^{10} \text{ TeV} \rightarrow m_q \sim \mathcal{O}(\text{MeV})$ from $\gamma_q \approx 1.75$

$m_q \sim \mathcal{O}(\text{GeV})$ from $\gamma_q \approx 1.9$
Plans for partial compositeness on the lattice

For SU(3) theories \( \mathcal{O}_q \sim \psi \psi \psi \sim \) baryons with scaling dim. \( [\mathcal{O}_q] = \frac{9}{2} - \gamma_q \)

Plan \( N_F = 4 + 6 \) predictions \( \gamma_q = -\frac{d \log Z_{\mathcal{O}_q}(\mu)}{d \log \mu} \) from RI/MOM non-pert. renorm.

and from ratios of gradient-flowed operators \( \propto t^{\gamma_q/2} \) [arXiv:1806.01385]

\[ m_q \sim v_{\text{EW}} \left( \frac{\text{TeV}}{\Lambda_{\text{UV}}} \right)^{4-2\gamma_q} \]
Gravitational waves from early-universe phase transition

First-order chiral transition $\rightarrow$ stochastic background of gravitational waves

Massless $N_F = 4$ transition is first-order


Then compute properties of transition: latent heat, nucleation rate, etc.
Recap and outlook

Lattice field theory is a broadly applicable tool to study strongly coupled near-conformal theories

Near-conformality useful for new strong dynamics
—Light scalar observed, so far consistent with dilaton-EFT
—Reduced $S$ parameter
—Natural scale separation for flavour physics

Ongoing investigations of mass-split $N_F = 4 + 6$ system
—$S$ parameter and $I = 2$ scattering
—Baryon scaling dimensions for partial compositeness
—Finite-temperature transitions $\rightarrow$ gravitational waves
Thanks for your attention!

Any further questions?

Funding and computing resources
In lattice QCD, isosinglet scalar mass $M_S \gtrsim 2M_P$ → significant mixing with $I = 0$ two-pion scattering states

**arXiv:1607.05900**
Backup: Direct comparison of QCD-like and near-conformal $0^{++}$

\[ N_f = 4 \]

\[ N_f = 8 \]

$N_F = 8$ scalar much lighter than $M_\sigma \approx 2M_\pi$ for QCD-like $N_F = 4$
Monotonic step-scaling ($\sim -\beta$) function and increasing $M_\rho/M_\pi$ are evidence against conformal IR fixed point
Backup: Width of \( N_F = 8 \) vector resonance

\[
F_\rho = \sqrt{2} F_\pi
\]

\[
g_{\rho\pi\pi} = \frac{M_\rho}{\sqrt{2} F_\pi}
\]

Riazuddin–Fayyazuddin, 1966

Kawarabayashi–Suzuki, 1966

Can derive from current algebra, hidden local symmetry, chiral EFT

Can derive from current algebra, hidden local symmetry, chiral EFT

Confirm first KSRF relation, then apply second

\[
\Gamma_\rho = \frac{g_{\rho\pi\pi}^2 M_\rho}{48\pi} \simeq 450 \text{ GeV}/\sqrt{\xi} \quad \text{—— hard to see at LHC}
\]
$L_s$ copies of 4d gauge fields (expensive with $L_s = 16$!)

Localized fermions have renormalized mass $m = m_f + m_{\text{res}}$

with residual mass $m_{\text{res}} \ll m_f$ from overlap around $L_s/2$

$L_s \to \infty \quad \longrightarrow \quad$ exact chiral symmetry at non-zero lattice spacing
Backup: $N_F = 10$ step-scaling function (SSF)

SSF $\sim$ negative of $\beta$ function integrated over $s = 2 \times$ scale change.

Lattice calculations use finite-volume gradient flow schemes parameterized by $c$ (here $c = 0.3$).

Evidence for conformal IR fixed point at strong $g_c^2 \sim 13$ in $c = 0.3$ scheme.

Change to $c = 0.25 \rightarrow \beta_2 = 0$ for $g_c^2 \approx 11$ with larger systematic uncertainties.