Lattice studies of supersymmetric Yang–Mills in 2+1 dimensions

David Schaich (Liverpool)

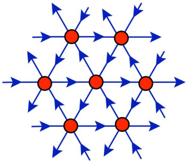
Relativistic Fermions in Flatland
ECT* Trento 9 July 2021

arXiv:1810.09282 arXiv:2010.00026 and more to come

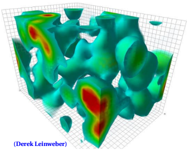
with S. Catterall, J. Giedt, R. G. Jha, A. Sherletov & T. Wiseman

Overview and plan

2+1 dimensions is a promising frontier
for practical lattice studies of supersymmetric QFTs



Why: Lattice supersymmetry motivation



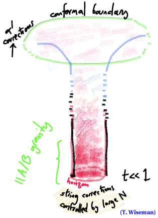
How: Lattice formulation highlights

What: Recent, ongoing & planned 3d work

Maximally supersymmetric Yang–Mills (SYM)

Half-maximal SYM

Supersymmetric QCD



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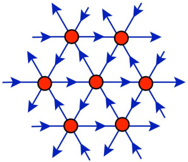

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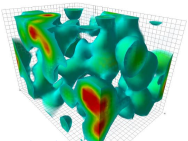
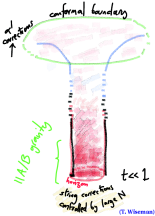


(Derek Leinweber)

How: Lattice formulation highlights

What: Recent, ongoing & planned 3d work



(T. Wiseman)

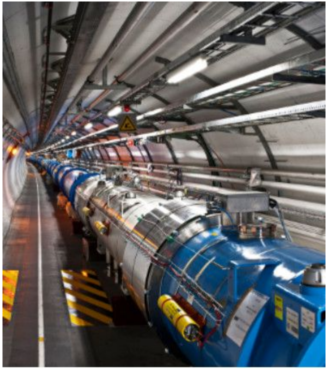
These slides: davidschaich.net/talks/2107ECT.pdf

Interaction encouraged — complete coverage unnecessary

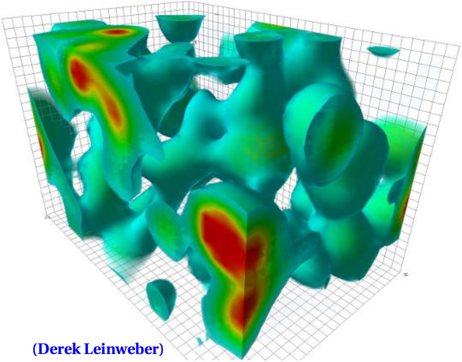
Motivations

Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

BSM

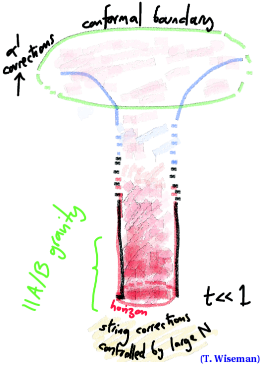


QFT



(Derek Leinweber)

Holography



(T. Wiseman)

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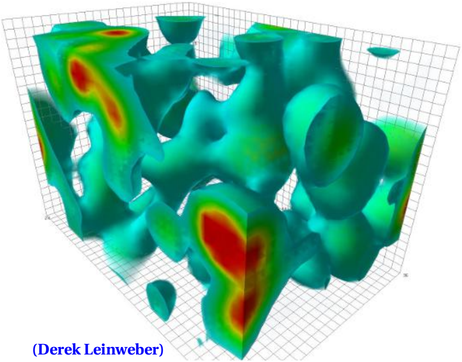
Lattice field theory promises first-principles predictions
for strongly coupled supersymmetric QFTs

2+1 dimensions

Rich field theory
and holographic
dynamics

QFT



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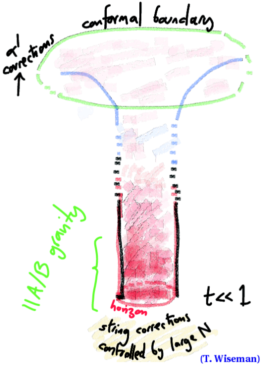
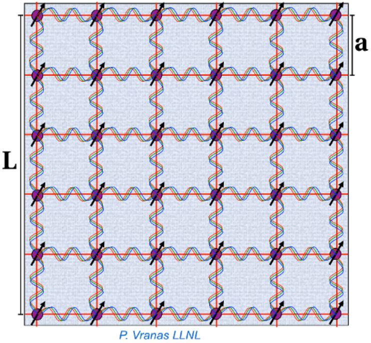


(T. Wiseman)

Quick reminder: Lattice regularization in the QFT context

Formally $\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete, euclidean space-time
↖Gauge invariant, non-perturbative, d-dimensional



Spacing between lattice sites ("a")
$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $a \to 0$ $(L/a \to \infty)$

Discrete $\longrightarrow$ continuous symmetries ✓

Supersymmetry must be broken on the lattice

Supersymmetry is a space-time symmetry, $(\mathrm{I} = 1, \cdots, \mathcal{N})$
adding spinor generators Q_α^{I} and $\overline{Q}_{\dot\alpha}^{\mathrm{I}}$ to translations, rotations, boosts

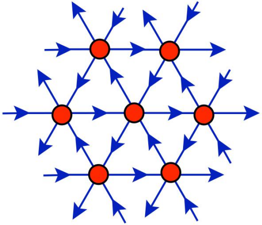
$\left\{Q_\alpha^{\mathrm{I}}, \overline{Q}_{\dot\alpha}^{\mathrm{J}}\right\} = 2\delta^{\mathrm{IJ}}\sigma_{\alpha\dot\alpha}^{\mu} P_\mu$ broken in discrete space-time
$\longrightarrow$ relevant susy-violating operators

Supersymmetry need not be *completely* broken on the lattice

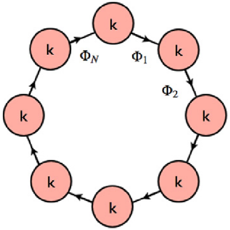
Preserve susy sub-algebra in discrete lattice space-time

$\Longrightarrow$ correct continuum limit with little or no fine tuning

Equivalent constructions from 'topological' twisting and dim'l deconstruction



Review:
Catterall–Kaplan–Ünsal,
arXiv:0903.4881



Need $Q = 2^d$ supersymmetries in d dimensions

$d = 3 \longrightarrow$ Super-Yang–Mills (SYM) with $Q = 8$ or (maximal) $Q = 16$

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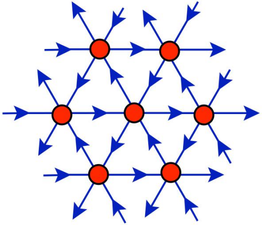

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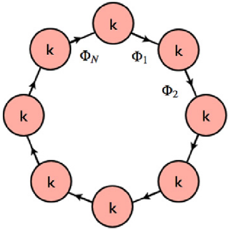

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3d maximal SYM in a nutshell

May be easiest to grok as dimensional reduction of 4d $\mathcal{N} = 4$ SYM
(famous testing ground for dualities, amplitudes & more)

All fields massless and in adjoint rep. of SU(N) gauge group

4d: Gauge field A_μ plus 6 scalars Φ^{IJ}

$\mathcal{N} = 4$ four-component fermions $\Psi^{\mathrm{I}} \longleftrightarrow$ 16 supersymmetries Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot\alpha}$

Global SU(4) $\sim$ SO(6) R symmetry

3d: Gauge field A_μ plus 7 scalars Φ

$\mathcal{N} = 8$ two-component fermions $\Psi \longleftrightarrow$ 16 supersymmetries

Global SO(8) $\supset$ SO(4) $\sim$ SU(2) $\times$ SU(2) R symmetry

Symmetries relate kinetic, Yukawa and Φ^4 terms $\longrightarrow$ single coupling $\lambda = g^2 N$

Twisting maximal SYM

Intuitive 4d picture — expand 4×4 matrix of supersymmetries

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot\alpha} & \overline{Q}^2_{\dot\alpha} & \overline{Q}^3_{\dot\alpha} & \overline{Q}^4_{\dot\alpha} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu} \gamma_\mu \gamma_\nu + \overline{\mathcal{Q}}_\mu \gamma_\mu \gamma_5 + \overline{\mathcal{Q}} \gamma_5$$

$$\longrightarrow \mathcal{Q} + \mathcal{Q}_a \gamma_a + \mathcal{Q}_{ab} \gamma_a \gamma_b$$

with $a, b = 1, \cdots, 5$

R-symmetry index $\times$ Lorentz index $\Longrightarrow$ reps of 'twisted rotation group'

$$\text{SO}(4)_{\text{tw}} \equiv \text{diag}\left[\text{SO}(4)_{\text{euc}} \otimes \text{SO}(4)_R\right] \qquad \qquad \text{SO}(4)_R \subset \text{SO}(6)_R$$

Change of variables $\longrightarrow$ $\mathcal{Q}$s transform with integer 'spin' under $\text{SO}(4)_{\text{tw}}$

Twisting maximal SYM

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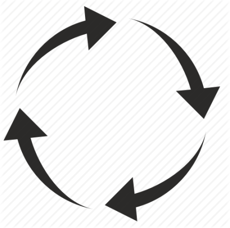
Discrete space-time

Can preserve closed sub-algebra

$$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$$



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with $a, b = 1, \cdots, 5$

Discrete space-time

Can preserve closed sub-algebra

$$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$$

Twisting maximal SYM

Reducing to 3d

$\{\mathcal{Q}, \mathcal{Q}_a, \mathcal{Q}_{ab}\} \longrightarrow \{\mathcal{Q}, \mathcal{Q}_5, \mathcal{Q}_a, \mathcal{Q}_{a5}, \mathcal{Q}_{ab}\}$ with $a, b = 1, \cdots, 4$

Twisted rotation group now

$$\text{SO}(3)_{\text{tw}} \equiv \text{diag}\left[\text{SO}(3)_{\text{euc}} \otimes \text{SO}(3)_R\right] \qquad \text{SO}(3)_R \subset \text{SO}(4)_R$$

Two closed supersymmetry sub-algebras

$$\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0 \qquad \{\mathcal{Q}_5, \mathcal{Q}_5\} = 2\mathcal{Q}_5^2 = 0$$

Completing the twist

Fields also transform with integer spin under $\mathrm{SO}(4)_{\text{tw}}$ — no spinors

$$\Psi \text{ and } \overline{\Psi} \longrightarrow \eta,\ \psi_a \text{ and } \chi_{ab}$$

$$A_\mu \text{ and } \Phi^{\mathrm{I}} \longrightarrow \text{complexified gauge field } \mathcal{A}_a \text{ and } \overline{\mathcal{A}}_a$$
$$\longrightarrow \mathrm{U}(N) = \mathrm{SU}(N) \otimes \mathrm{U}(1) \text{ gauge theory}$$

✓ $\mathcal{Q}$ interchanges bosonic $\longleftrightarrow$ fermionic d.o.f. with $\mathcal{Q}^2 = 0$

$$\begin{aligned}
&\mathcal{Q}\,\mathcal{A}_a = \psi_a && \mathcal{Q}\,\psi_a = 0 \\
&\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} && \mathcal{Q}\,\overline{\mathcal{A}}_a = 0 \\
&\mathcal{Q}\,\eta = d && \mathcal{Q}\,d = 0
\end{aligned}$$

$\nwarrow$ bosonic auxiliary field with e.o.m. $d = \overline{\mathcal{D}}_a \mathcal{A}_a$

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Dimensional reduction rearranges fermions and takes $\mathcal{A}_5, \overline{\mathcal{A}}_5 \longrightarrow \varphi, \overline{\varphi}$

Lattice maximal SYM

Lattice theory looks nearly the same despite breaking $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

Covariant derivatives $\longrightarrow$ finite difference operators

Complexified gauge fields $\mathcal{A}_a$ $\longrightarrow$ gauge links $\mathcal{U}_a \in \mathfrak{gl}(N, \mathbb{C})$

$$\mathcal{Q}\,\mathcal{A}_a \longrightarrow \mathcal{Q}\,\mathcal{U}_a = \psi_a \qquad\qquad \mathcal{Q}\,\psi_a = 0$$

$$\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad\qquad \mathcal{Q}\,\overline{\mathcal{A}}_a \longrightarrow \mathcal{Q}\,\overline{\mathcal{U}}_a = 0$$

$$\mathcal{Q}\,\eta = d \qquad\qquad \mathcal{Q}\,d = 0$$

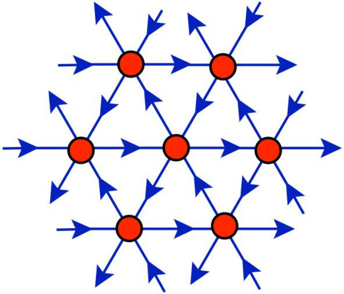
Geometry: η on sites, ψ_a on links, etc.

Supersymmetric lattice action $(\mathcal{Q}S = 0)$ from $\mathcal{Q}^2 \cdot = 0$ and Bianchi identity

$$S = \frac{N}{4\lambda_{\text{lat}}}\text{Tr}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de}\right]$$

$d+1$ links in d dimensions $\longrightarrow$ A_d^* lattice

A_d^* $\sim$ d-dimensional analog of 2d triangular lattice



Basis vectors linearly dependent and non-orthogonal

Large S_{d+1} point group symmetry

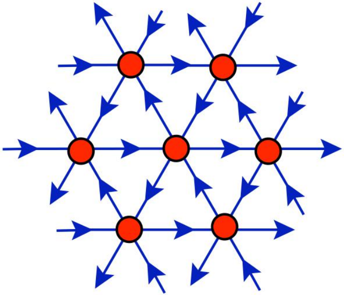
S_{d+1} irreps precisely match onto irreps of twisted $\text{SO}(d)_{\text{tw}}$. 4d example:

$$\psi_a \longrightarrow \psi_\mu, \ \ \overline{\eta} \qquad \text{is} \qquad \mathbf{5} \longrightarrow \mathbf{4} \oplus \mathbf{1}$$

$$\chi_{ab} \longrightarrow \chi_{\mu\nu}, \ \ \overline{\psi}_\mu \qquad \text{is} \qquad \mathbf{10} \longrightarrow \mathbf{6} \oplus \mathbf{4}$$

$S_{d+1} \longrightarrow \text{SO}(d)_{\text{tw}}$ in continuum limit restores $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

$d+1$ links in d dimensions $\longrightarrow$ A_d^* lattice



A_d^* $\sim$ d-dimensional analog of 2d triangular lattice

Basis vectors linearly dependent and non-orthogonal

Large S_{d+1} point group symmetry

Twisted maximal SYM on A_d^* lattice is elegant formulation
not yet practical for numerical calculations

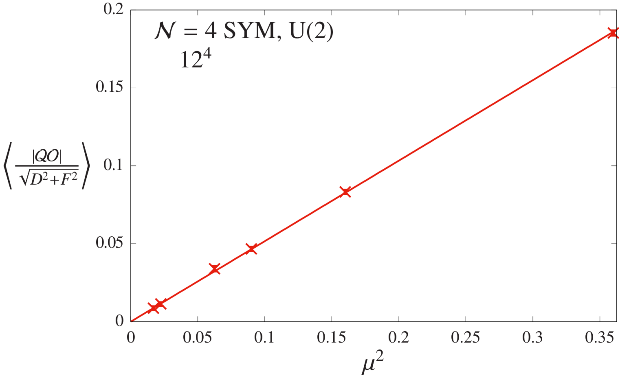
Must regulate zero modes and flat directions, especially in U(1) sector

Deformations to stabilize lattice calculations

1) Add SU(N) scalar potential $\propto \mu^2 \sum_a \text{Tr}\left[\left(\mathcal{U}_a\overline{\mathcal{U}}_a - \mathbb{I}_N\right)^2\right]$

Softly breaks susy $\longrightarrow$ $\mathcal{Q}$-violating operators vanish $\propto \mu^2 \to 0$

Test via Ward identity violations
$\mathcal{Q}\left[\eta\mathcal{U}_a\overline{\mathcal{U}}_a\right] \neq 0$



Deformations to stabilize lattice calculations

2) Constrain U(1) plaquette determinant $\sim G\sum_{a<b}\left(\det\mathcal{P}_{ab}-1\right)$

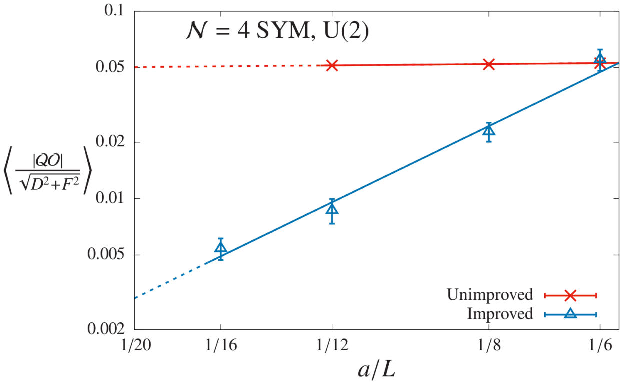
Implemented supersymmetrically by modifying auxiliary field equations of motion

Test via Ward identity violations

$$\mathcal{Q}\left[\eta\,\mathcal{U}_a\overline{\mathcal{U}}_a\right]\neq 0$$

Log–log axes

$\longrightarrow$ violations $\propto (a/L)^2$

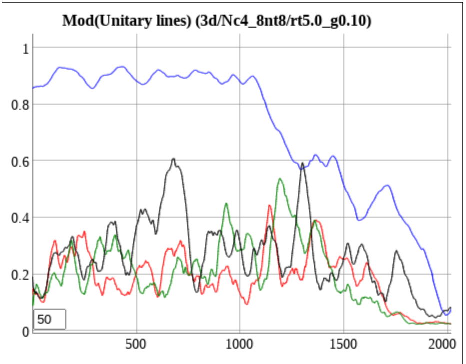


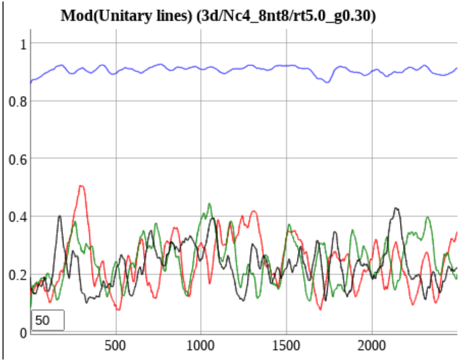
Deformations to stabilize lattice calculations

Enable naive dimensional reduction (4d code with $N_x = 1$)

3) Potential $\propto \mathrm{Tr}\left[(\varphi - \mathbb{I}_N)^\dagger (\varphi - \mathbb{I}_N)\right]$ to break center symmetry in reduced dir(s)

(∼Kaluza–Klein rather than Eguchi–Kawai reduction)





Public code for supersymmetric lattice field theories

so that the full improved action becomes

$$S_{\rm imp} = S'_{\rm exact} + S_{\rm closed} + S'_{\rm soft} \tag{18}$$

$$S'_{\rm exact} = \frac{N}{4\lambda_{\rm lat}} \sum_n \mathrm{Tr}\left[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\right] - S_{\rm det}$$

$$S_{\rm det} = \frac{N}{4\lambda_{\rm lat}} G \sum_n \mathrm{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \mathrm{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right]$$

$$S_{\rm closed} = -\frac{N}{16\lambda_{\rm lat}} \sum_n \mathrm{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right],$$

$$S'_{\rm soft} = \frac{N}{4\lambda_{\rm lat}}\mu^2 \sum_n\sum_a\left(\frac{1}{N}\mathrm{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2$$

$\gtrsim$100 inter-node data transfers in the fermion operator — non-trivial. . .

Public parallel code to reduce barriers to entry: github.com/daschaich/susy

Evolved from MILC QCD code, user guide in arXiv:1410.6971

3d maximal SYM thermodynamics

arXiv:2010.00026

Formulate on $r_1 \times r_2 \times r_\beta$ (skewed) 3-torus

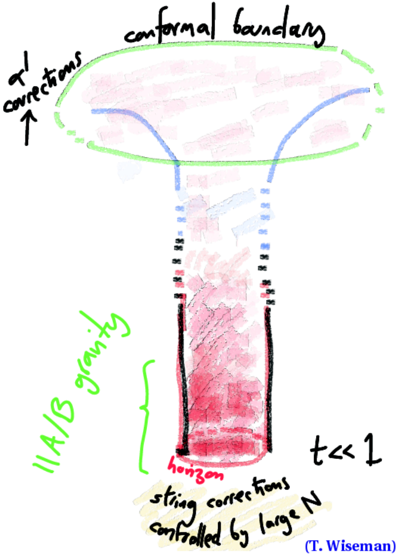
Thermal boundary conditions

$\longrightarrow$ dimensionless temperature $t = \dfrac{T}{\lambda} = \dfrac{1}{r_\beta}$

Low temperatures t at large N

$\Updownarrow$

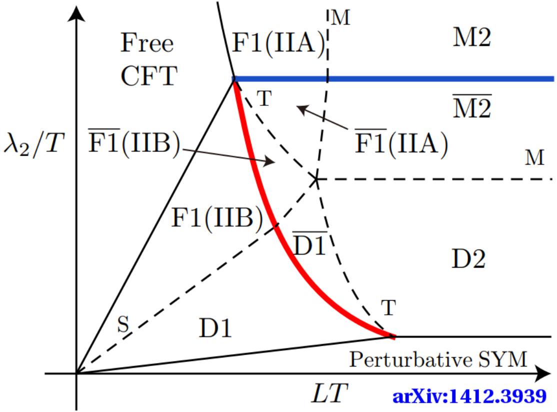
Black branes in dual supergravity



3d maximal SYM phase diagram

Holography $\longrightarrow$ rich low-t phase diagram conjectured

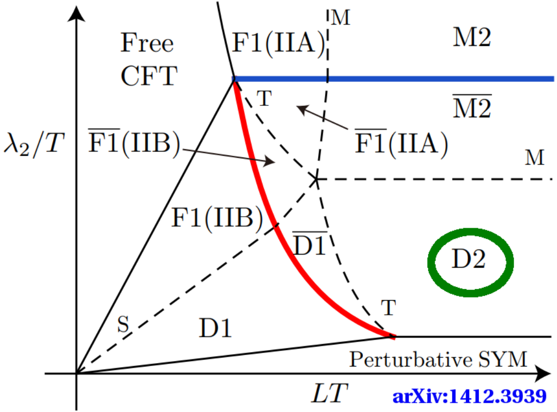
(simpler 2d case studied in arXiv:1709.07025)



3d maximal SYM phase diagram

Holography $\longrightarrow$ rich low-t phase diagram conjectured

For now consider simplest homogeneous black D2-branes $\longrightarrow$ $r_1 = r_2 = r_\beta$

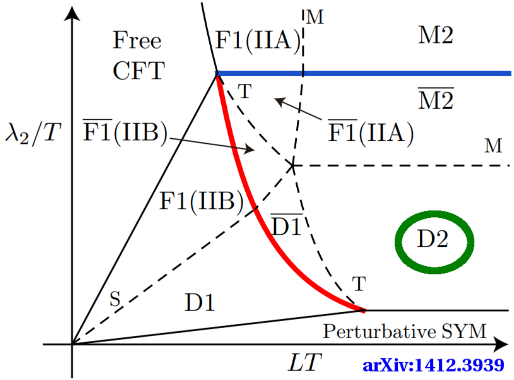


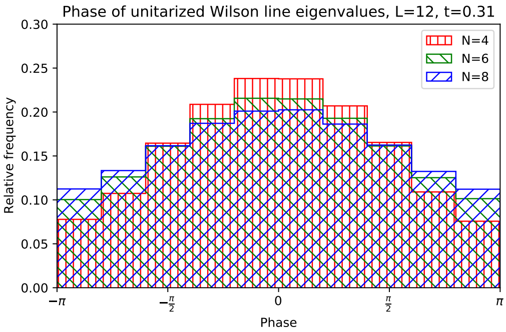
arXiv:1412.3939

Homogeneous D2 phase

Lattice volume L^3, continuum limit $L \to \infty$ with fixed $t = 1/r_\beta$

Homogeneous D2-branes $\longleftrightarrow$ uniform Wilson line eigenvalue phases at large N



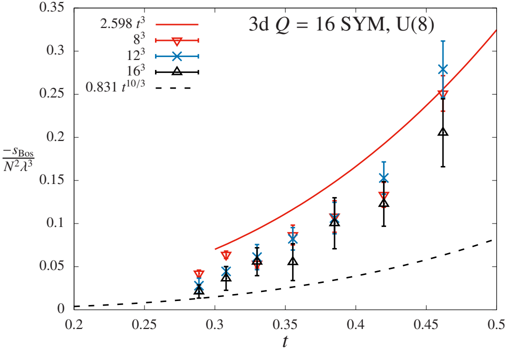


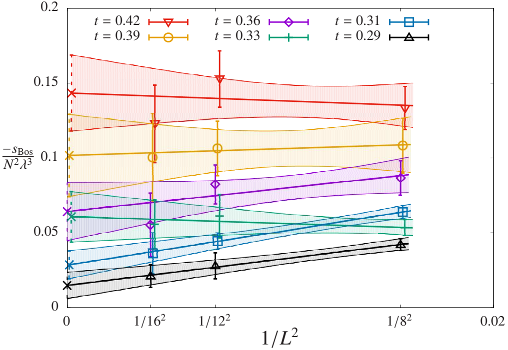
Holographic black brane energies and continuum extrapolation

Lattice volume L^3 with fixed $N = 8$

$\longrightarrow$ results approach leading holographic expectation $\propto t^{10/3}$ for low $t \lesssim 0.4$

Carry out first $L \to \infty$ continuum extrapolations (not yet attempted for 2d)



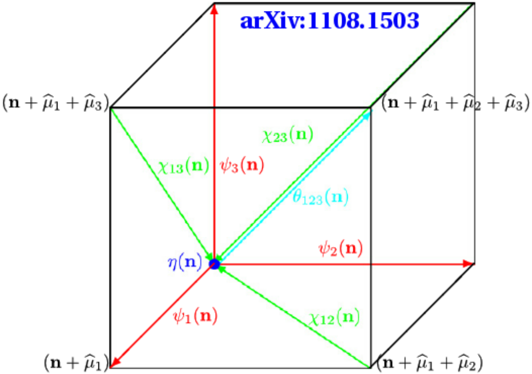


Work in progress: Half-maximal ($Q = 8$) SYM

Slight simplification of twisted formulation

$Q = 8$ supercharges $\{\mathcal{Q}, \mathcal{Q}_a, \mathcal{Q}_{ab}, \mathcal{Q}_{abc}\}$ with $a, b = 1, \cdots, 3$

$\longrightarrow$ site / link / plaquette / cube fermions $\{\eta, \psi_a, \chi_{ab}, \theta_{abc}\}$ on simple cubic lattice



Work in progress: Half-maximal ($Q = 8$) SYM

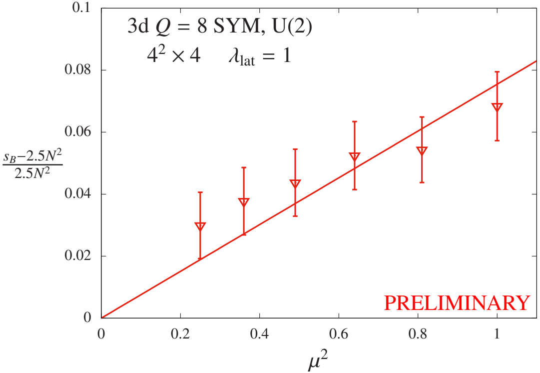
Slight simplification of twisted formulation

$Q = 8$ supercharges $\{\mathcal{Q}, \mathcal{Q}_a, \mathcal{Q}_{ab}, \mathcal{Q}_{abc}\}$ with $a, b = 1, \cdots, 3$

$\longrightarrow$ site / link / plaquette / cube fermions $\{\eta, \psi_a, \chi_{ab}, \theta_{abc}\}$ on simple cubic lattice

Parallel code developed
(Angel Sherletov)

Tests passed
$\longrightarrow$ larger-scale calculations



Work coming up: Supersymmetric QCD

Add 'quarks' and squarks $\longrightarrow$ investigate electric–magnetic dualities,
dynamical supersymmetry breaking and more

Quiver construction based on twisted SYM [arXiv:1505.00467]
preserves susy sub-algebra to reduce fine-tuning

Quiver superQCD from twisted SYM

First check 3d SYM $\longrightarrow$ 2d superQCD   then new 4d SYM $\longrightarrow$ 3d superQCD

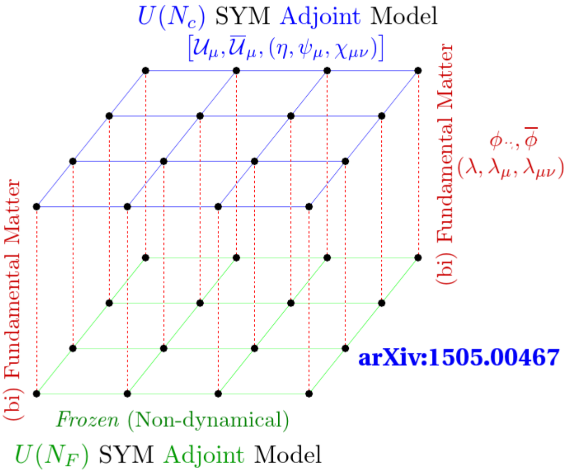
2-slice lattice SYM
with U(N) $\times$ U(F) gauge group

Adj. fields on each slice

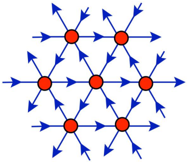
Bi-fundamental in between

Decouple U(F) slice

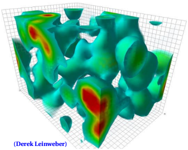
$\longrightarrow$ U(N) SQCD in $d-1$ dims.
with F fund. hypermultiplets



Recap: An exciting time for lattice supersymmetry

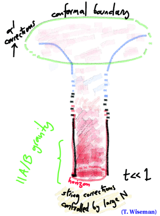


2+1 dimensions is a promising frontier
for practical lattice studies of supersymmetric QFTs



Preserving susy sub-algebra enables lattice calculations,
public code available

3d maximal SYM thermodynamics consistent with holography



Work in progress on 3d $Q = 8$ SYM $\longrightarrow$ superQCD

Phase diagrams, sign problems and much more for the future

Thanks for your attention! Any further questions?

Collaborators

Simon Catterall, Joel Giedt, Raghav Jha, Angel Sherletov, Toby Wiseman

Funding and computing resources

UK Research and Innovation

LIVERPOOL THEORETICAL PHYSICS

DEPARTMENT OF ENERGY · UNITED STATES OF AMERICA

USQCD

Supplement: 2d maximal SYM phase diagram

arXiv:1709.07025

Dimensionally reduce to 2d $\mathcal{N} = (8, 8)$ SYM on $(r_L \times r_\beta)$ torus with four scalar $\mathcal{Q}$

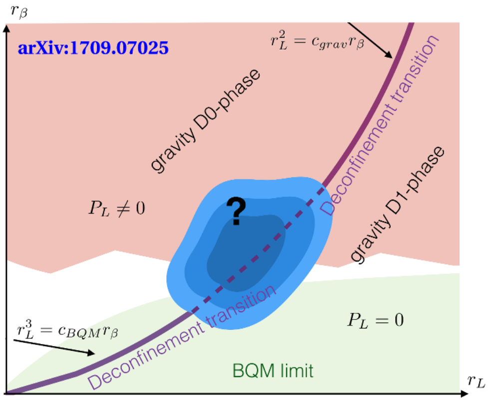
Low temperatures $t = 1/r_\beta \longleftrightarrow$ black holes in dual supergravity

For decreasing r_L at large N

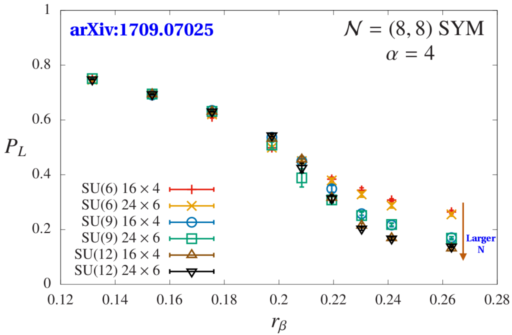
homogeneous black string (D1)
$\longrightarrow$ localized black hole (D0)

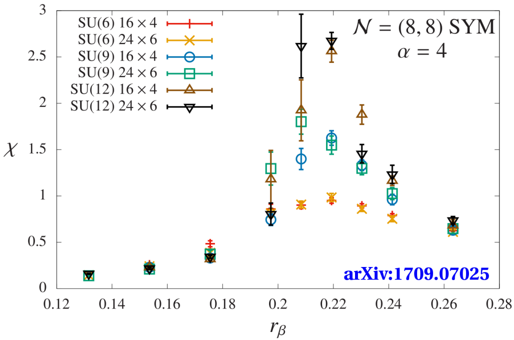
$\updownarrow$

"spatial deconfinement"
signalled by Wilson line P_L



Spatial deconfinement transition signals





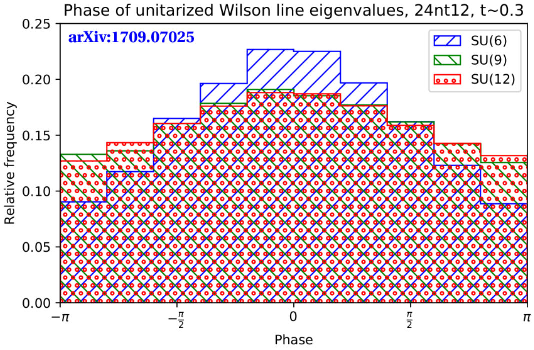
Peaks in Wilson line susceptibility match change in its magnitude |PL|,

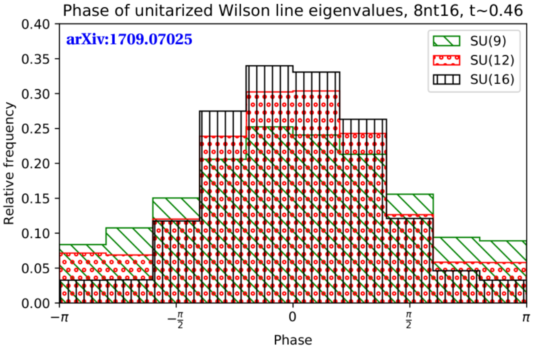
grow with size of SU(N) gauge group, comparing $N = 6, 9, 12$

Agreement for 16×4 vs. 24×6 lattices (aspect ratio $\alpha = r_L / r_\beta = 4$)

Check Wilson line eigenvalues

Wilson line eigenvalue phases sensitive to ‘spatial deconfinement’





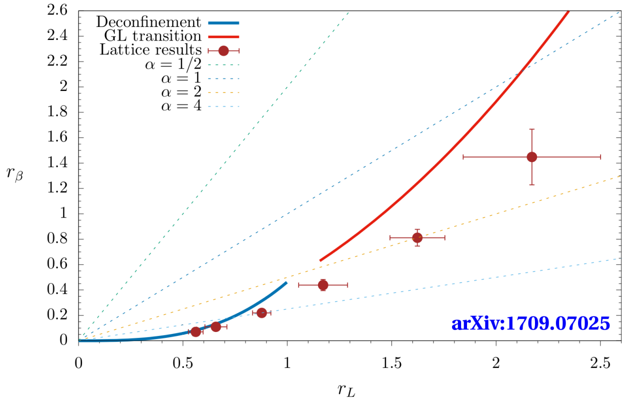
Left: $\alpha = 2$ distributions more uniform as *N* increases $\longrightarrow$ D1 black string

Right: $\alpha = 1/2$ distributions more compact as *N* increases $\longrightarrow$ D0 black hole

Lattice results for 2d $\mathcal{N} = (8, 8)$ SYM phase diagram

Good agreement with bosonic QM at high temperatures

Harder to control low-temperature uncertainties (larger $N > 16$ should help)



Overall consistent with holography

Comparing multiple lattice sizes and $6 \leq N \leq 16$

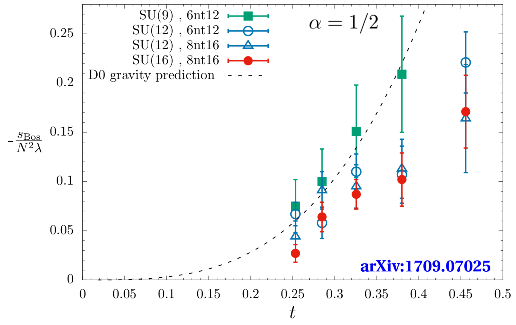
Controlled extrapolations are work in progress

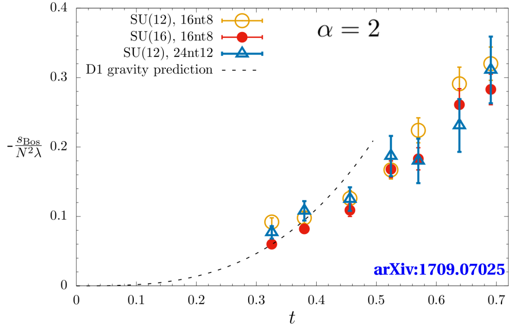
Check holographic black hole energies

Lattice results consistent with leading expectation for sufficiently low $t \lesssim 0.4$

Similar behavior $\longrightarrow$ difficult to distinguish phases

$\propto t^{3.2}$ for small-r_L D0 phase $\qquad \propto t^{3}$ for large-r_L D1 phase





Supplement: Sign problems

Recall typical algorithms sample field configurations Φ with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$\longrightarrow$ "sign problem" if action $S[\Phi]$ can be negative or complex

Lattice SYM has complex pfaffian $\text{pf}\,\mathcal{D} = |\text{pf}\,\mathcal{D}|\, e^{i\alpha}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\ \mathcal{O}\ e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]}\ \text{pf}\,\mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

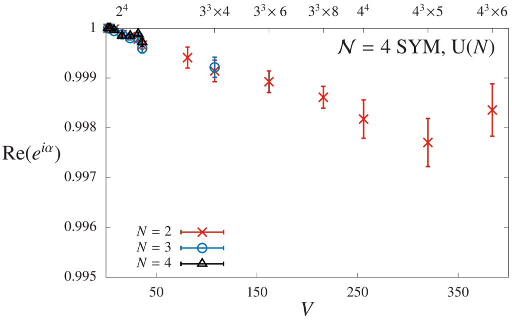
We phase quench $\text{pf}\,\mathcal{D} \longrightarrow |\text{pf}\,\mathcal{D}|$, need to reweight $\langle \mathcal{O} \rangle = \dfrac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{\text{pq}}}{\left\langle e^{i\alpha} \right\rangle_{\text{pq}}}$

$\Longrightarrow \left\langle e^{i\alpha} \right\rangle_{\text{pq}} = \dfrac{\mathcal{Z}}{\mathcal{Z}_{\text{pq}}}$ quantifies severity of sign problem

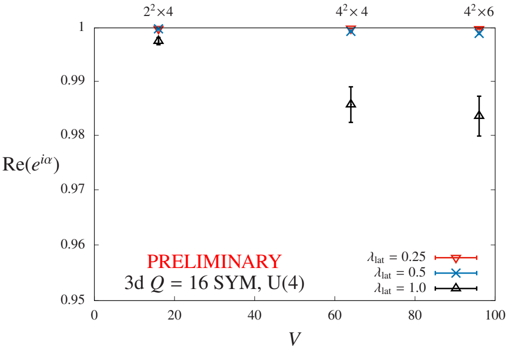
Lattice maximal SYM sign problems

Fix $\lambda_{\text{lat}} \longrightarrow$ pfaffian nearly real positive for all accessible volumes

4d



3d

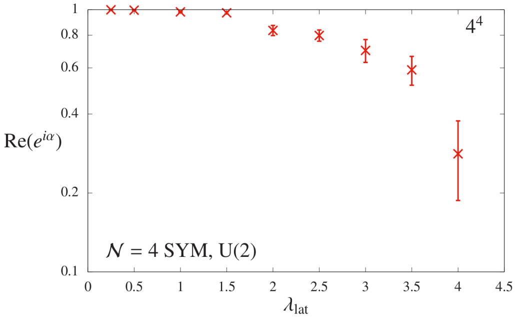


Lattice maximal SYM sign problems

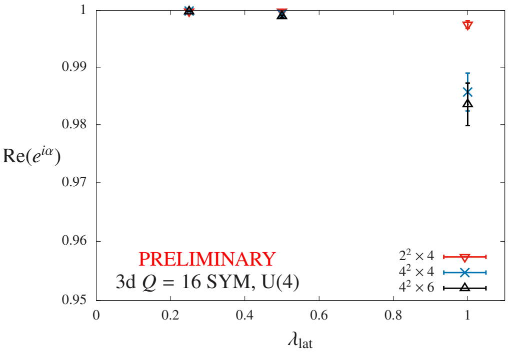
Fix volume $\longrightarrow$ 4d signal-to-noise becomes obstruction for $\lambda_{\text{lat}} \gtrsim 4$

3d temperatures studied so far $\longleftrightarrow$ $\lambda_{\text{lat}} \leq 1$ with no problem

4d



3d



Backup: Breakdown of Leibniz rule on the lattice

$\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$ is problematic

$\Longrightarrow$ try finite difference $\partial\phi(x) \longrightarrow \Delta\phi(x) = \frac{1}{a}\left[\phi(x+a) - \phi(x)\right]$

Crucial difference between ∂ and Δ

$$\Delta\left[\phi\eta\right] = a^{-1}\left[\phi(x+a)\eta(x+a) - \phi(x)\eta(x)\right]$$
$$= \left[\Delta\phi\right]\eta + \phi\Delta\eta + a\left[\Delta\phi\right]\Delta\eta$$

Full supersymmetry requires Leibniz rule $\partial\left[\phi\eta\right] = \left[\partial\phi\right]\eta + \phi\partial\eta$

only recoverd in $a \to 0$ continuum limit for any local finite difference

Backup: Breakdown of Leibniz rule on the lattice

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only recoverd in $a \to 0$ continuum limit for any local finite difference

Supersymmetry vs. locality 'no-go' theorems

by Kato–Sakamoto–So [arXiv:0803.3121] and Bergner [arXiv:0909.4791]

Complicated constructions to balance locality vs. supersymmetry

Non-ultralocal product operator $\longrightarrow$ lattice Leibniz rule but not gauge invariance

D'Adda–Kawamoto–Saito, arXiv:1706.02615

Cyclic Leibniz rule $\longrightarrow$ partial lattice supersymmetry but only (0+1)d QM so far

Kadoh–Kamei–So, arXiv:1904.09275

Backup: Complexified gauge field from twisting

Combining A_μ and $\Phi^{\mathrm{I}} \longrightarrow \mathcal{A}_a$ and $\overline{\mathcal{A}}_a$

produces $\mathrm{U}(N) = \mathrm{SU}(N) \otimes \mathrm{U}(1)$ gauge theory

Complicates lattice action but needed so that $\mathcal{Q}\,\mathcal{A}_a = \psi_a$

Further motivation: Under $\mathrm{SO}(d)_{\mathrm{tw}} = \mathrm{diag}\left[\mathrm{SO}(d)_{\mathrm{euc}} \otimes \mathrm{SO}(d)_R\right]$

$$\begin{aligned} A_\mu &\sim \text{vector} \otimes \text{scalar} = \text{vector} \\ \Phi^{\mathrm{I}} &\sim \text{scalar} \otimes \text{vector} = \text{vector} \end{aligned}$$

Easiest to see in 5d (then dimensionally reduce)

$$\mathcal{A}_a = A_a + i\Phi_a \longrightarrow (A_\mu, \phi) + i(\Phi_\mu, \overline{\phi})$$

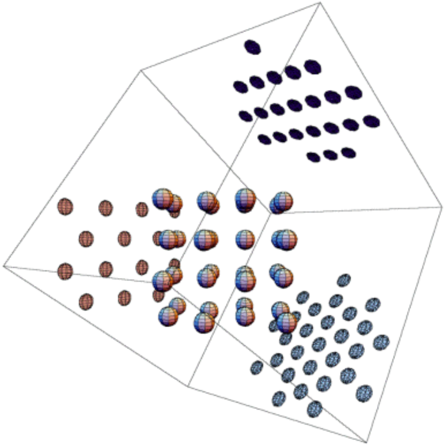
Backup: A_4^* lattice from five dimensions

Again dimensionally reduce, treating all five gauge links symmetrically

Start with hypercubic lattice
in 5d momentum space

Symmetric constraint $\sum_a \partial_a = 0$
projects to 4d momentum space

Result is A_4 lattice
$\longrightarrow$ dual A_4^* lattice in position space



Backup: Restoration of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ supersymmetries

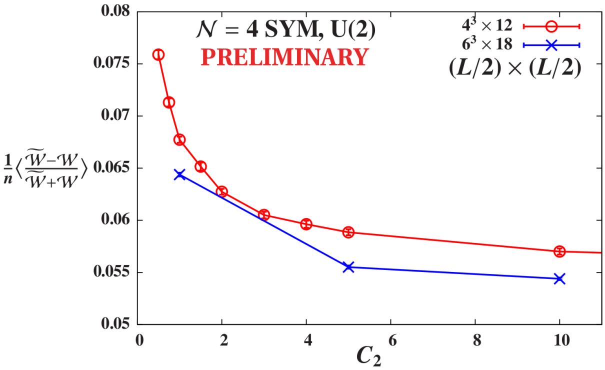
"$\mathcal{Q}$ + discrete $R_a \subset \mathrm{SO}(4)_{\mathrm{tw}}$ = $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$"

[arXiv:1306.3891]

Test R_a on Wilson loops

$$\widetilde{\mathcal{W}}_{ab} \equiv R_a \mathcal{W}_{ab}$$

Tune coeff. c_2 of d^2 term in action
for fastest restoration
towards continuum limit



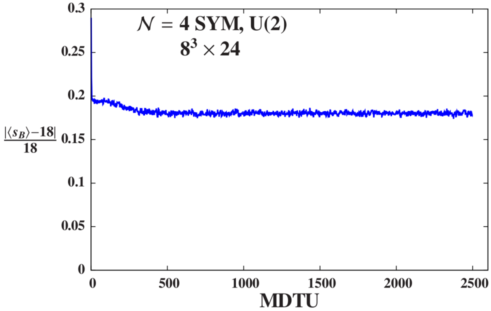
Backup: Problem with SU(N) flat directions

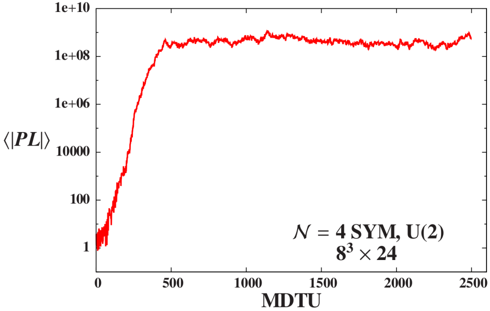
$\mu^2/\lambda_{\text{lat}}$ too small $\longrightarrow$ $\mathcal{U}_a$ can move far from continuum form $\mathbb{I}_N + \mathcal{A}_a$

Example: $\mu = 0.2$ and $\lambda_{\text{lat}} = 2.5$ on $8^3 \times 24$ volume

Left: Bosonic action stable $\sim$18% off its supersymmetric value

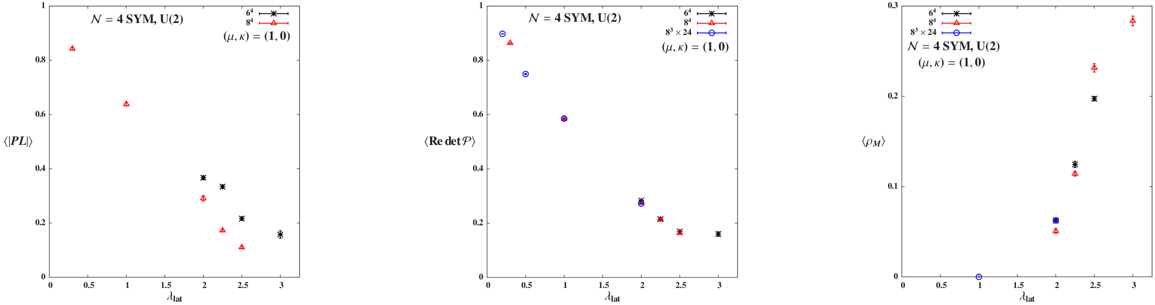
Right: (Complexified) Polyakov loop wanders off to $\sim 10^9$





Backup: Problem with U(1) flat directions

Monopole condensation $\longrightarrow$ confined lattice phase not present in continuum



Around the same $2\lambda_{\mathrm{lat}} \approx 2\ldots$

Left: Polyakov loop falls towards zero

Center: Plaquette determinant falls towards zero

Right: Density of U(1) monopole world lines becomes non-zero

Backup: Naively regulating U(1) flat directions

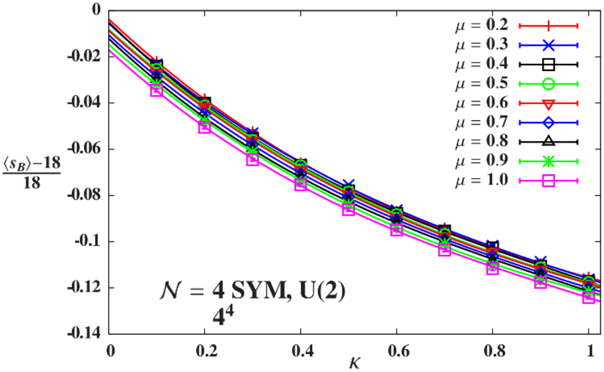
In earlier work we added another soft $\mathcal{Q}$-breaking term

$$S_{\text{soft}} = \frac{N}{4\lambda_{\text{lat}}}\mu^2 \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - 1\right)^2 + \kappa \sum_{a<b} \left|\det \mathcal{P}_{ab} - 1\right|^2$$

More sensitivity to κ than to μ^2

Showing $\mathcal{Q}$ Ward identity from bosonic action

$$\langle s_B \rangle = 9N^2/2$$

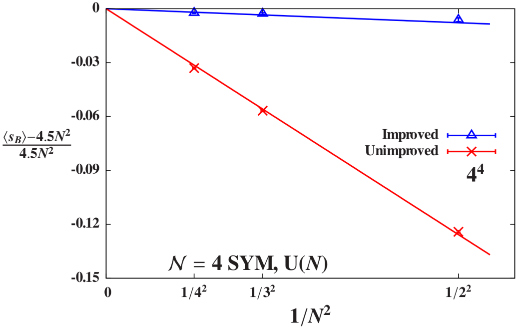


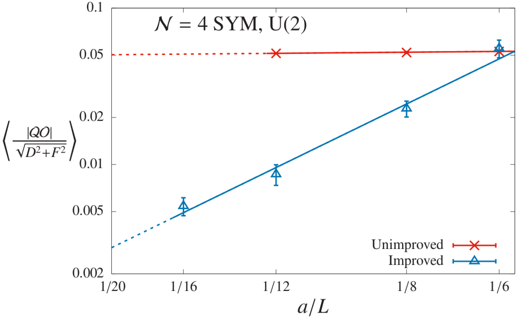
Backup: Better regulating U(1) flat directions

$$S = \frac{N}{4\lambda_{\text{lat}}}\left[\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\left\{\overline{\mathcal{D}}_a\mathcal{U}_a + G\sum_{a<b}\left[\det\mathcal{P}_{ab} - 1\right]\mathbb{I}_N\right\} - \frac{1}{2}\eta d\right) - \frac{1}{4}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V\right]$$

$\mathcal{Q}$ Ward identity violations scale $\propto 1/N^2$ (**left**) and $\propto (a/L)^2$ (**right**)

$\sim$ effective '$\mathcal{O}(a)$ improvement' since $\mathcal{Q}$ forbids all dim-5 operators





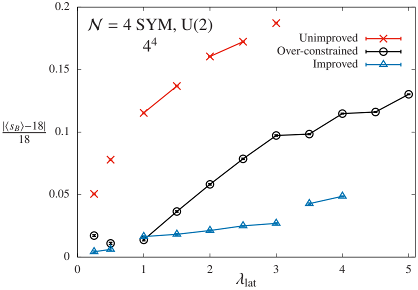
Backup: Supersymmetric moduli space modification [arXiv:1505.03135]

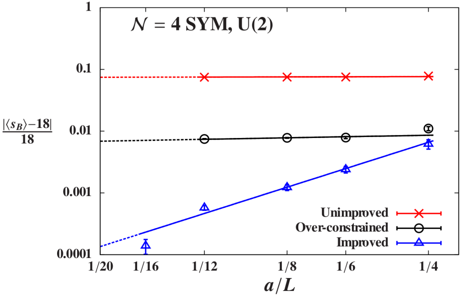
Method to impose $\mathcal{Q}$-invariant constraints on generic site operator $\mathcal{O}(n)$

Modify auxiliary field equations of motion $\longrightarrow$ moduli space

$$d(n) = \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n) \qquad \longrightarrow \qquad d(n) = \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(n) + G\mathcal{O}(n)\mathbb{I}_N$$

Including both U(1) and SU(N) $\in \mathcal{O}(n)$ over-constrains system





Backup: Dimensional reduction to 2d $\mathcal{N} = (8, 8)$ SYM

Naive for now: 4d $\mathcal{N} = 4$ SYM code with $N_x = N_y = 1$

$A_4^* \longrightarrow A_2^*$ (triangular) lattice

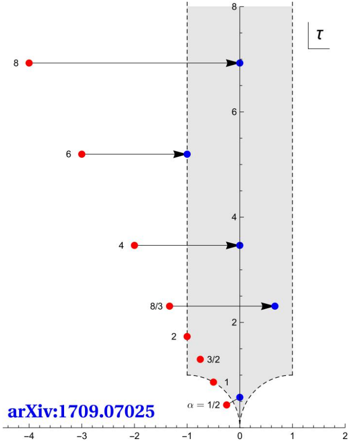
Torus **skewed** depending on $\alpha = L/N_t$

Modular transformation into fundamental domain

$\longrightarrow$ some skewed tori actually rectangular

Also need to stabilize compactified links

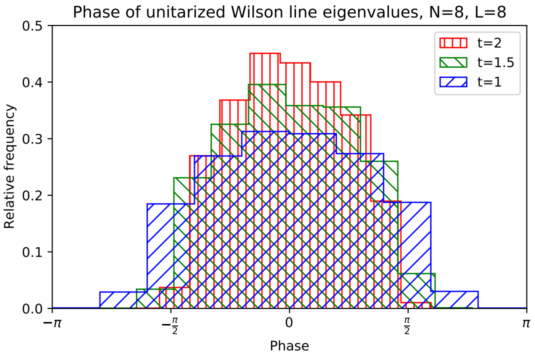
to ensure broken center symmetries

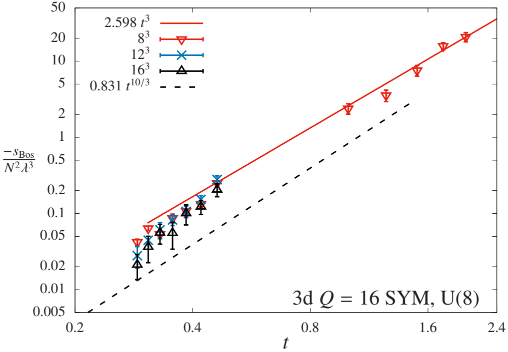


Backup: High-temperature ($t \gtrsim 1$) 3d maximal SYM

Wilson line eigenvalue phases localized rather than uniform (**left**)

Thermodynamics consistent with weak-coupling expectation $\propto t^3$ (**right**)



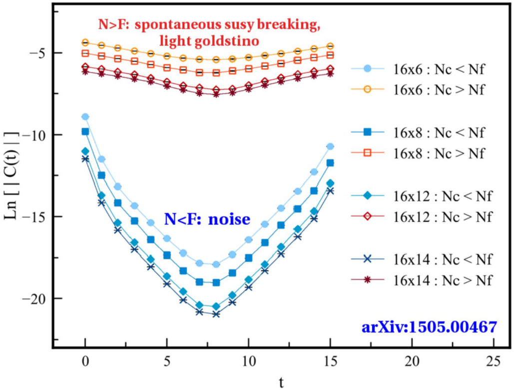


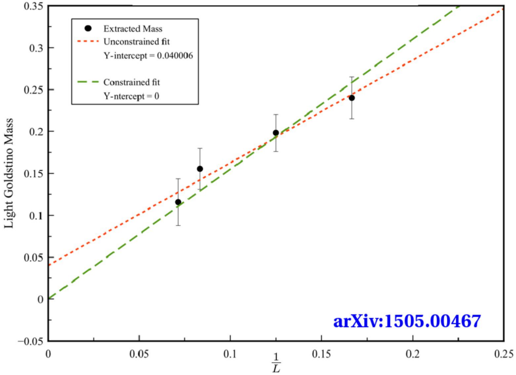
Backup: Dynamical susy breaking in 2d lattice superQCD

U(N) superQCD with F fundamental hypermultiplets

Observe spontaneous susy breaking only for $N > F$, as expected

Catterall–Veernala, arXiv:1505.00467





Backup: More on dynamical susy breaking

Spontaneous susy breaking means $\langle 0 \,| H |\, 0 \rangle > 0$ or equivalently $\langle \mathcal{Q}\mathcal{O} \rangle \neq 0$

Twisted superQCD auxiliary field e.o.m. $\longleftrightarrow$ Fayet–Iliopoulos D-term potential

$$d = \overline{\mathcal{D}}_a \mathcal{U}_a + \sum_{i=1}^{F} \phi_i \overline{\phi}_i - r\mathbb{I}_N \qquad \longleftrightarrow \qquad \mathrm{Tr}\left[\left(\sum\nolimits_i \phi_i \overline{\phi}_i - r\mathbb{I}_N\right)^2\right] \in H$$

Have $F \times N$ scalar vevs to zero out $N \times N$ matrix

$\longrightarrow N > F$ suggests susy breaking, $\langle 0 \,| H |\, 0 \rangle > 0 \longleftrightarrow \langle \mathcal{Q}\eta \rangle = \langle d \rangle \neq 0$