Lattice strong dynamics for composite Higgs sectors

David Schaich

Swansea Theory Seminar, 11 June 2021

arXiv:1807.08411 arXiv:1809.02624 arXiv:2007.01810

and work in progress with the Lattice Strong Dynamics Collaboration

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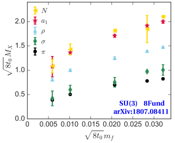
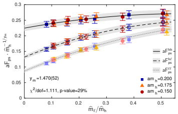
Overview and plan

Lattice field theory is a broadly applicable tool
to study strongly coupled near-conformal theories

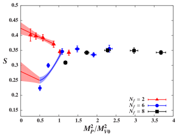


Composite Higgs motivation for near-conformal lattice studies

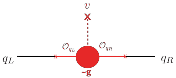


Light scalar and low-energy effective theory connection

Mass-split systems for improved control



More: S parameter, partial compositeness, . . .



These slides: davidschaich.net/talks/2106Swansea.pdf

Interaction encouraged — complete coverage unnecessary

Motivation from composite Higgs sectors

Large Hadron Collider priority

Study fundamental nature of the Higgs

Composite Higgs sector
can stabilize electroweak scale

New strong dynamics must differ from QCD

—Flavour-changing neutral currents
—Electroweak precision observables
—SM-like Higgs boson with $M \approx 0.5 v_{\text{EW}}$

Near-conformality for composite Higgs

New strong dynamics must differ from QCD

—Flavour-changing neutral currents
—Electroweak precision observables
—SM-like Higgs boson with $M \approx 0.5 v_{\mathrm{EW}}$

Near-conformal dynamics
can help with all three issues

Near-conformality $\longrightarrow$ natural scale separation, novel IR dynamics

UV conformal chirally broken IR

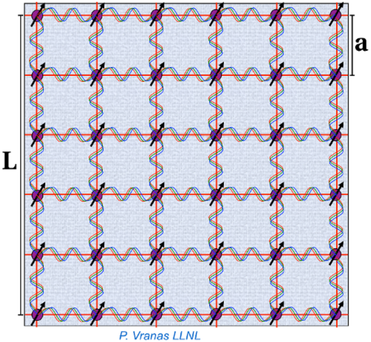
Λ_{UV} fermion masses Λ_{IR} Higgs dynamics

Can't rely on intuition from QCD or $\mathcal{N} = 4$ SYM $\longrightarrow$ lattice calculations

Background: Lattice field theory in a nutshell

Formally $\quad \langle \mathcal{O} \rangle = \dfrac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]}$

Regularize by formulating theory in finite, discrete, euclidean space-time
↖Gauge invariant, non-perturbative, *d*-dimensional



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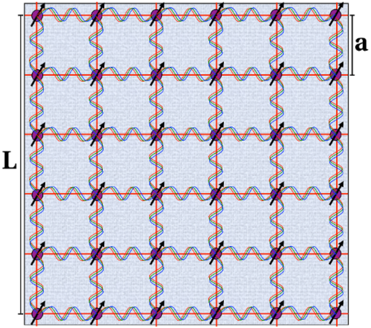

P. Vranas LLNL

Spacing between lattice sites ("a")
$\longrightarrow$ UV cutoff scale $1/a$

Remove cutoff: $\quad a \to 0 \quad (L/a \to \infty)$

Discrete $\longrightarrow$ continuous symmetries $\checkmark$

Numerical lattice field theory calculations

High-performance computing $\longrightarrow$ evaluate up to $\sim$billion-dimensional integrals

(Dirac operator as $\sim 10^9 \times 10^9$ matrix)

Results to be shown, and work in progress, require state-of-the-art resources

Many thanks to USQCD–DOE, DiRAC–STFC–UKRI, and computing centres!

Lassen @Livermore

USQCD @JLab

DiRAC @Cambridge

Numerical lattice field theory algorithms

Importance sampling Monte Carlo

Algorithms sample field configurations with probability $\frac{1}{\mathcal{Z}} e^{-S[\Phi]}$

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\Phi \;\; \mathcal{O}(\Phi) \;\; e^{-S[\Phi]} \longrightarrow \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(\Phi_i) \;\; \text{with stat. uncertainty} \;\; \propto \frac{1}{\sqrt{N}}$$

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Lattice calculation requires specific theory $\longleftrightarrow$ lattice action $S[\Phi]$

Our strategy aims to gain generic insights into near-conformal strong dynamics

Lattice Strong Dynamics Collaboration

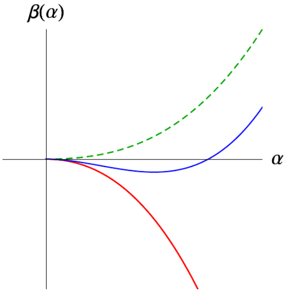
Argonne	Xiao-Yong Jin, James Osborn
Bern	Andy Gasbarro
Boston	Casey Berger, Rich Brower, Evan Owen, Claudio Rebbi
Colorado	Anna Hasenfratz, Ethan Neil, Curtis Peterson
UC Davis	Joseph Kiskis
Livermore	Dean Howarth, Pavlos Vranas
Liverpool	Chris Culver, DS
Michigan	Enrico Rinaldi
Nvidia	Evan Weinberg
Oregon	Graham Kribs
Siegen	Oliver Witzel
Trieste	James Ingoldby
Yale	Thomas Appelquist, Kimmy Cushman, George Fleming

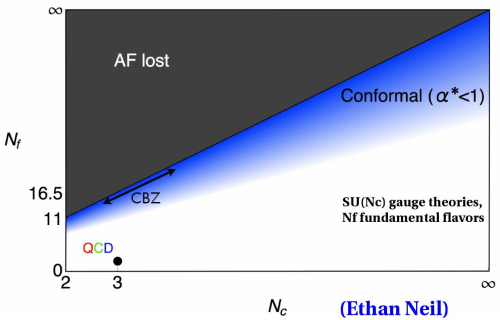
Exploring the range of possible phenomena in strongly coupled field theories

Choosing near-conformal theories to analyze for generic insights

For SU(3) gauge group, observe near-conformal strong dynamics
for $8 \lesssim N_F \lesssim 10$ light fundamental fermions

Intermediate between $N_F = 2$ QCD
and weakly coupled Banks–Zaks IR fixed point for $N_F \simeq 16$ (massless)





From new strong dynamics to electroweak symmetry breaking

For SU(3) with N_F fundamental fermions, chiral symmetry breaking is

$$\mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R \longrightarrow \mathrm{SU}(N_F)_V$$

Lattice studies of strong sector apply to two distinct model interpretations

1) Electroweak symmetry breaks directly

$$\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset \mathrm{SU}(N_F)_L \times \mathrm{SU}(N_F)_R \longrightarrow \mathrm{U}(1)_{\text{em}} \subset \mathrm{SU}(N_F)_V$$

2) Electroweak symmetry breaks radiatively via vacuum misalignment

$$\mathrm{SU}(N_F)_V \supset \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \longrightarrow \mathrm{U}(1)_{\text{em}}$$

From new strong dynamics to electroweak symmetry breaking

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Lattice studies of strong sector apply to two distinct model interpretations

1) Electroweak symmetry breaks directly

$\Longrightarrow$ Symmetry breaking scale $F^2 = v_{\text{EW}}^2 = 246$ GeV

$\Longrightarrow$ Higgs boson as 0^{++} isosinglet scalar ('σ')

2) Electroweak symmetry breaks radiatively via vacuum misalignment

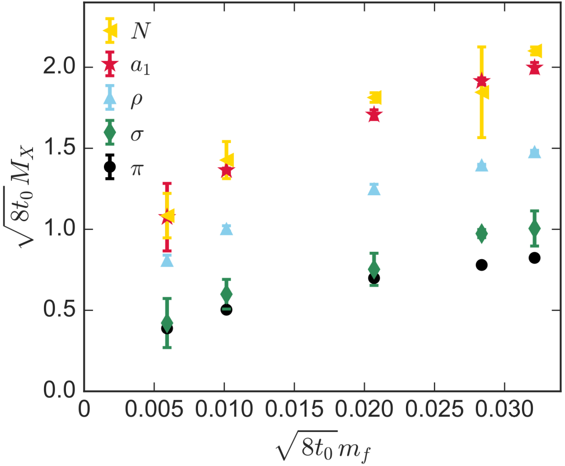
$\Longrightarrow$ Symmetry breaking scale $F^2 = v_{\text{EW}}^2/\xi$ with $\xi \lesssim 0.1$

$\Longrightarrow$ Higgs boson as pseudo-Goldstone ('π') [affected by 0^{++}]

$N_F = 8$ light composite spectrum

arXiv:1807.08411

Light 0^{++} scalar observed, $M_\sigma \approx M_\pi \lesssim M_\rho/2$ qualitatively different than QCD



Improved staggered fermions

Masses in units of lattice scale $\sqrt{8t_0}$

$L \geq 5.3/(a \cdot M_\pi) \longrightarrow$ up to $64^3 \times 128$

[$96^3 \times 192$ with $a \cdot m_f = 0.00056$ in progress]

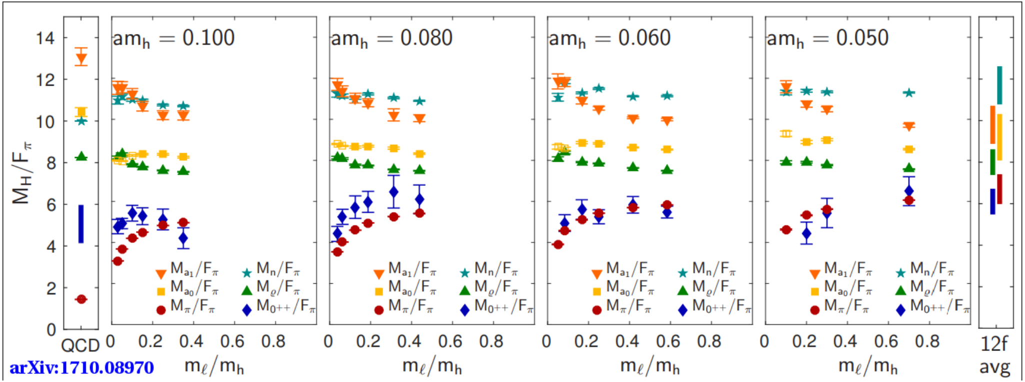
Large 0^{++} uncertainties from mixing with vacuum

Light scalar is generic feature of near-conformal dynamics

Recently observed by many groups considering various theories

✓ SU(3) with $N_F = 8$ fundamental

✓ SU(3) with $N_F = 12$ fundamental



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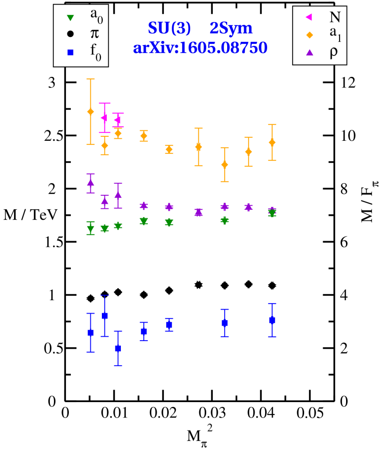
✓ SU(3) with $N_F = 12$ fundamental

✓ SU(3) with $N_F = 2$ sextet



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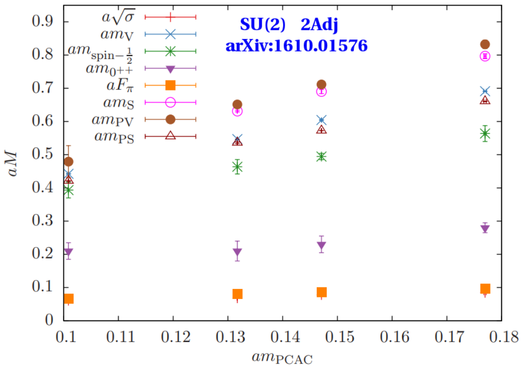
✓ SU(3) with $N_F = 2$ sextet

✓ SU(2) with $N_F = 2$ adjoint



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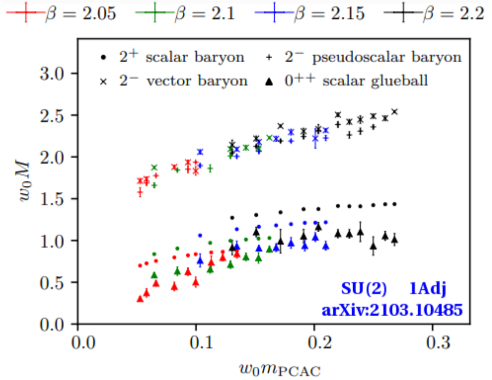
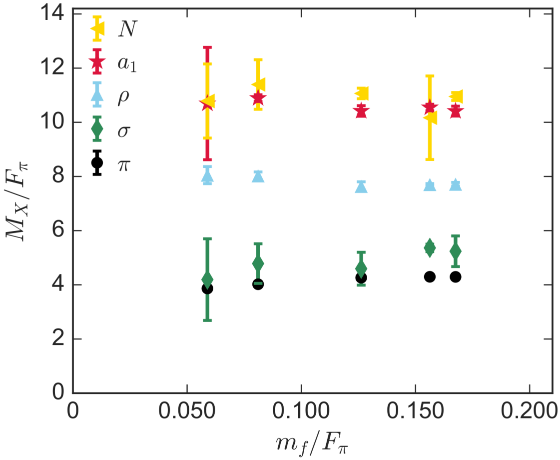
✓ SU(2) with $N_F = 2$ adjoint

✓ SU(2) with $N_F = 1$ adjoint



From lattice results to phenomenology — a gap to bridge

$M_\rho / F_\pi \approx 8$ naively implies resonance $\sim$2 TeV$/\sqrt{\xi}$ (with broad $\Gamma_\rho / M_\rho \approx 0.2$)



This may be too naive

1) Need $M_\pi / F_\pi \to 0$ as $m_f \to 0$

2) Need exactly 3 massless pions, other $N_F^2 - 4 = 60$ stay massive

Need effective field theory (EFT) for chiral extrapolation

Low-energy chiral effective theories

Chiral perturbation theory (χPT) is EFT of pions familiar from QCD

Need to extend χPT to incorporate light scalar,
with power-counting rules to organize interactions

Several candidate EFTs currently being explored

—Completely generic (no assumptions, many parameters)
[Soto–Talavera–Tarrus; Hansen–Langaeble–Sannino; Catà–Müller]

—Based on linear sigma model [LSD, arXiv:1809.02624]

—Treating scalar as 'dilaton', pseudo-Goldstone of broken scale invariance
[Matsuzaki–Yamawaki; Golterman–Shamir; Appelquist–Ingoldby–Piai]

Testing low-energy EFTs

—Completely generic (no assumptions, many parameters)

—Based on linear sigma model

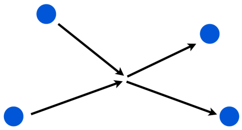
—Treating scalar as ‘dilaton’, pseudo-Goldstone of broken scale invariance

Work in progress to test and compare EFTs — limited lattice results available

Focus on well-developed dilaton-χPT with relatively few parameters

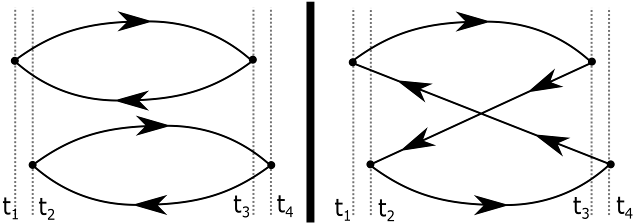
[state of the art by Appelquist–Ingoldby–Piai, arXiv:2012.09698]

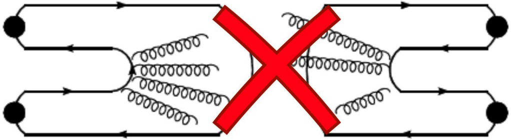
Compute more results to fit $\longrightarrow$ two-pion elastic scattering



$I = 2$ s-wave pion scattering

Same-sign $\pi^{\pm}\pi^{\pm}$ scattering avoids challenging 'disconnected' contributions
related to $W^{\pm}W^{\pm}$ scattering via EFTs





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Well-established path from finite-volume interaction energy to low-momentum scattering length $a_{\pi\pi}$ and effective range $r_{\pi\pi}$

$$k^2 = E_{\pi\pi}^2/4 - M_\pi^2$$

$$\frac{1}{\pi L}\left[\sum_{\vec{\jmath}\neq 0}^{\Lambda}\frac{4\pi^2}{4\pi^2|\vec{\jmath}|^2 - k^2 L^2} - 4\pi\Lambda\right] = \frac{1}{a_{\pi\pi}} + \frac{1}{2}M_\pi^2 r_{\pi\pi}\left(\frac{k^2}{M_\pi^2}\right) + \mathcal{O}\left(\frac{k^4}{M_\pi^4}\right)$$

$N_F = 8$ scattering results

arXiv:2106.XXXXX

Unhappy six-param. fit to tree-level dilaton-χPT

$[\chi^2/\text{dof} = 56/18 \longrightarrow \text{p-value} = 10^{-5}]$

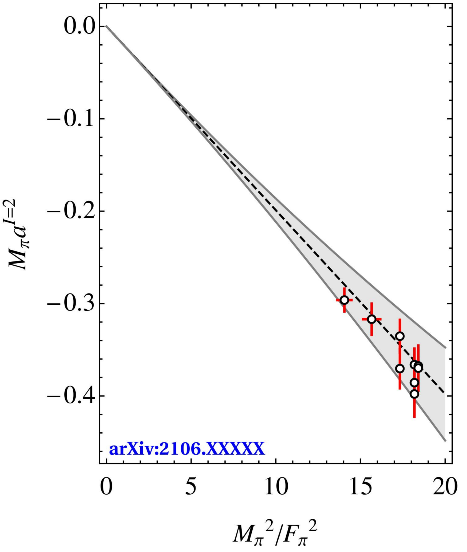
Parameters suppress dilaton effects:

$$M_\pi a_{\pi\pi} = -\frac{M_\pi^2}{16\pi^2 F_\pi^2}\left[1 - \mathcal{O}\left(10^{-3}\right)\frac{M_\pi^2}{M_\sigma^2}\right]$$

Grey band estimates size of NLO effects

to be considered

$I = 0$ scattering another future possibility



Splitting $N_F = 10 \longrightarrow 4 + 6$ for improved control

arXiv:2007.01810

Consider four light flavours (mass $\widetilde{m}_\ell$) and six heavy flavours (mass $\widetilde{m}_h$)

[switch to domain-wall fermions — better symmetries, larger costs]

In UV, effectively massless $N_F = 10 \longrightarrow$ flow toward conformal IR fixed point

Around $\Lambda_{IR} \sim \widetilde{m}_h$ heavy flavours decouple

Subsequent 4-flavour symmetry breaking

$\longrightarrow$ composite Higgs sensitive to 10-flavour IR fixed point

Splitting $N_F = 10 \longrightarrow 4 + 6$ for improved control

arXiv:2007.01810

Consider four light flavours (mass $\widetilde{m}_\ell$) and six heavy flavours (mass $\widetilde{m}_h$)

UV — conformal — chirally broken → IR

Λ_{UV} fermion masses Λ_{IR} Higgs dynamics

Flow toward IR fixed point $\longrightarrow$ bare gauge coupling $\beta = 6/g_0^2$ irrelevant

$$\frac{\Lambda_{UV}}{\Lambda_{IR}} \sim \frac{1}{a \cdot \widetilde{m}_h} \quad \longrightarrow \quad a \to 0 \text{ continuum limit is } a \cdot \widetilde{m}_h \to 0 \text{ (with } \widetilde{m}_\ell / \widetilde{m}_h \text{ fixed)}$$

Chiral limit is $\widetilde{m}_\ell / \widetilde{m}_h \to 0$ and then no free parameters remain

$N_F = 4 + 6$ continuum limit and hyperscaling

Continuum limit is $a \cdot \widetilde{m}_h \to 0$ (with $\widetilde{m}_\ell / \widetilde{m}_h$ fixed)

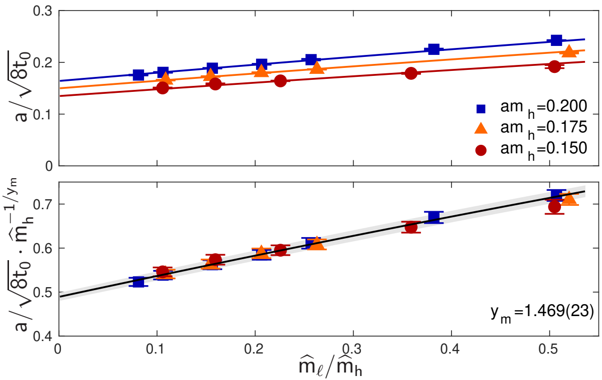
Lattice spacing decreases
as heavy mass decreases

Obeys conformal hyperscaling relation

$$\frac{a}{\sqrt{8t_0}} = \widetilde{m}_h^{1/y_m} \Phi_a(\widetilde{m}_\ell / \widetilde{m}_h)$$

Fit quadratic ansatz for $\Phi_a(\widetilde{m}_\ell / \widetilde{m}_h)$
$\longrightarrow$ mass anomalous dimension

$$\gamma_m = y_m - 1 = 0.47(2)$$



$N_F = 4 + 6$ hyperscaling and mass anomalous dimension

Spectrum also exhibits hyperscaling

Here pseudoscalar decay constants

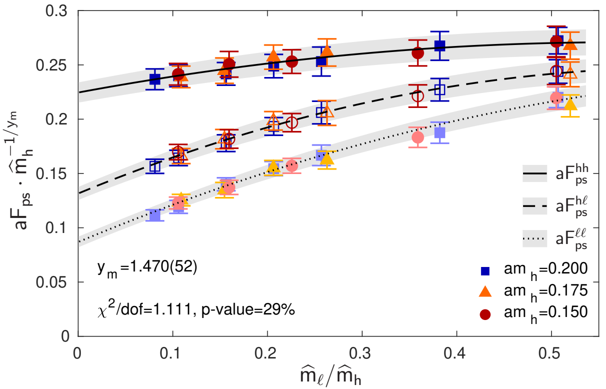
[light–light, heavy–light, heavy–heavy]

$$a \cdot F = \widetilde{m}_h^{1/y_m} \Phi_F(\widetilde{m}_\ell / \widetilde{m}_h)$$

Fit polynomial ansatz for $\Phi_F(\widetilde{m}_\ell / \widetilde{m}_h)$

$\longrightarrow$ mass anomalous dimension

$$\gamma_m = y_m - 1 = 0.47(5)$$



Large anomalous dim's from strong dynamics phenomenologically desirable

Testing dilaton-χPT with $N_F = 4 + 6$

Dilaton-χPT relation:

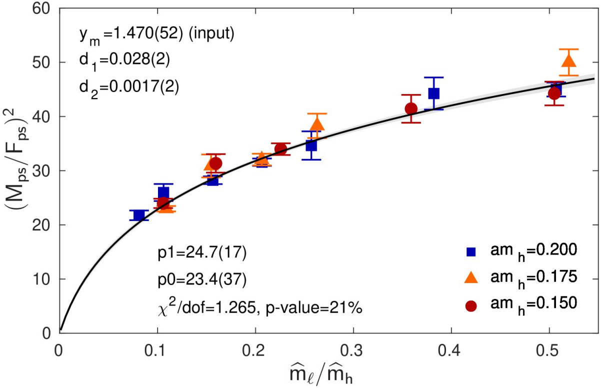
$$\frac{M_\pi^2}{F_\pi^2} = \frac{1}{y_m d_1} W_0\left(\frac{y_m d_1}{d_2} \frac{\widetilde{m}_\ell/\widetilde{m}_h}{\Phi_a(0)\sqrt{8t_0}}\right)$$

in terms of Lambert W-function

Fix y_m from hyperscaling

Good fit suggests light scalar

(still to be analyzed directly)

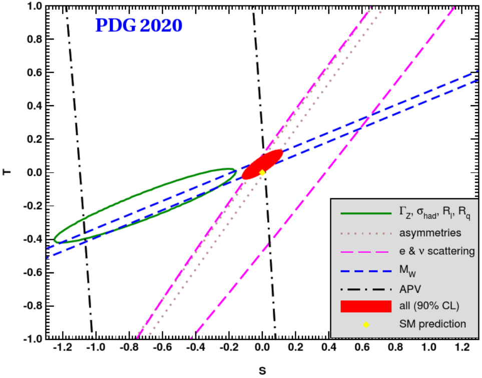


$I = 2$ scattering getting underway via DiRAC

Domain-wall fermions also assist S param. and partial compositeness analyses

Electroweak precision observable — the S parameter

Constrain Higgs sector from vector-minus-axial vacuum polarization $\Pi_{V-A}(Q)$



Experimental $S = -0.01 \pm 0.10$

vs. QCD-like $S \approx 0.43\sqrt{\xi}$

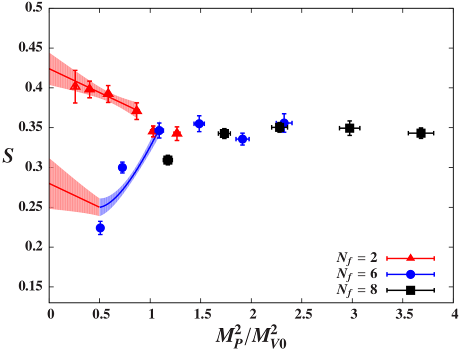
Related to χPT low-energy constant L_{10}

[cf. arXiv:2010.01920]

Domain-wall fermion symmetries important

S parameter on the lattice

$$\mathcal{L}_\chi \supset -\frac{S}{32\pi^2} g_1 g_2 B_{\mu\nu} \mathrm{Tr}\left[U \mathcal{T}_3 U^\dagger W^{\mu\nu}\right] \longrightarrow$$



Prior LSD study of $N_F = 2, 6, 8$ [arXiv:1405.4752]

$N_F = 4 + 6$ getting underway via DiRAC

$S/\sqrt{\xi} = 0.42(2)$ for $N_F = 2$ matches QCD ✓

Significant reduction from larger N_F,

chiral extrapolation again challenging

V–A vacuum polarization also contributes to Higgs potential [arXiv:1903.02535]

Anomalous dimensions for partial compositeness

Old challenge for new strong dynamics

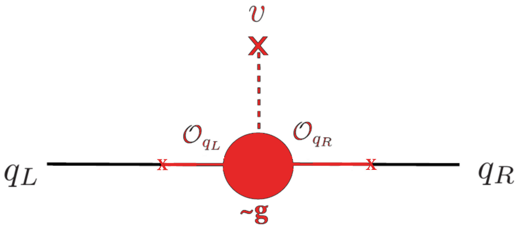
Quark & lepton masses $\sim \frac{\overline{q}q\overline{\psi}\psi}{\Lambda_{UV}^2}$ vs. flavour-changing NCs $\sim \frac{\overline{q}q\overline{q}q}{\Lambda_{UV}^2}$

Partial compositeness alternative

Linear mixing with composite partners

$$\mathcal{L} \supset \lambda \overline{q} \mathcal{O}_q + \text{h.c.}$$

$$\longrightarrow m_q \sim v_{\text{EW}} \left(\frac{\text{TeV}}{\Lambda_{UV}}\right)^{4-2\gamma_q}$$



Large mass hierarchy $\longleftrightarrow$ $\mathcal{O}(1)$ anomalous dimensions

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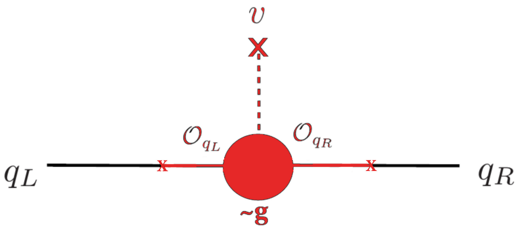


Large mass hierarchy $\longleftrightarrow$ $\mathcal{O}(1)$ anomalous dimensions

Example: $\Lambda_{UV} = 10^{10}$ TeV $\longrightarrow$ $m_q \sim \mathcal{O}(\text{MeV})$ from $\gamma_q \approx 1.75$

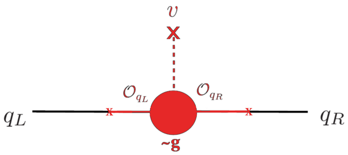
$m_q \sim \mathcal{O}(\text{GeV})$ from $\gamma_q \approx 1.9$

Plans for partial compositeness on the lattice

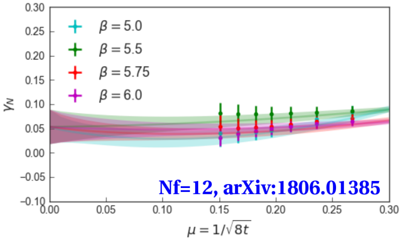
For SU(3) theories $\mathcal{O}_q \sim \psi\psi\psi \sim$ baryons with scaling dim. $[\mathcal{O}_q] = \frac{9}{2} - \gamma_q$

Plan $N_F = 4 + 6$ predictions $\gamma_q = -\dfrac{d \log Z_{\mathcal{O}_q}(\mu)}{d \log \mu}$ from RI/MOM non-pert. renorm.

and from ratios of gradient-flowed operators $\propto t^{\gamma_q/2}$ [arXiv:1806.01385]



$$m_q \sim v_{\mathrm{EW}} \left(\frac{\mathrm{TeV}}{\Lambda_{UV}}\right)^{4-2\gamma_q}$$



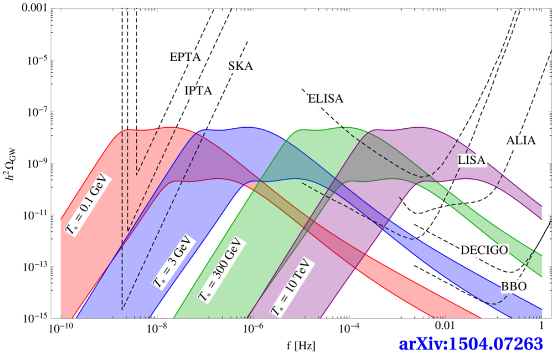
Gravitational waves from early-universe phase transition

First-order confinement transition $\longrightarrow$ stochastic background of grav. waves

Massless $N_F = 4$ transition is first-order

Work in progress
to map $N_F = 4 + 6$ phase diagram

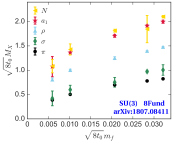
Then compute properties of transition:
latent heat, nucleation rate, etc.



arXiv:1504.07263

Recap and outlook

Lattice field theory is a broadly applicable tool
to study strongly coupled near-conformal theories

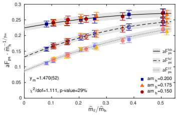


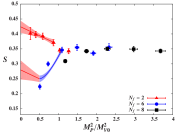
Near-conformality useful for new strong dynamics

—Light scalar observed, so far consistent with dilaton-χPT

—Reduced S parameter

—Natural scale separation for flavour physics



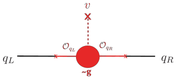


Ongoing investigations of mass-split $N_F = 4 + 6$ system

—S parameter and $I = 2$ scattering

—Baryon scaling dimensions for partial compositeness

—Finite-temperature transitions $\longrightarrow$ gravitational waves



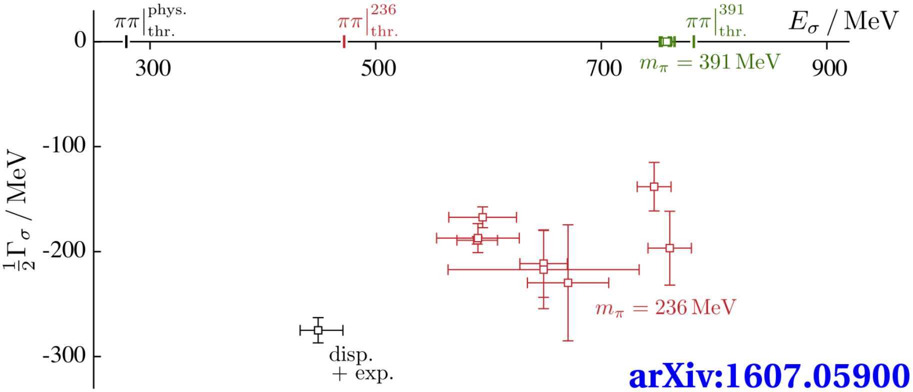
Thanks for your attention!

Any further questions?

Funding and computing resources

UK Research
and Innovation

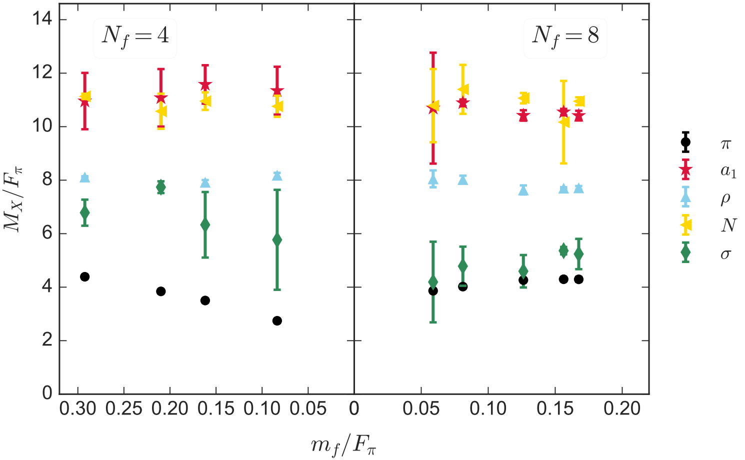
Backup: Singlet scalar in QCD spectrum



In lattice QCD, isosinglet scalar mass $M_S \gtrsim 2M_P$

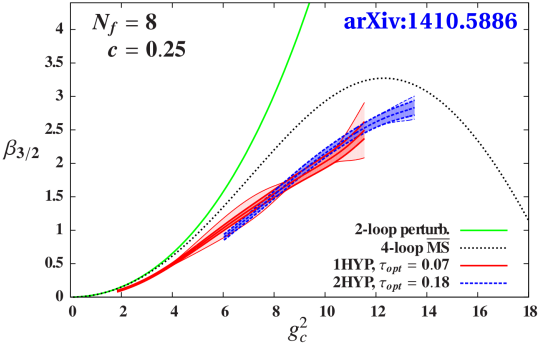
$\longrightarrow$ significant mixing with $I = 0$ two-pion scattering states

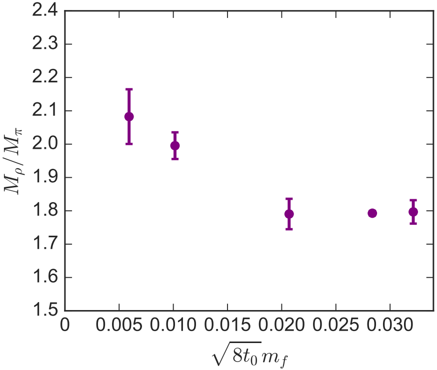
Backup: Direct comparison of QCD-like and near-conformal 0^{++}



$N_F = 8$ scalar much lighter than $M_\sigma \approx M_\pi/2$ for QCD-like $N_F = 4$

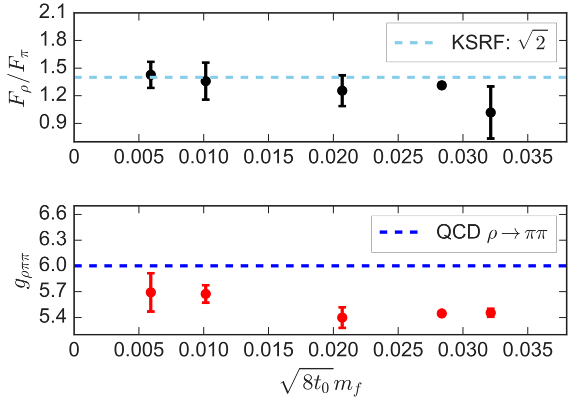
Backup: $N_F = 8$ spontaneous chiral symmetry breaking





Monotonic step-scaling ($\sim -\beta$) function and increasing M_ρ / M_π are evidence against conformal IR fixed point

Backup: Width of $N_F = 8$ vector resonance



$$F_\rho = \sqrt{2} F_\pi \qquad g_{\rho\pi\pi} = \frac{M_\rho}{\sqrt{2} F_\pi}$$

Kawarabayashi–Suzuki, 1966

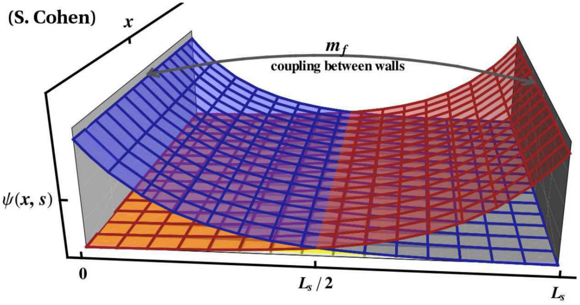
Riazuddin–Fayyazuddin, 1966

Can derive from current algebra, hidden local symetry, chiral EFT

Confirm first KSRF relation, then apply second

$\longrightarrow$ vector width $\Gamma_\rho = \dfrac{g^2_{\rho\pi\pi} M_\rho}{48\pi} \simeq 450\text{ GeV}/\sqrt{\xi}$ — hard to see at LHC

Backup: Domain-wall fermions

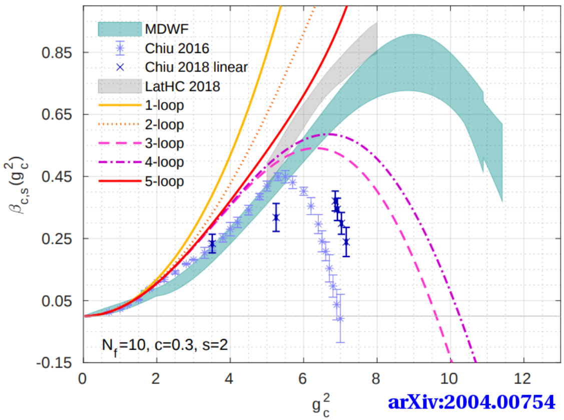


L_s copies of 4d gauge fields (expensive with $L_s = 16$!)

Localized fermions have renormalized mass $m = m_f + m_{res}$
with residual mass $m_{res} \ll m_f$ from overlap around $L_s/2$

$L_s \to \infty \longrightarrow$ exact chiral symmetry at non-zero lattice spacing

Backup: $N_F = 10$ step-scaling function (SSF)



SSF $\sim$ negative of β function
integrated over $s = 2\times$ scale change

Lattice calculations use
finite-volume gradient flow schemes
parameterized by c (here $c = 0.3$)

Evidence for conformal IR fixed point at strong $g_c^2 \sim 13$ in $c = 0.3$ scheme

$\beta_2 = 0$ for $g_c^2 \approx 11$ in $c = 0.25$ scheme, with larger systematic uncertainties