Lattice gauge theory at the electroweak scale

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Strong dynamics at the electroweak scale
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Overview and plan

Lattice gauge theory is a broadly applicable tool to study strongly coupled systems.

Especially important when QCD-based intuition may be unreliable.

A high-level summary of lattice gauge theory

$\beta$ functions and anomalous dimensions

Light scalar from near-conformal dynamics

More possible topics for discussion

- Electroweak S parameter
- Composite dark matter
- Multi-rep. composite Higgs UV completions

...
The essence of lattice gauge theory

Lattice discretization is a non-perturbative regularization of QFT

Formulate theory on finite, discrete euclidean space-time \( \rightarrow \) the lattice

Spacing between lattice sites ("\( a \)") \( \rightarrow \) UV cutoff scale \( 1/a \)

Removing cutoff: \( a \rightarrow 0 \) (with \( L/a \rightarrow \infty \))

Finite number of degrees of freedom \( (\sim 10^9) \)

\( \rightarrow \) numerically compute observables via importance sampling

\[
\langle \mathcal{O} \rangle = \frac{1}{V} \int \mathcal{D}U \ \mathcal{O}(U) \ e^{-S[U]} \quad \rightarrow \quad \frac{1}{N} \sum_{k=1}^{N} \mathcal{O}(U_k)
\]
Features of lattice gauge theory

- Fully non-perturbative predictions from first principles (lagrangian)
- Fully gauge invariant—no gauge fixing required
- Applies directly in four dimensions
- Euclidean SO(4) rotations & translations (→ Poincaré symmetry) recovered automatically in the $a \to 0$ continuum limit
Limitations of lattice gauge theory

Need UV completion, (usually) include only strong sector

Finite volume (usually) needs to contain all correlation lengths
→ unphysically large masses extrapolated to chiral limit via EFT

Chiral symmetry of lattice fermion operator complicated

Obstructions to chiral gauge theories, real-time dynamics, susy
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**Lattice fermion discretizations**

Tension between chiral symmetry vs. ‘doubling’ of lattice fermions

**Naive** $\rightarrow$ 16$F$ continuum fermions from $F$ lattice fields, large $U(4F)_V \times U(4F)_A$ chiral symmetry

**Staggered** $\rightarrow$ 4$F$ continuum fermions, $U(F)_V \times U(F)_A$ chiral symm.

**Wilson** $\rightarrow$ $F$ continuum fermions, no chiral symmetry

**Domain wall** $\rightarrow$ $F$ continuum fermions, lattice “remnant” $SU(F)_V \times SU(F)_A$ chiral symmetry
Symmetries of lattice fermions

Different lattice symmetries for fixed $N_F$ continuum fermions

**Domain wall**

$SU(N_F)_V \times SU(N_F)_A$

**Staggered**

$U(N_F/4)_V \times U(N_F/4)_A$

**Wilson**

None

All $\rightarrow$ same UV continuum limit ('lattice universality')

Possibility different lattice symmetries $\rightarrow$ different IR dynamics?

Example of 3d $O(n)$ scalar model
Lattice gauge theory beyond QCD

Lattice calculations especially important for non-QCD strong dynamics

Exploratory investigations of representative systems

→ elucidate generic dynamical phenomena, connect with EFT


Executive Summary

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Identify the new physics of dark matter
- Understand cosmic acceleration: dark energy and inflation
- Explore the unknown: new particles, interactions, and physical principles.
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Non-QCD strong dynamics

Two main directions (not mutually exclusive)

Near-conformal dynamics from many fermionic d.o.f.
  → large number of fundamental fermions or a few in a larger rep

Different symmetries from different gauge group or reps
  → (pseudo)real reps for cosets $\text{SU}(n)/\text{Sp}(n)$ or $\text{SU}(n)/\text{SO}(n)$

Today focus on near-conformality

Study a few representative systems, look for similarities/difference vs. QCD

Start with non-perturbative $\beta$ function
\( \beta \) function motivation

\[ \beta = \frac{d g^2}{d \log \mu^2} \rightarrow \text{scale dependence of running coupling} \]

Perturbative

\[ \beta(g^2) = -\frac{g^4(\mu^2)}{16\pi^2} \left[ b_1 + b_2 \frac{g^2(\mu^2)}{16\pi^2} \right] + \mathcal{O}(g^8) \]

Asymptotic freedom in UV \( \rightarrow b_1 = \frac{1}{3} [11C_2(G) - 4N_F T(R)] > 0 \)

\( b_2 < 0 \) might give non-trivial conformal fixed point in IR

Banks & Zaks make argument rigorous for \( b_1 \approx 0 \)
Lattice $g^2$ for non-perturbative $\beta$ function

First step: Define measurable $g^2$ with scale given by lattice size $L$

Use Yang–Mills gradient flow
(integrating infinitesimal smoothing operation)

Local observables measured after “flow time” $t$
depend on original fields within $r \sim \sqrt{8t}$

Flowed energy density $E(t) = -\frac{1}{2} \text{Tr} \left[ G_{\mu\nu}(t) G^{\mu\nu}(t) \right]$
perturbatively gives $g^2_{\text{MS}}(\mu) \propto t^2 E(t)$ with $\mu = 1/\sqrt{8t}$

Tie to lattice size by defining $g^2_c(L, a)$ at fixed $c = L/\sqrt{8t}$
(scheme dependent as expected)
Step scaling for non-perturbative $\beta$ function

Next step: Scale change $L \rightarrow sL$ gives discrete $\beta$ function

\[
\beta_s(g_c^2; L) = \frac{g_c^2(sL; a) - g_c^2(L; a)}{\log(s^2)} \quad \text{for} \quad s \rightarrow 1 \\
= -\beta \left(g^2(\mu^2)\right)
\]

$N_F = 12$ staggered fermions, bare coupling $\beta_F \approx 12/g_0^2$

With $s = 3/2$ have

$L = 12 \rightarrow 18 \quad 16 \rightarrow 24$

$20 \rightarrow 30 \quad 24 \rightarrow 36$

$s = 2$ and $4/3$ also accessible

\[g_c^2 \text{ for all } L \text{ cross around } g_c^2 \approx 7 \rightarrow \beta_s(g_c^2; L) = 0\]

Does $\beta_s$ remain zero as $L \rightarrow \infty$?
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$g_c^2$ for all $L$ cross around $g_c^2 \approx 7 \rightarrow \beta_s(g_c^2; L) = 0$

Does $\beta_s$ remain zero as $L \rightarrow \infty$?
Continuum extrapolation

Final step: Extrapolate \((a/L) \rightarrow 0\) to obtain continuum \(\beta_s(g_c^2)\)

\[ N_F = 12 \] staggered results seemed broadly consistent

Even for different schemes and scale changes \(s\)

Slope at fixed point \(g_\star^2 \approx 7.3\)

\[ \rightarrow \gamma_g^\star = -0.26(2) \] (scheme independent)

Simple \((a/L)^2 \rightarrow 0\) extrapolations fine near gaussian UV fixed point

May need \(g_c^2(L; a) - g_\star^2 \propto L^{\gamma_g^\star}\) finite-size scaling near IR fixed point...
Current status of staggered $N_F = 12 \beta$ function

Developing tension between two independent staggered analyses
$\implies$ not yet consensus about $N_F = 12$ fixed point

Same lattice symmetries
$\implies$ same fixed point

Despite details of lattice action, analyses

Main difference is larger $sL \leq 56$ vs. 36

Tension related to $(a/L)^2 \to 0$ extrapolations vs. finite-size scaling?
\( \beta \) function wrap-up: Challenge I

\( \beta \) function becomes very small as \( N_F \) increases

Order of magnitude decrease for \( N_F = 8 \) (left) vs. \( N_F = 12 \) (right)

Hard to distinguish slow running vs. no running on finite lattices
**β function wrap-up: Challenge II**

Different symmetries of lattice fermions

→ IR fixed points in different universality classes?

Recently reported tensions between staggered vs. domain wall results

→ currently developing story
Anomalous dimension motivation

At IR fixed point, universal anomalous dimensions $\gamma^*$

$\rightarrow$ scheme-independent critical exponents characterizing CFT

Large $\gamma$ wanted for fermion mass generation by new strong dynamics

(hopefully discussed in previous talk)

Near-conformality $\rightarrow$ scheme and scale dependence negligible?

Plan: Focus on staggered $N_F = 12$ IRFP

- Already saw $\gamma^*_g \approx -0.26$ from slope of $\beta$ function
- Extract mass anomalous dimension $\gamma^*_m$ from Dirac eigenmodes
- Extract $\gamma^*_m$ and $\gamma^*_g$ from spectrum finite-size scaling
- Prospects for baryon anomalous dim. for partial compositeness
$\gamma_m^*$ from Dirac eigenvalue mode number $\nu(\lambda)$

$\mathcal{L} \supset \overline{\psi} (\Slash{D} + m) \psi \quad \rightarrow \quad \Slash{D}$ eigenvalues sensitive to $\gamma_m^* = 3 - d[\overline{\psi}\psi]$

Histogram of eigenvalues
$\rightarrow$ spectral density $\rho(\lambda)$

Integral is mode number

$$\nu(\lambda) = 2V \int_0^\lambda \rho(\omega) d\omega$$

Conformal FP: $\rho(\lambda) \propto \lambda^\alpha \quad \rightarrow \quad \nu(\lambda) \propto \lambda^{1+\alpha}$

Mode number RG invariant
$\rightarrow \quad 1 + \gamma_m^* = \frac{4}{1 + \alpha}$

(Del Debbio & Zwicky)
Scale-dependent $\gamma_{\text{eff}}(\lambda)$ from eigenmodes

$\lambda$ defines energy scale $\longrightarrow \nu(\lambda)$ gives effective $\gamma_{\text{eff}}(\lambda)$ at that scale

**UV:** Asymp. freedom $\Rightarrow \gamma_{\text{eff}}(\lambda) \to 0$

or $\alpha(\lambda) \to 3$

**IRFP** $\Rightarrow \gamma_{\text{eff}}(\lambda) \xrightarrow{\lambda \to 0} \gamma_m^*$

$\langle \bar{\psi} \psi \rangle \propto \rho(0) \neq 0 \Rightarrow \alpha \to 0$, breakdown of $\rho(\lambda) \propto \lambda^\alpha$

Monitor $\gamma_{\text{eff}}(\lambda)$ evolution from perturbative UV to strongly coupled IR
\( \gamma_{\text{eff}}(\lambda) \) from eigenmodes for \( N_F = 12 \)

Fit \( \nu(\lambda) \propto \lambda^{1+\alpha} \) in small range of \( \lambda \) \[ 1 + \gamma_{\text{eff}}(\lambda) = \frac{4}{1 + \alpha(\lambda)} \]

\( \nu(\lambda) \) computed stochastically

Include fit ranges in error bands

Multiple \( L^4 \) volumes overlaid,
\( L \)-sensitive data dropped

All systems have \( \rho(0) = 0 \)

Strong dependence on irrelevant bare coupling \( \beta_F \approx 12/g_0^2 \)

\( \gamma_{\text{eff}} \) increasing with \( \lambda \) \( \sim \) “backward flow” at strong coupling
$\gamma_m^*(\lambda)$ from eigenmodes for $N_F = 12$

Extrapolate $\lim_{\lambda \to 0} \gamma_{\text{eff}}(\lambda) = \gamma_m^*$ at conformal IR fixed point

Zoom in on largest volumes, couplings closest to $g^2_\star$ (in this scheme)

Joint quadratic extrapolation $\longrightarrow \gamma_m^* = 0.24(3)$

Uncertainty dominated by $\lambda \to 0$ extrapolation

Single fit for some range of $\lambda > 0$ would give precise result

but generally not $\gamma_m^*$ at the $\lambda \to 0$ IR fixed point
Wilson RG picture of finite-size scaling

Fermion mass $m$ is relevant coupling; gauge coupling $\beta_F$ is irrelevant.

Increase $m$ and decrease RG flow ($L$)

$\longrightarrow$ same point on renormalized trajectory (RT)

Universal flow along RT

Correlation lengths depend on scaling variable

$$x \equiv L \frac{m^{1/(1+\gamma^*_m)}}{}$$

Assuming RG flow quickly reaches RT
Naive finite-size scaling for $N_F = 12$

Correlation lengths depend on scaling variable $x \equiv L \, m^{1/(1+\gamma_m^*)}$

$\longrightarrow \gamma_m^*$ from optimizing curve collapse of $M_H L = f_H(x)$

Curve collapse $\longrightarrow$ non-universal $\gamma_m^*$ from different observables

Conformality requires universal $\gamma^*$

$\longrightarrow$ corrections to scaling from near-marginal gauge coupling?
Corrections to finite-size scaling

Slowly running gauge coupling \( \rightarrow \) RG flow may not reach RT
\( \rightarrow \) non-universal results from curve collapse

Leading correction to scaling:

\[
M_H L = f_H(x, gm^\omega)
\]

where \( \omega = -\gamma^*_g/(1 + \gamma^*_m) \)

Two-loop \( \overline{\text{MS}} \): small \( \omega \approx 0.2 \)

Hard to extract both \( \gamma_m^* \) and \( \gamma_g^* \) from curve collapse analyses

\( \rightarrow \) simplify \( f_H(x, gm^\omega) \approx f_H(x) \left[ 1 + c_g m^\omega \right] \)
Consistent corrected finite-size scaling for $N_F = 12$

Approximate $M_{HL} \approx f_H(x) \left[ 1 + c_g m^\omega \right]$

$\longrightarrow$ consistent $\gamma_m^*$ from all observables and $\beta_F$

Quality of curve collapse also improves

Can attempt combined analyses of multiple data sets...
Combined finite-size scaling analyses for $N_F = 12$

Approximate $M_H L \approx f_H(x) \left[ 1 + c_g m^\omega \right]$

$\rightarrow$ consistent $\gamma^*_m$ from all observables and $\beta_F$

Combined analyses of multiple data sets better constrain $\gamma^*_m$ and $\gamma^*_g$

Result from green points: $\gamma^*_m = 0.235(15)$ and $\gamma^*_g \approx -0.5$
Baryon anomalous dim. for partial compositeness

SM fermions $q$ couple linearly to $O_q \sim \psi\psi\psi$ of new strong dynamics

$$m_q \sim v \left( \frac{\text{TeV}}{\Lambda_F} \right)^{4-2\gamma_3}$$

with $\gamma_3 = \frac{9}{2} - d[\psi\psi\psi]$.

Large mass hierarchy $\leftrightarrow O(1)$ anomalous dimensions

Example: With $\Lambda_F = 10^{10}$ TeV, $O(\text{MeV})$ quarks need $\gamma_3 \approx 1.75$
$O(\text{GeV})$ quarks need $\gamma_3 \approx 1.9$

Compute $\gamma_O = -\frac{d \log Z_O(\mu)}{d \log \mu}$,

$Z_O(\mu)$ from standard lattice RI/MOM non-perturbative renormalization.
Baryon anomalous dim. for partial compositeness

Compute \( \gamma_{O} = -\frac{d \log Z_{O}(\mu)}{d \log \mu} \),

\( Z_{O}(\mu) \) from standard lattice RI/MOM non-perturbative renormalization

\( N_{F} = 10, 12 \) DWF pilot studies starting, re-using \( \beta \) function work
All near-conformal lattice studies so far observe light singlet scalar qualitatively different from QCD.
Light scalar in 8-flavor SU(3) spectrum

Flavor-singlet scalar degenerate with pseudo-Goldstones down to lightest masses that fit into $64^3 \times 128$ lattices

Both $M_S$ and $M_P$ less than half the vector mass $M_V$, hierarchy growing as we approach the chiral limit $\rightarrow$ qualitatively different from QCD

Controlled chiral extrapolations need EFT that includes scalar...
Vector resonance generically QCD-like

Without EFT, roughly constant ratio \( \frac{M_V}{F_P} \approx 8 \sim \frac{M_V}{2 \text{ TeV}} \sqrt{\xi} \)

[ NB: expect \( \frac{M_P}{F_P} \rightarrow 0 \) in chiral limit! ]

We measure \( F_V \approx F_P \sqrt{2} \) (KSRF relation, suggesting vector domin.)

Applying second KSRF relation \( g_{VPP} \approx \frac{M_V}{(F_P \sqrt{2})} \)

\( \rightarrow \) vector width \( \Gamma_V \approx \frac{g_{VPP}^2 M_V}{48 \pi} \approx 450 \text{ GeV} \) — hard to see at LHC
QCD-like non-singlet scalar $a_0$ for $N_F = 8$

May be relevant for holographic approaches...

![Graph](image)

Earlier work with domain wall fermions farther from chiral limit

$\rightarrow$ non-singlet scalar $a_0$ heavier than vector, $M_{a_0} \gtrsim M_V$
QCD-like non-singlet scalar $a_0$ for $N_F = 12$

$N_f = 12$ comparison

Staggered $N_F = 12$ results also show $M_{a_0} \gtrsim M_V$

Analyses complicated by staggered spin–flavor mixing

(Revised by Oliver Witzel)

[LatKMI PRD86 (2012) 059903]
[LatKMI PRL 111 (2013) 162001]
Work in progress: Constraining EFT

There are many candidate EFTs that include PNGBs + light scalar
(linear $\sigma$ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki; Golterman–Shamir; Hansen–Langaebel–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

Now computing $2 \rightarrow 2$ elastic scattering of PNGBs & scalar,
scalar form factor of PNGB

Subsequent step: Analog of $\pi K$ scattering in mass-split system
$S$ parameter on the lattice

\[ \mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \text{Tr} \left[ U_{\tau 3} U^\dagger W^{\mu\nu} \right] \rightarrow \gamma, Z \]

Lattice vacuum polarization calculation provides $S = -16\pi^2\alpha_1$

Non-zero masses and chiral extrapolation needed to avoid sensitivity to finite lattice volume

$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Larger $N_F \rightarrow$ significant reduction

Extrapolation to correct zero-mass limit becomes more challenging
Vacuum polarization from current correlator

\[ S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{VA}(Q^2) - \Delta S_{SM}(M_H) \]

\[ \gamma, Z \xrightarrow{\text{new}} Q \xrightarrow{\gamma, Z} \gamma, Z \]

\[ \Pi_{VA}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[ \left\langle \mathcal{V}^{\alpha}(x) V^{\nu b}(0) \right\rangle - \left\langle A^{\mu\alpha}(x) A^\nu{}^b(0) \right\rangle \right] \]

\[ \Pi^{\mu\nu}(Q) = \left( \delta^{\mu\nu} - \frac{\hat{Q}^\mu \hat{Q}^\nu}{\hat{Q}^2} \right) \Pi(Q^2) - \frac{\hat{Q}^\mu \hat{Q}^\nu}{\hat{Q}^2} \Pi^L(Q^2) \quad \hat{Q} = 2 \sin(Q/2) \]

- Renormalization constant \( Z \) evaluated non-perturbatively
- Chiral symmetry of domain wall fermions \( \implies Z = Z_A = Z_V \)
  \[ Z = 0.85 \ [2f]; \ 0.73 \ [6f]; \ 0.70 \ [8f] \]
- Conserved currents \( \mathcal{V} \) and \( A \) ensure that lattice artifacts cancel
Composite dark matter

Many possibilities: (arXiv:1604.04627)
dark baryon, dark nuclei, dark pion, dark quarkonium, dark glueball...

Deconfined charged fermions $\rightarrow$ relic density
Confined SM-singlet dark baryon $\rightarrow$ direct detection via form factors

For QCD-like SU(3) baryon, direct detection $\rightarrow$ $M_{DM} \gtrsim 20$ TeV
due to leading magnetic moment interaction (arXiv:1301.1693)
A lower bound for stealth dark matter

SU(4) bosonic baryons forbid leading magnetic moment and sub-leading charge radius interactions in non-rel. EFT

EM polarizability is unavoidable — compute it on the lattice

\[ \text{lower bound on the direct detection rate} \]

Nuclear cross section \( \propto Z^4/A^2 \), these results specific to Xenon

Uncertainties dominated by nuclear matrix element

Shaded region is complementary constraint from particle colliders
Future plans: Colliders and gravitational waves

Other composite dark-sector states can be discovered at colliders

Additional lattice input can help predict production and decays

Confinement transition in early universe may produce gravitational waves

First-order transition $\rightarrow$ colliding bubbles

Lattice calculations needed to predict properties of transition
Multi-rep finite-temperature phase diagram

SU(4) gauge theory with $N_4 = 2$ fund. and $N_6 = 2$ two-index-symm.

Step towards composite Higgs model with $N_4 = 3$ and $N_6 = 2.5$

Simultaneous first-order chiral/deconfinement transitions for both reps
Multi-rep mesonic spectrum

Looks broadly consistent with large-$N$ rescalings of QCD

**Left:** \( M_V / F_P \sim 8 \sqrt{\frac{3}{4}} \frac{1}{\sqrt{2}} \approx 4.9 \)

**Right:** Narrower vector resonance widths expected for larger $N$
Recap: An exciting time for lattice gauge theory

Lattice gauge theory is a broadly applicable tool to study strongly coupled systems and BSM physics.

Exploring generic features of representative systems beyond QCD

- $\beta$ functions and anomalous dimensions
- Light scalar from near-conformal dynamics
- Low-energy constants including $S$ parameter
- Composite dark matter and more...

Thank you!
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Thank you!

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Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations $U$ with probability $\frac{1}{Z} e^{-S[U]}$

HMC is Markov process based on Metropolis–Rosenbluth–Teller

Fermions $\rightarrow$ extensive action computation

$\Rightarrow$ Global updates using fictitious molecular dynamics

1. Introduce fictitious “MD time” $\tau$
   and stochastic canonical momenta for fields

2. Inexact MD evolution along trajectory in $\tau$
   $\rightarrow$ new four-dimensional field configuration

3. Accept/reject test on MD discretization error

(Image credit: Claudio Rebbi)
Backup: Lattice QCD for BSM

High-precision non-perturbative QCD calculations reduce uncertainties and help resolve potential new physics

- Hadronic matrix elements & form factors for flavor physics
  Sub-percent precision for easiest observables (arXiv:1607.00299)

- Hadronic contributions to $(g - 2)_\mu$
  Targeting $\sim 0.1\%$ precision for vac. pol., $\sim 10\%$ for light-by-light (arXiv:1311.2198)

- $m_c, m_b$ and $\alpha_s(m_Z)$ to $\sim 0.1\%$ for Higgs couplings (arXiv:1404.0319)


- Nucleon electric dipole moment, form factors (arXiv:1701.07792)
Backup: $\gamma_{\text{eff}}(\lambda)$ from eigenmodes for $N_F = 8$

Fit $\nu(\lambda) \propto \lambda^{1+\alpha}$ in small range of $\lambda$ \hfill \longrightarrow \hfill 1 + \gamma_{\text{eff}}(\lambda) = \frac{4}{1 + \alpha(\lambda)}$

$v(\lambda)$ computed stochastically

Include fit ranges in error bands

Multiple $L^4$ volumes overlaid, $L$-sensitive data dropped

All systems have $\rho(0) = 0$

Appears to evolve slowly across wide range of scales, qualitatively different from $N_F = 12$ and QCD-like $N_F = 4$
Exploring the range of possible phenomena in strongly coupled gauge theories
Backup: 8-flavor SU(3) infrared dynamics

- $\beta$ function monotonic up to fairly strong $g^2 \sim 14$
  No sign of approach towards conformal IR fixed point $[\beta(g_\star^2) = 0]$

- Ratio $M_V/M_P$ increases monotonically as masses decrease
  as expected for spontaneous chiral symmetry breaking ($S\chi$SB)
  Mass-deformed conformal hyperscaling predicts constant ratio

Strengthen conclusion by matching to low-energy EFT
  $\rightarrow$ must go beyond QCD-like $\chi$PT to include light scalar...
Backup: Technical challenge for scalar on lattice

Only new strong sector included in the lattice calculations

⇒ flavor-singlet scalar mixes with the vacuum

leads to noisy data and relatively large uncertainties

Fermion propagator computation relatively expensive

“Disconnected diagrams” formally need propagators at all $L^4$ sites

In practice estimate stochastically to control computational costs
Lattice QCD $\rightarrow$ isosinglet scalar much heavier than pion

Generally $M_S \gtrsim 2M_P$ $\rightarrow$ $M_S > M_V$ for heavy quarks

For a large range of quark masses $m$

it mixes significantly with two-pion scattering states
Backup: Qualitative picture of light scalar

Light scalar likely related to near-conformal dynamics

\[ \rightarrow \text{possibly dilaton, PNGB of approximate scale symmetry?} \]

QCD-like chiral breaking

Conformal hyperscaling

\[ M_\rho \]

\[ M_{TT} \]

\[ M_{0^{++}} \]

Chirally broken, near conformal

\[ M_{0^{++}} \text{ light relative to } M_\rho \]

(Anna Hasenfratz)
Backup: 2 → 2 elastic scattering on the lattice

Measure both $E_{PP}$ and $M_P \quad \implies \quad k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

s-wave scattering phase shift: $\cot \delta_0(k) = \frac{1}{\pi k L} \cdot S \left( \frac{k^2 L^2}{4\pi} \right)$

where regularized $\zeta$ function $S(\eta) = \sum_{j \neq 0} \frac{1}{j^2 - \eta} - 4\pi \Lambda$

Effective range expansion:

$$k \cot \delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left( \frac{k^2}{M_P^2} \right) + O \left( \frac{k^4}{M_P^4} \right)$$
Backup: Initial $2 \rightarrow 2$ elastic scattering results

Simplest case: Analog of QCD $I = 2 \pi \pi$ scattering
(no fermion-line-disconnected diagrams)

Simplest observable: Scattering length $a_{PP} \approx 1/(k \cot \delta)$

Left: $M_P a_{PP}$ vs. $M_P^2/F_P^2$ curiously close to leading-order $\chi$PT

Right: Divide by fermion mass $m \rightarrow$ tension with $\chi$PT as expected
(predicts constant at LO; involves 8 LECs at NLO)
\( \chiPT \) omits the light scalar and suffers from large expansion parameter

\[
5.8 \leq \frac{2N_F Bm}{16\pi^2 F^2} \leq 41.3 \quad \text{for} \quad 0.00125 \leq m \leq 0.00889
\]

\( \sim 50\sigma \) shift in \( F \) between linear extrapolation vs. NLO \( \chiPT \)

Poor fit quality, especially for NLO joint fit \( (\chi^2/d.o.f. > 10^4) \)
Backup: NLO chiral perturbation theory formulas

\[ M_P^2 = 2Bm \left[ 1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ 128\pi^2 \left( 2L_6^r - L_4^r + \frac{2L_8^r - L_5^r}{N_F} \right) + \frac{\log \left( \frac{2Bm}{\mu^2} \right)}{N_F^2} \right\} \right] \]

\[ F_P = F \left[ 1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ 64\pi^2 \left( L_4^r + \frac{L_5^r}{N_F} \right) - \frac{1}{2} \log(2Bm/\mu^2) \right\} \right] \]

\[ M_{P\alpha PP} = \frac{-2Bm}{16\pi F^2} \left[ 1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ -256\pi^2 \left( \left[ 1 - \frac{2}{N_F} \right] (L_4^r - L_6^r) \\
+ \frac{L_0^r + 2L_1^r + 2L_2^r + L_3^r}{N_F} \right) \right. \\
\left. + \frac{2 - N_F + 2N_F^2 + N_F^3}{N_F^3} \log \left( \frac{2Bm}{\mu^2} \right) \right\} \right] \]
Backup: Thermal freeze-out for relic density

Requires coupling between ordinary matter and dark matter

\[ T \gtrsim M_{DM} : \quad DM \leftrightarrow SM \]

Thermal equilibrium

\[ T \lesssim M_{DM} : \quad DM \rightarrow SM \]

Rapid depletion of \( \Omega_{DM} \)

Hubble expansion

\[ \rightarrow \text{dilution} \rightarrow \text{freeze-out} \]

2 → 2 scattering relates coupling and mass as

\[ 200\alpha \sim \frac{M_{DM}}{100 \text{ GeV}} \]

Strong \( \alpha \sim 16 \quad \rightarrow \text{‘natural’ mass scale} \quad M_{DM} \sim 300 \text{ TeV} \)

Smaller \( M_{DM} \gtrsim 1 \text{ TeV} \) possible from 2 → \( n \) scattering or asymmetry
Backup: Two roads to natural asymmetric dark matter

**Idea:** Dark matter relic density related to baryon asymmetry

\[ \Omega_D \approx 5\Omega_B \]
\[ \implies M_D n_D \approx 5M_B n_B \]

- \( n_D \sim n_B \implies M_D \sim 5M_B \approx 5 \text{ GeV} \)
  High-dim. interactions relate baryon# and DM# violation

- \( M_D \gg M_B \implies n_B \gg n_D \sim \exp \left[ -\frac{M_D}{T_s} \right] \quad T_s \sim 200 \text{ GeV} \)
  EW sphaleron processes above \( T_s \) distribute asymmetries

Both require coupling between ordinary matter and dark matter
Backup: Composite dark matter interactions

Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5:** Magnetic moment $\rightarrow (\bar{\psi} \sigma^{\mu\nu} \psi) F_{\mu\nu}/\Lambda$
- **Dimension 6:** Charge radius $\rightarrow (\bar{\psi} \psi) v_\mu \partial_\nu F_{\mu\nu}/\Lambda^2$
- **Dimension 7:** Polarizability $\rightarrow (\bar{\psi} \psi) F_{\mu\nu} F_{\mu\nu}/\Lambda^3$

Higgs exchange via scalar form factors

Higgs couples through $\langle B \mid m_\psi \bar{\psi} \psi \mid B \rangle$ ($\sigma$ terms)

Needed for Big Bang nucleosynthesis
(\rightarrow rapid charged ‘meson’ decay)

Non-perturbative form factors $\Rightarrow$ lattice calculations
Backup: SU(3) direct detection constraints

**Solid:** Predicted event rate for SU(3) model vs. DM mass $M_B$

**Dashed:** Sub-leading charge radius contribution
suppressed $\sim 1/M_B^2$ compared to magnetic moment

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![Graph showing predicted event rate vs. DM mass](image)

XENON100 (arXiv:1207.5988) excludes $M_B \lesssim 10$ TeV

More recent LUX, PandaX $\rightarrow M_B \gtrsim 25$ TeV

SU($N$) with even $N \geq 4$ forbids mag. moment...
Backup: Stealth dark matter model details

### Mass terms

$$m_V (F_1 F_2 + F_3 F_4) + y (F_1 \cdot HF_4 + F_2 \cdot H^\dagger F_3) + \text{h.c.}$$

### Vector-like masses

Evade Higgs-exchange direct detection bounds

### Higgs couplings

⇒ charged meson decay before Big Bang nucleosyn.

Both required
Backup: Effective Higgs interaction

$M_H = 125$ GeV $\rightarrow$ Higgs exchange can dominate direct detection

$$\sigma_H \propto \left| \frac{\mu_{DM,N}}{M_H^2} \right| y_\psi \langle DM \left| \overline{\psi} \psi \right| DM \rangle \left| y_q \langle N \left| \overline{q} q \right| N \rangle \right|^2$$

Quark $y_q = \frac{m_q}{\nu}$

Dark $y_\psi = \alpha \frac{m_\psi}{\nu}$ suppressed by $\alpha \equiv \left. \frac{\nu}{m_\psi} \frac{\partial m_\psi(h)}{\partial h} \right|_{h = \nu} = \frac{y\nu}{y\nu + m_V}$

Can determine scalar form factors using Feynman–Hellmann theorem

$$\langle DM \left| \overline{\psi} \psi \right| DM \rangle = \frac{\partial M_{DM}}{\partial m_\psi}$$
Backup: Stealth dark matter EM form factors

Lightest SU(4) dark baryon

Scalar $\rightarrow$ no magnetic moment

$\pm$ charge symmetry $\rightarrow$ no charge radius

Small $\alpha$ $\rightarrow$ Higgs exchange suppressed

Polarizability $\rightarrow$ lower bound on direct-detection cross section

Compute on lattice as dependence of $M_{\text{DM}}$ on external field $\mathcal{E}$
Backup: Stealth dark matter mass scales

Lattice studies focus on \( m_\psi \sim \Lambda_{DM} \) (effective theories least reliable)

\[ m_\psi \sim \Lambda_{DM} \] could arise dynamically

Smaller \( m_\psi \rightarrow \) stronger collider constraints

\[ M_f \sim \Lambda \] Could arise dynamically

(plot from G. Kribs)

\[ M_{Pl} \]
Backup: Stealth dark matter at colliders

Spectrum significantly different from typical susy

Very little missing $E_T$

Main constraints from much lighter charged "$\Pi$" states

Rapid $\Pi$ decays, $\Gamma \propto m_f^2$

Best current constraints recast LEP stau searches

LHC can search for $t\bar{b} + \bar{t}b$ from $\Pi^+\Pi^-$ Drell–Yan
Backup: Philosophy of mixed-mass approach

\[ N_F = N_\ell + N_h \] fermions, light \( m_\ell \to 0 \) at fixed heavy \( m_h > 0 \)

\[ \rightarrow \] approximate conformality without extra PNGBs

Smaller \( m_h \) \[ \rightarrow \] larger range of scales for approximately conformality

Real-space RG flow lines
(from UV to IR)

“IRFP” in UV
\[ \rightarrow \] work on either side