Lattice gauge theory beyond the standard model

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and now for something completely different
Overview

Lattice gauge theory is a broadly applicable tool to study strongly coupled systems.

Lattice calculations are especially important when QCD-based intuition may be unreliable.

- A high-level summary of lattice gauge theory
- High-precision non-perturbative QCD (briefly)
- Non-QCD strong dynamics
  - Near-conformal dynamics for composite Higgs
  - Bosonic baryons for composite dark matter
The essence of lattice gauge theory

Lattice discretization is a non-perturbative regularization of QFT

Formulate theory on a finite, discrete euclidean spacetime $\rightarrow$ the lattice

Spacing between lattice sites ("a") introduces UV cutoff scale $1/a$

Remove cutoff by taking continuum limit:

$$a \rightarrow 0 \text{ with } L/a \rightarrow \infty$$

Finite number of degrees of freedom ($\sim 10^9$)

$\rightarrow$ numerically compute observables via importance sampling

$$\langle O \rangle = \frac{1}{Z} \int D\Phi \ O(\Phi) \ e^{-S[\Phi]} \ \rightarrow \ \frac{1}{N} \sum_{k=1}^{N} O(\Phi_k)$$
Features of lattice gauge theory

- Fully non-perturbative predictions from first principles (lagrangian)
- Fully gauge invariant—no gauge fixing required
- Applies directly in four dimensions
- Euclidean SO(4) rotations & translations (→ Poincaré symmetry) recovered automatically in the $a \to 0$ continuum limit
Limitations of lattice gauge theory

- Lattice action discretizes UV-complete lagrangian, (usually) including only strong sector
- Super-Poincaré symmetry (usually) not recovered automatically
- Finite volume (usually) needs to contain all correlation lengths $\rightarrow$ unphysically large masses extrapolated to chiral limit via EFT
- Obstructions to real-time dynamics, chiral gauge theories
Lattice QCD for BSM

High-precision non-perturbative QCD calculations reduce uncertainties and help resolve potential new physics

- Hadronic matrix elements & form factors for flavor physics
  Sub-percent precision for easiest observables (arXiv:1607.00299)

- Hadronic contributions to $(g - 2)_\mu$ (arXiv:1311.2198)
  Targeting $\sim 0.1\%$ precision for vac. pol., $\sim 10\%$ for light-by-light

- $m_c, m_b$ and $\alpha_s(m_Z)$ to $\sim 0.1\%$ for Higgs couplings (arXiv:1404.0319)


- Nucleon electric dipole moment, form factors (arXiv:1701.07792)
Lattice gauge theory beyond QCD

Lattice calculations especially important for non-QCD strong dynamics

These are exploratory investigations of representative systems to elucidate generic dynamical phenomena & connect with EFT


Executive Summary

- Use the Higgs boson as a new tool for discovery
- Pursue the physics associated with neutrino mass
- Identify the new physics of dark matter
- Understand cosmic acceleration: dark energy and inflation
- Explore the unknown: new particles, interactions, and physical principles.
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Non-QCD strong dynamics

Two main directions (not mutually exclusive)

- Near-conformal dynamics from many fermionic d.o.f.
  \[ \rightarrow \text{large number of fundamental fermions or a few in a larger rep} \]

- Different symmetries from different gauge group or reps
  \[ \rightarrow \text{(pseudo)real reps for cosets SU}(n)/\text{Sp}(n) \text{ or SU}(n)/\text{SO}(n) \]

All near-conformal lattice studies so far observe a very light singlet scalar qualitatively different from QCD

Example: SU(3) with \( N_F = 8 \) fund.
work by LSD Collaboration

Exploring the range of possible phenomena in strongly coupled gauge theories
Light scalar in 8-flavor spectrum

Flavor-singlet scalar degenerate with pseudo-Goldstones down to lightest fermion masses we can fit into $64^3 \times 128$ lattices

Both $M_S$ and $M_P$ are less than half the vector mass $M_V$, and the hierarchy is growing as we approach the chiral limit

This is very different from QCD

Controlled chiral extrapolations need EFT that includes scalar...
Another generic feature: broad vector resonance

Without EFT, roughly constant ratio $M_V/F_P \simeq 8 \sim M_V \simeq 2 \text{ TeV}/\sin \theta$

[ NB: expect $M_P/F_P \to 0$ in chiral limit! ]

We measure $F_V \approx F_P \sqrt{2}$ (KSRF relation, suggesting vector domin.)

Applying second KSRF relation $g_{VPP} \approx M_V/(F_P \sqrt{2})$

gives vector width $\Gamma_V = \frac{g_{VPP}^2 M_V}{48 \pi} \simeq 450 \text{ GeV} — \text{hard to see at LHC}$
Work in progress: Constraining EFT

There are many candidate EFTs that include PNGBs + light scalar
(linear $\sigma$ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki; Golterman–Shamir; Hansen–Langaeble–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

We are now computing $2 \rightarrow 2$ elastic scattering of PNGBs & scalar, as well as scalar form factor of PNGB

Subsequent step: Analog of $\pi K$ scattering in mass-split system
Composite dark matter

Many possibilities: (arXiv:1604.04627)
dark baryon, dark nuclei, dark pion, dark quarkonium, dark glueball. . .

- Deconfined charged fermions produce relic density
- Confined SM-singlet dark baryon detectable via form factors

For QCD-like SU(3) model, direct detection constrains $M_{DM} \gtrsim 20$ TeV due to leading magnetic moment interaction (arXiv:1301.1693)
A lower bound for stealth dark matter

SU(4) bosonic baryons forbid both leading magnetic moment and sub-leading charge radius interactions in non-rel. EFT. The polarizability is unavoidable — compute it on the lattice to place a lower bound on the direct detection rate.

Nuclear cross section $\propto Z^4/A^2$, these results specific to Xenon.

Uncertainties dominated by nuclear matrix element.

Shaded region is complementary constraint from particle colliders.
Future plans: Colliders and gravitational waves

Other composite dark-sector states can be discovered at colliders.

Additional lattice input can help predict production and decays.

Confinement transition in early universe may produce gravitational waves.

First-order transition $\rightarrow$ colliding bubbles.

Lattice calculations needed to predict properties of transition.
Outlook: An exciting time for lattice gauge theory

Lattice gauge theory is a broadly applicable tool to study strongly coupled systems and BSM physics

- High-precision non-perturbative QCD helps resolve new physics
- Exploring generic features of representative systems beyond QCD
  - Near-conformal dynamics with connections to composite Higgs
  - SU(4) bosonic baryons make composite dark matter stealthier

Thank you!

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Thank you!

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Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations $\Phi_k$ with probability $\frac{1}{Z} e^{-S[\Phi_k]}$

HMC is a Markov process, based on Metropolis–Rosenbluth–Teller (MRT)

Fermions $\rightarrow$ extensive action computation, so best to update entire system at once

Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension (“MD time” $\tau$) and stochastic canonical momenta for all field variables

2. Run inexact MD evolution along a trajectory in $\tau$ to generate new four-dimensional field configuration

3. Apply MRT accept/reject test to MD discretization error

/Image credit: Claudio Rebbi/
Backup: Light scalars beyond QCD
Not so shocking in mass-deformed IR-conformal theories

More surprising in systems apparently exhibiting spontaneous chiral symmetry breaking

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Backup: 8-flavor SU(3) infrared dynamics

- $\beta$ function is monotonic up to fairly strong couplings
  - No sign of approach towards conformal IR fixed point $[\beta(g^2_*) = 0]$

- Ratio $M_V / M_P$ increases monotonically as masses decrease
  - as expected for spontaneous chiral symmetry breaking ($S\chi$SB)
  - Mass-deformed conformal hyperscaling predicts constant ratio

Want to strengthen conclusion by matching to low-energy EFT, but must go beyond QCD-like $\chi$PT to include light scalar...
Backup: Technical challenge for scalar on lattice

Only the new strong sector is included in the lattice calculation

\[ \Rightarrow \text{The flavor-singlet scalar mixes with the vacuum} \]

Leads to noisy data and relatively large uncertainties

Fermion propagator computation is relatively expensive

“Disconnected diagrams” formally need propagators at all \( L^4 \) sites

In practice estimate stochastically to control computational costs
In lattice QCD, the isosinglet scalar is much heavier than the pion. Generally $M_S \gtrsim 2M_P$, and for heavy quarks $M_S > M_V$.

For a large range of quark masses $m$ it mixes significantly with two-pion scattering states.
Backup: Qualitative picture of light scalar

Light scalar likely related to near-conformal dynamics
(unconfirmed interpretation as PNGB of approx. scale symmetry)

QCD-like chiral breaking

Conformal hyperscaling

Chirally broken, near conformal

\( M_{\rho} \)
\( M_{TT} \)
\( M_{0^{++}} \)

\( M_{0^{++}} \) light relative to \( M_{\rho} \)

(Anna Hasenfratz)
In earlier work with domain wall fermions at heavier fermion masses, the non-singlet scalar is heavier than the vector, $M_{a_0} \sim M_V$.

Staggered analyses in progress, but more complicated.
Backup: $2 \rightarrow 2$ elastic scattering on the lattice

Measure both $E_{PP}$ and $M_P \rightarrow k = \sqrt{(E_{PP}/2)^2 - M_P^2}$

s-wave scattering phase shift: $\cot \delta_0(k) = \frac{1}{\pi k L} S \left( \frac{k^2 L^2}{4\pi} \right)$

with regularized $\zeta$ function $S(\eta) = \sum_{j \neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi \Lambda$

Effective range expansion:

$$k \cot \delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left( \frac{k^2}{M_P^2} \right) + O \left( \frac{k^4}{M_P^4} \right)$$
First looking at analog of QCD $\pi\pi$ scattering in $I = 2$ channel (simplest case with no fermion-line-disconnected diagrams)

Simplest observable is scattering length $a_{PP} \approx 1/(k \cot \delta)$

$M_P a_{PP}$ vs. $M_P^2/F_P^2$ curiously close to leading-order $\chi$PT prediction

Dividing by fermion mass $m$ reveals expected tension with $\chi$PT which predicts $M_P a_{PP}/m = \text{const.}$ at LO and involves 8 LECs at NLO
In addition to omitting the light scalar, \( \chi \)PT also suffers from large expansion parameter

\[
5.8 \leq \frac{2N_FBm}{16\pi^2 F^2} \leq 41.3 \quad \text{for} \quad 0.00125 \leq m \leq 0.00889
\]

Big (~50\( \sigma \)) shift in \( F \) from linear extrapolation vs. NLO \( \chi \)PT

Fit quality is not good, especially for NLO joint fit with \( \chi^2 / \text{d.o.f.} > 10^4 \)
Backup: NLO chiral perturbation theory formulas

\[ M_P^2 = 2Bm \left[ 1 + \frac{2N_F Bm}{16 \pi^2 F^2} \right] \left\{ 128 \pi^2 \left( 2L_6^r - L_4^r + \frac{2L_8^r - L_5^r}{N_F} \right) + \frac{\log \left( \frac{2Bm}{\mu^2} \right)}{N_F^2} \right\} \]

\[ F_P = F \left[ 1 + \frac{2N_F Bm}{16 \pi^2 F^2} \right] \left\{ 64 \pi^2 \left( L_4^r + \frac{L_5^r}{N_F} \right) - \frac{1}{2} \log(2Bm/\mu^2) \right\} \]

\[ M_P a_{PP} = \frac{-2Bm}{16 \pi F^2} \left[ 1 + \frac{2N_F Bm}{16 \pi^2 F^2} \right] \left\{ -256 \pi^2 \left( \left[ 1 - \frac{2}{N_F} \right] (L_4^r - L_6^r) + \frac{L_0^r + 2L_1^r + 2L_2^r + L_3^r}{N_F} \right) - 2 \frac{N_F - 1}{N_F^3} \right\} \]

\[ + 2 - N_F + 2N_F^2 + \frac{N_F^3}{N_F^3} \log \left( \frac{2Bm}{\mu^2} \right) \]

David Schaich (U. Bern)    Lattice BSM    Planck 2017, 22 May
Backup: The $S$ parameter on the lattice

$$\mathcal{L}_\chi \supset \frac{\alpha_1}{2} g_1 g_2 B_{\mu\nu} \text{Tr} \left[ U_{\tau 3} U^\dagger W^{\mu\nu} \right] \rightarrow \gamma, Z \text{ new} \gamma, Z$$

Lattice vacuum polarization calculation provides $S = -16\pi^2 \alpha_1$

Non-zero masses and chiral extrapolation needed to avoid sensitivity to finite lattice volume

$S = 0.42(2)$ for $N_F = 2$
matches scaled-up QCD

Moving away from QCD with larger $N_F$ produces significant reductions

Extrapolation to correct zero-mass limit becomes more challenging
Backup: Vacuum polarization is just current correlator

\[ S = 4\pi N_D \lim_{Q^2 \to 0} \frac{d}{dQ^2} \Pi_{V-A}(Q^2) - \Delta S_{SM}(M_H) \]

\[ \gamma, Z \xrightarrow{\text{new}} Q \xrightarrow{\text{new}} \gamma, Z \]

\[ \Pi_{V-A}^{\mu\nu}(Q) = Z \sum_x e^{iQ \cdot (x + \hat{\mu}/2)} \text{Tr} \left[ \left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0) \right\rangle - \left\langle A^{\mu a}(x) A^{\nu b}(0) \right\rangle \right] \]

\[ \Pi^{\mu\nu}(Q) = \left( \delta^{\mu\nu} - \frac{\hat{Q}^\mu \hat{Q}^\nu}{\hat{Q}^2} \right) \Pi(Q^2) - \frac{\hat{Q}^\mu \hat{Q}^\nu}{\hat{Q}^2} \Pi^L(Q^2) \quad \hat{Q} = 2 \sin (Q/2) \]

- Renormalization constant \( Z \) evaluated non-perturbatively
- Chiral symmetry of domain wall fermions \( \implies Z = Z_A = Z_V \)

\[ Z = 0.85 \ [2f]; \quad 0.73 \ [6f]; \quad 0.70 \ [8f] \]

- Conserved currents \( \mathcal{V} \) and \( A \) ensure that lattice artifacts cancel
Backup: Thermal freeze-out for relic density

\[ T \gtrsim M_{DM} : \text{DM} \leftrightarrow \text{SM} \]
Thermal equilibrium

\[ T \lesssim M_{DM} : \text{DM} \rightarrow \text{SM} \]
Rapid depletion of \( \Omega_{DM} \)

Hubble expansion \( \rightarrow \) dilution
leads to freeze-out

Requires coupling between ordinary matter and dark matter

Mass and coupling of pure thermal relic are related:
\[ \frac{M_{DM}}{100 \text{ GeV}} \sim 200\alpha \]

With strong \( \alpha \sim 16 \), ‘natural’ mass scale is \( M_{DM} \sim 300 \text{ TeV} \)

At smaller masses \( M_{DM} \gtrsim 1 \text{ TeV} \)
thermal relic could be just part of total relic density
Basic idea: Dark matter relic density related to baryon asymmetry

\[ \Omega_D \approx 5\Omega_B \]
\[ \implies M_D n_D \approx 5M_B n_B \]

- \( n_D \sim n_B \implies M_D \sim 5M_B \approx 5 \text{ GeV} \)
  High-dimensional interactions relate baryon\# and DM\# violation

- \( M_D \gg M_B \implies n_B \gg n_D \sim \exp \left[ -\frac{M_D}{T_s} \right] \)
  Sphaleron transitions above \( T_s \approx 200 \text{ GeV} \) distribute asymmetries

Both require coupling between ordinary matter and dark matter
Backup: Composite dark matter interactions

Photon exchange via electromagnetic form factors

Interactions suppressed by powers of confinement scale $\Lambda \sim M_{DM}$

- **Dimension 5**: Magnetic moment $\rightarrow (\overline{\psi} \sigma^{\mu\nu} \psi) F_{\mu\nu} / \Lambda$
- **Dimension 6**: Charge radius $\rightarrow (\overline{\psi} \psi) \nu_\mu \partial_\nu F_{\mu\nu} / \Lambda^2$
- **Dimension 7**: Polarizability $\rightarrow (\overline{\psi} \psi) F^{\mu\nu} F_{\mu\nu} / \Lambda^3$

Higgs exchange via scalar form factors

Effective Higgs interaction of composite DM needed for correct Big Bang nucleosynthesis

Higgs couples through $\langle B \mid m_\psi \overline{\psi} \psi \mid B \rangle$ ($\sigma$ terms)

All form factors arise non-perturbatively $\Rightarrow$ lattice calculations
Backup: SU(3) direct detection constraints

Solid lines are predictions for total number of events XENON100 would observe for SU(3) model with dark baryon mass $M_B$

Dashed lines are subleading charge radius contribution suppressed $\sim 1/M_B^2$ relative to magnetic moment contribution

$M_{\chi} = M_B$ [TeV]

XENON100 results
(arXiv:1207.5988)
exclude $M_B \lesssim 10$ TeV

SU(N) with even $N \geq 4$ forbids mag. moment...
Backup: Stealth dark matter model details

Mass terms $\sim m_V (F_1 F_2 + F_3 F_4) + y (F_1 \cdot H F_4 + F_2 \cdot H^\dagger F_3) + h.c.$

Both vector-like masses $m_V$ and Higgs couplings $y$ are **required**
- Higgs couplings ensure rapid meson decay in early universe
- Vector-like masses avoid bounds on direct detection via Higgs exchange
Backup: Effective Higgs interaction

With $M_H = 125$ GeV, Higgs exchange may dominate spin-independent direct detection cross section

$$\sigma_{SI} \propto \left| \frac{\mu_{B,N}}{M_H^2} y_\psi \langle B | \overline{\psi} \psi | B \rangle \, y_q \langle N | \overline{q} q | N \rangle \right|^2$$

For quarks $y_q = \frac{m_q}{v} \implies y_q \langle N | \overline{q} q | N \rangle \propto \frac{M_N}{v} \frac{\langle N | m_q \overline{q} q | N \rangle}{M_N}$

For dark constituent fermions $\psi$

there is an additional model parameter, $y_q = \alpha \frac{m_\psi}{v}$

In both cases the scalar form factor is most easily determined using the Feynman–Hellmann theorem

$$\frac{\langle B | m_\psi \overline{\psi} \psi | B \rangle}{M_B} = \frac{m_\psi}{M_B} \frac{\partial M_B}{\partial m_\psi}$$
Backup: Stealth dark matter EM form factors

Lightest SU(4) composite dark baryon

 Scalar particle $\rightarrow$ no magnetic moment

 $+/-$ charge symmetry $\rightarrow$ no charge radius

Higgs exchange can be negligibly small

Polarizability places lower bound on direct-detection cross section

Compute on lattice as dependence of $M_{DM}$ on external field $\mathcal{E}$
Lattice calculations have focused on $m_\psi \simeq \Lambda_D$, the regime where analytic estimates are least reliable.

This mass scale has some theoretical motivation.

In addition, collider constraints tighten as mass decreases.

Could arise dynamically

(plot from G. Kribs)
Backup: Stealth dark matter collider detection

Spectrum significantly different from MSSM-inspired models

Very little missing $E_T$ at colliders

Main constraints from much lighter charged “$\Pi$” states

Rapid $\Pi$ decays with $\Gamma \propto m_f^2$

Best current constraints recast stau searches at LEP

LHC can also search for $t\bar{b} + \bar{t}b$ from $\Pi^+\Pi^-$ Drell–Yan production