Light scalar from lattice strong dynamics

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and work in progress with the Lattice Strong Dynamics Collaboration
Overview

- Several groups are using lattice gauge theory to explore strongly coupled systems with non-QCD-like dynamics.

- These studies find remarkably light scalars in many IR-conformal and near-conformal systems.

- Using 8-flavor SU(3) gauge theory as a representative example, we are studying more quantities to constrain the low-energy EFT.
Light scalars from beyond-QCD lattice calculations

Not so shocking in mass-deformed IR-conformal theories

More surprising in systems apparently exhibiting spontaneous chiral symmetry breaking
Exploring the range of possible phenomena in strongly coupled field theories

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$N_f = 8$ scalar spectrum & EFT

Bad Honnef, 14 Feb. 2017
Infrared dynamics of 8-flavor SU(3) gauge theory

- $\beta$ function is monotonic up to fairly strong couplings
  No sign of approach towards conformal IR fixed point $[\beta(g^2_*) = 0]$

- Ratio $M_V/M_P$ increases monotonically as masses decrease as expected for spontaneous chiral symmetry breaking ($S\chi$SB)
  Mass-deformed conformal hyperscaling predicts constant ratio

Want to strengthen conclusion by matching to low-energy EFT, but must go beyond QCD-like $\chi$PT to include light scalar...
Light scalar in 8-flavor spectrum

Flavor-singlet scalar degenerate with pseudo-Goldstones down to lightest fermion masses we can reach on $64^3 \times 128$ lattices

Both $M_S$ and $M_P$ are less than half the vector mass $M_V$, and the hierarchy is growing as we approach the chiral limit

This is very different from QCD

Controlled chiral extrapolations need EFT that includes scalar...
2 TeV vector resonance with width $\Gamma_V \simeq 450$ GeV

$M_V/F_P \simeq 8 \implies M_V \simeq 2$ TeV if we assume roughly constant ratio

[ NB: $S_\chi$SB implies $M_P/F_P \to 0$ in chiral limit! ]

We measure $F_V \approx F_P \sqrt{2}$ (KSRF relation, suggesting vector domin.)

Applying second KSRF relation $g_{VPP} \approx M_V/(F_P \sqrt{2})$

gives vector width $\Gamma_V = \frac{g_{VPP}^2 M_V}{48\pi} \simeq 450$ GeV — hard to see at LHC
Work in progress: Constraining EFT

There are many candidate EFTs that include PNGBs + light scalar
(linear $\sigma$ model; Goldberger–Gristein–Skiba; Soto–Talavera–Tarrus; Matsuzaki–Yamawaki; Golterman–Shamir; Hansen–Langaebel–Sannino; Appelquist–Ingoldby–Piai)

Need lattice computations of more observables to test EFTs

We are now computing $2 \rightarrow 2$ elastic scattering of PNGBs & scalar, as well as scalar form factor of PNGB

Example: Scalar exchange in $\pi\pi$ scattering vs. $\pi$ scalar form factor
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Subsequent step: Analog of $\pi K$ scattering in mass-split system
Initial 2 → 2 elastic scattering results

First looking at analog of QCD $I = 2 \pi\pi$ scattering
(simplest case with no fermion-line-disconnected diagrams)

Simplest observable is scattering length $a_{PP} \approx 1/(k \cot \delta)$

$M_P a_{PP}$ vs. $M_P^2/F_P^2$ curiously close to leading-order $\chi$PT prediction

Dividing by fermion mass $m$ reveals expected tension with $\chi$PT which predicts $M_P a_{PP}/m = \text{const.}$ at LO and involves 8 LECs at NLO
Recapitulation and outlook

- 8-flavor SU(3) gauge theory is a representative system with a light scalar likely related to near-conformal dynamics

- Growing hierarchy between scalar and broad $\sim 2\text{TeV}$ vector

- Chiral extrapolations need an EFT beyond QCD-like $\chi$PT

- We are now studying elastic scattering to test potential EFTs
Thank you!
Thank you!

Lattice Strong Dynamics Collaboration

In particular: George Fleming, Andrew Gasbarro, Xiao-Yong Jin, Enrico Rinaldi, Evan Weinberg

Funding and computing resources
Backup: Essence of numerical lattice calculations

Evaluate observables from functional integral via importance sampling Monte Carlo

\[ \langle O \rangle = \frac{1}{Z} \int D\Phi \; O(\Phi) \; e^{-S[\Phi]} \]

\[ \rightarrow \frac{1}{N} \sum_{i=1}^{N} O(\Phi_i) \text{ with uncert. } \propto \sqrt{\frac{1}{N}} \]

\( \Phi \) are field configurations in discretized euclidean spacetime

\( S[\Phi] \) is the lattice action, which should be real and non-negative so that \( \frac{1}{Z} e^{-S} \) can be treated as a probability distribution

The hybrid Monte Carlo algorithm samples \( \Phi \) with probability \( \propto e^{-S} \)
Spacing between lattice sites ("a") introduces UV cutoff scale $1/a$.

Lattice cutoff preserves hypercubic subgroup of full Poincaré symmetry.

Remove cutoff by taking continuum limit: $a \to 0$ with $L/a \to \infty$.

The lattice action $S$ is defined by the bare lagrangian at the UV cutoff set by the lattice spacing.

After generating and saving an ensemble $\{\Phi_n\}$ distributed $\propto e^{-S}$, it is usually quick and easy to measure many observables $\langle \mathcal{O} \rangle$.

Changing the action (generally) requires generating a new ensemble.
Backup: Hybrid Monte Carlo (HMC) algorithm

Goal: Sample field configurations $\Phi_i$ with probability $\frac{1}{Z} e^{-S[\Phi_i]}$

HMC is a Markov process, based on Metropolis–Rosenbluth–Teller (MRT)

Fermions $\rightarrow$ extensive action computation, so best to update entire system at once

Use fictitious molecular dynamics evolution

1. Introduce a fictitious fifth dimension (“MD time” $\tau$) and stochastic canonical momenta for field variables

2. Run inexact MD evolution along a trajectory in $\tau$ to generate a new four-dimensional field configuration

3. Apply MRT accept/reject test to MD discretization error
Backup: Qualitative picture of lattice results

Light scalar likely related to near-conformal dynamics
(unconfirmed interpretation as PNGB of approx. scale symmetry)

QCD-like chiral breaking

Conformal hyperscaling

Chirally broken, near conformal

\( m_f = \text{8 scalar spectrum & EFT} \)

\( M_{\rho} \)
\( M_{\pi} \)
\( M_{0^{++}} \)

\( M_{0^{++}} \) light relative to \( M_{\rho} \)

(Anna Hasenfratz)
Backup: Technical challenge for scalar on lattice

Only the new strong sector is included in the lattice calculation

⇒ The flavor-singlet scalar mixes with the vacuum

Leads to noisy data and relatively large uncertainties

Fermion propagator computation is relatively expensive

“Disconnected diagrams” formally need propagators at all $L^4$ sites

In practice estimate stochastically to control computational costs
In lattice QCD, the isosinglet scalar is much heavier than the pion. Generally $M_S \gtrsim 2M_P$, and for heavy quarks $M_S > M_V$.

For a large range of quark masses $m$

it mixes significantly with two-pion scattering states.
In earlier work with domain wall fermions at heavier fermion masses the non-singlet scalar is heavier than the vector, $M_{a_0} > M_V$

Staggered analyses in progress, but more complicated
In addition to omitting the light scalar $\chi$PT also suffers from large expansion parameter

$$5.8 \leq \frac{2N_F Bm}{16\pi^2 F^2} \leq 41.3 \quad \text{for} \quad 0.00125 \leq m \leq 0.00889$$

Big ($\sim 50\sigma$) shift in $F$ from linear extrapolation vs. NLO $\chi$PT

Fit quality is not good, especially for NLO joint fit with $\chi^2$/d.o.f. $> 10^4$
Backup: NLO chiral perturbation theory formulas

\[ M_P^2 = 2Bm \left[ 1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ 128\pi^2 \left( 2L'_6 - L'_4 + \frac{2L'_8 - L'_5}{N_F} \right) + \frac{\log \left( \frac{2Bm}{\mu^2} \right)}{N_F^2} \right\} \right] \]

\[ F_P = F \left[ 1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ 64\pi^2 \left( L'_4 + \frac{L'_5}{N_F} \right) - \frac{1}{2} \log(2Bm/\mu^2) \right\} \right] \]

\[ M_P a_{PP} = -\frac{2Bm}{16\pi F^2} \left[ 1 + \frac{2N_F Bm}{16\pi^2 F^2} \left\{ -256\pi^2 \left[ \left( 1 - \frac{2}{N_F} \right) \left( L'_4 - L'_6 \right) + \frac{L'_0 + 2L'_1 + 2L'_2 + L'_3}{N_F} \right] - 2 \frac{N_F - 1}{N_F^3} \right. \\
\left. + \frac{2 - N_F + 2N_F^2 + N_F^3}{N_F^3} \log \left( \frac{2Bm}{\mu^2} \right) \right\} \right] \]
Backup: 2 → 2 elastic scattering on the lattice

Measure both $E_{PP}$ and $M_P$ \( \rightarrow k = \sqrt{(E_{PP}/2)^2 - M_P^2} \)

s-wave scattering phase shift: \( \cot \delta_0(k) = \frac{1}{\pi k L} \left( \frac{k^2 L^2}{4\pi} \right) \)

with regularized \( \zeta \) function \( S(\eta) = \sum_{j \neq 0}^{\Lambda} \frac{1}{j^2 - \eta} - 4\pi \Lambda \)

Effective range expansion:

\[
k \cot \delta_0(k) = \frac{1}{a_{PP}} + \frac{1}{2} M_P^2 r_{PP} \left( \frac{k^2}{M_P^2} \right) + \mathcal{O} \left( \frac{k^4}{M_P^4} \right)\]