# Lattice Strong Dynamics for Electroweak Symmetry Breaking

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# Motivation and outline

- Physics beyond the standard model may be strongly coupled
- Strongly-coupled gauge theories need not resemble QCD
- Lattice gauge theory can provide non-perturbative information

1. Brief review of dynamical electroweak symmetry breaking
   - Basics
   - Problems
   - Solutions?
2. New strong dynamics on the lattice
3. Lattice Strong Dynamics (LSD) Collaboration results
   - LSD philosophy and simulation details
   - Chiral condensate enhancement
   - $S$ parameter

For references, cf. arXiv:0812.2035 & Lattice 2010 plenary by L. Del Debbio

# (Extended) technicolor in a picture

Massless SU(2)
Gauge fields

$A_1 \quad A_2 \quad A_3$

**A new strong force**
Techni-quarks
Techni-gluons

Massless particle
fermion fields

$\Psi_e, \Psi_\mu, \ldots$

**Spontaneous chiral symmetry breaking by the strong dynamics**

$W^+ \quad Z^0 \quad W^-$


$m_\pi = 0$

$\langle \bar{T}T \rangle \neq 0$

$\bar{T}T\bar{\Psi}_e\Psi_e, \ldots$

$m_{W,Z} \sim F_\pi^{tc}$

**TC**

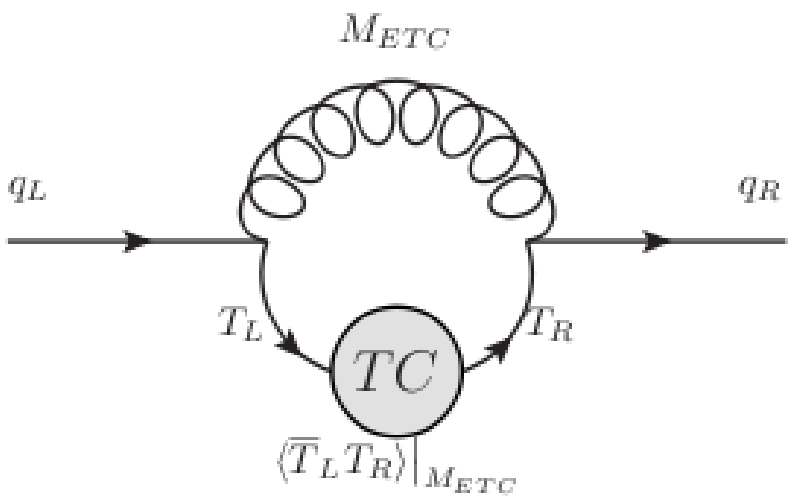
$m_e \sim \langle \bar{T}T \rangle, \ldots$

**ETC**

*P. Vranas, LLNL*

# (Extended) technicolor in words

- Replace scalar Higgs bosons with new strongly interacting sector
- $SU(N_c)$ gauge theory with $N_f$ "technifermions" $T$
- Chiral symmetry breaking from $\langle \overline{T}T \rangle \big|_{\Lambda_{TC}} \sim \Lambda_{TC}^3 \sim (1\ \text{TeV})^3$
  also breaks $SU(2)_L \times U(1)_Y \to U(1)_{em}$ with $v \sim F\sqrt{N_D}$
- SM fermions $q$ acquire masses from "extended" interactions




Integrate out ETC gauge bosons at scale $M_{ETC}$: $m_q \sim \dfrac{\langle \overline{T}T \rangle \big|_{M_{ETC}}}{M_{ETC}^2}$

# Wasn't technicolor ruled out a decade ago?

Technicolor models face three main challenges:

1. The $S$ parameter (a problem of TC)
2. FCNCs vs. SM fermion masses (a problem of ETC)
3. The top quark mass (a really big problem)

# Problem 1: The $S$ parameter (briefly)

$S$ measures BSM contributions to electroweak physics (more later)

- Experimentally, $S \lesssim 0$
- In TC, two contributions to $S$, **both positive**:

1. Techni-hadronic contribution $\sim 0.3 \frac{N_f}{2} \frac{N_c}{3}$ ("voodoo QCD")
2. Pseudo Nambu–Goldstone bosons' contribution ($\chi$PT)

$$\sim \frac{1}{12\pi} \left( \frac{N_f^2}{4} - 1 \right) \log \left( \frac{M_{\rho_T}^2}{M_{PNGB}^2} \right)$$

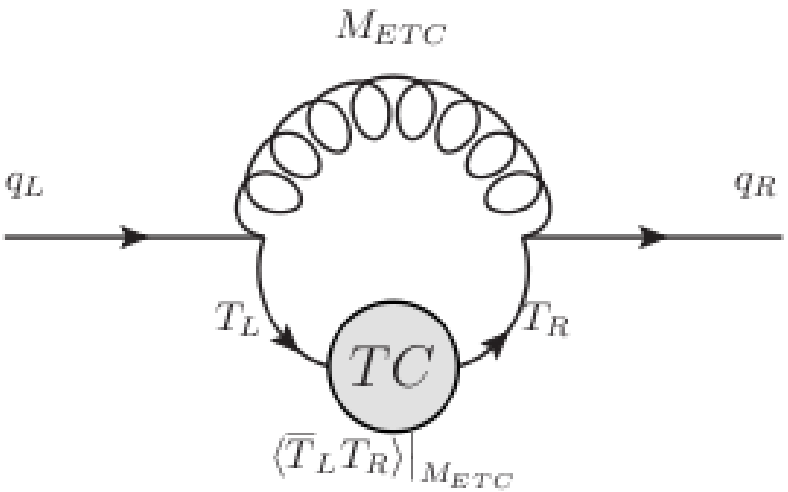
Apparent tension with experiment worsens as $N_c$, $N_f$ increase

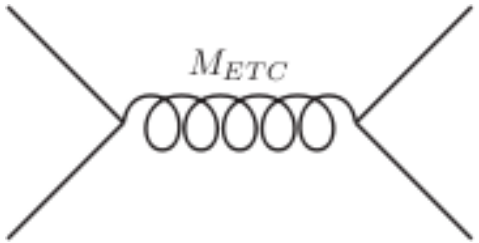
# Problem 2: FCNCs vs. SM fermion masses

Integrating out ETC gauge bosons produces four-fermion operators that provide both SM fermion masses and FCNCs

Masses: $\frac{(\overline{T}T)(\overline{q}q)}{M_{ETC}^2}$ FCNCs: $\frac{(\overline{q}q)(\overline{q}q)}{M_{ETC}^2}$

FCNCs required by CKM mixing, limit obtainable SM fermion masses.



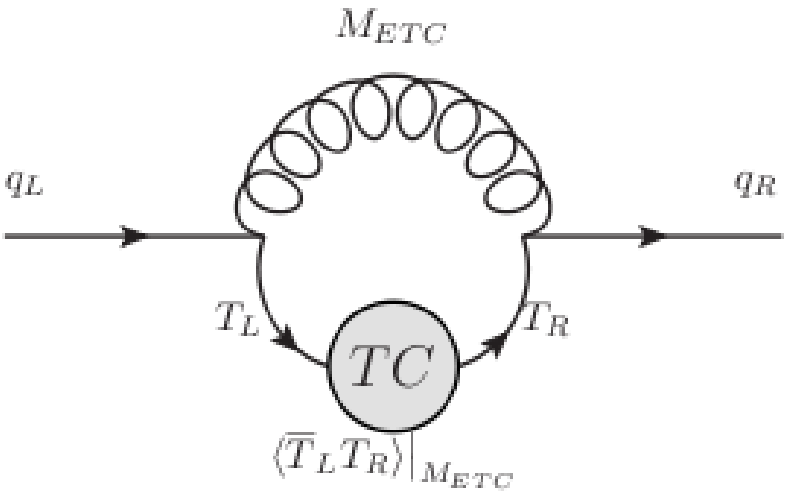
# Problem 2: FCNCs vs. SM fermion masses

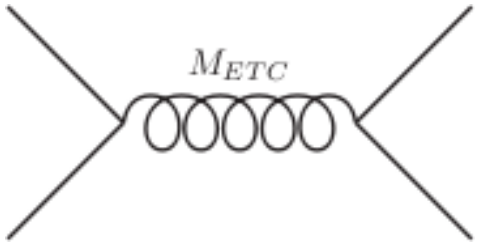
Example: $(\Delta M_K)|\epsilon_K| \lesssim 8 \times 10^{-19}$ TeV $\Rightarrow M_{ETC}^{(s)} \gtrsim 16,000$ TeV.

Using renormalization group and voodoo QCD $\gamma(\mu) \sim \mathcal{O}(\alpha(\mu)) \ll 1$,

$$\langle \overline{T}T \rangle\big|_{M_{ETC}} = \langle \overline{T}T \rangle\big|_{\Lambda_{TC}} \exp\left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma(\mu) \right) \approx \langle \overline{T}T \rangle\big|_{\Lambda_{TC}} \sim (1 \text{ TeV})^3$$

$$m_s \sim \frac{\langle \overline{T}T \rangle\big|_{M_{ETC}}}{\left(M_{ETC}^{(s)}\right)^2} \lesssim \frac{(1 \text{ TeV})^3}{(10^4 \text{ TeV})^2} \sim 0.1 \text{ MeV}$$



# “Walking” Technicolor

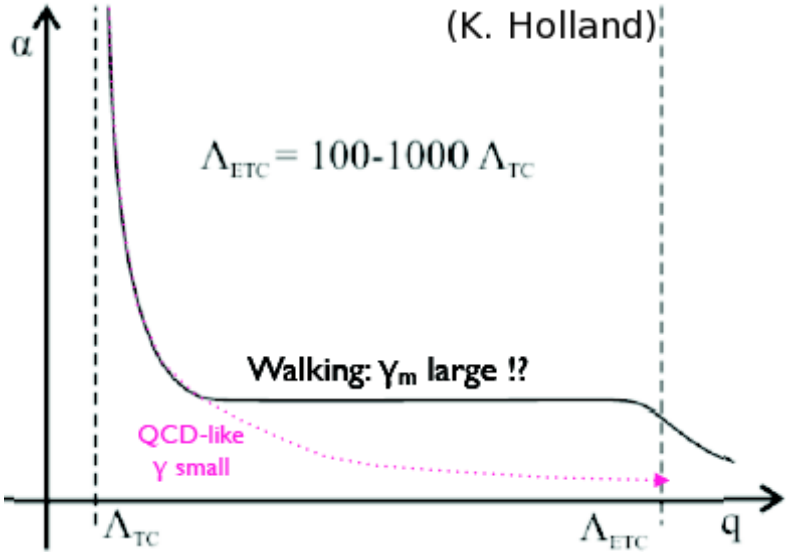
Suppose $\gamma(\mu) \sim 1$, so that

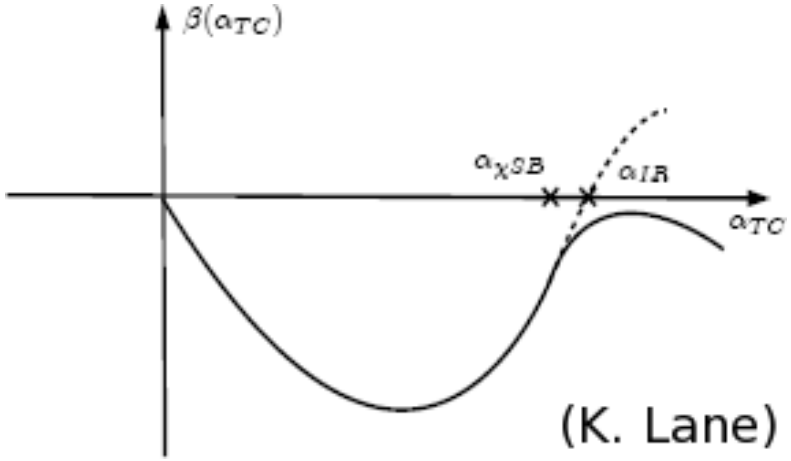
$$\langle \overline{T}T \rangle|_{M_{ETC}} = \langle \overline{T}T \rangle|_{\Lambda_{TC}} \exp\left( \int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma(\mu) \right) \approx \langle \overline{T}T \rangle|_{\Lambda_{TC}} \left( \frac{M_{ETC}}{\Lambda_{TC}} \right)^{\gamma}$$

$$m_s \lesssim 1 \text{ GeV}$$

$\gamma(\mu) \sim 1$ for $\Lambda_{TC} \lesssim \mu \lesssim M_{ETC}$

implies large, slowly-running (“walking”) coupling, small $\beta$ function.

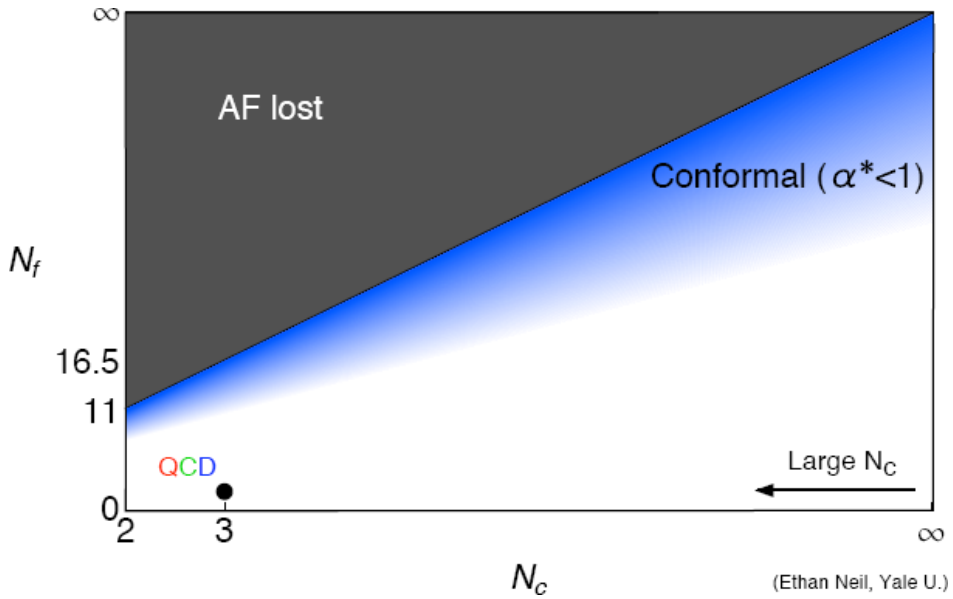


# Walking Technicolor: not just Wishful Thinking?

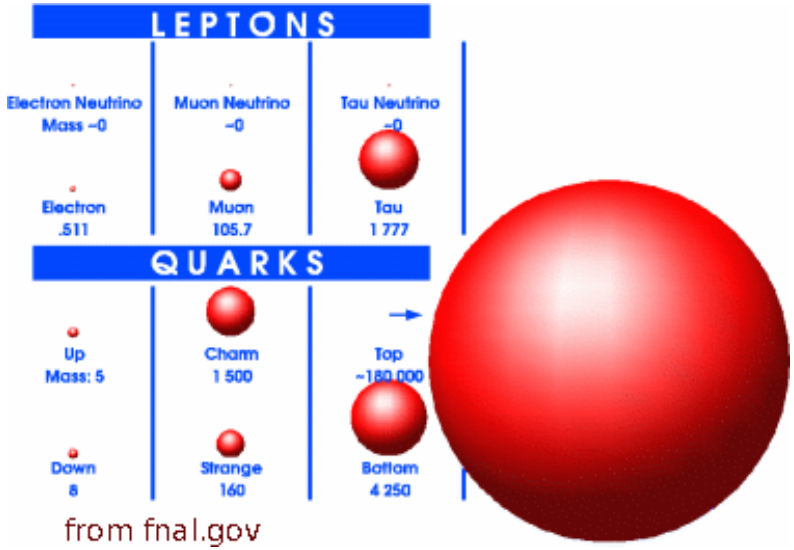
- Strongly-coupled gauge theories can look very different than QCD
- With many fermions, theory has perturbative IR fixed point; it is in a conformal phase with no spontaneous $\chi$SB
- The **conformal window** ranges from loss of asymptotic freedom to some (unknown) critical $N_f^c < N_f^{AF}$
- With $N_f \lesssim N_f^c$, may be approximately conformal (walking!) for some range of scales

Visualization of conformal window for $SU(N_c)$ fermions in fundamental rep:




Strong coupling $\Rightarrow$ lattice!

# Problem 3: The top quark mass




The top quark mass weighs like a nightmare on the brains of technicolor theorists...

...and is beyond the scope of this talk.

Even with $\gamma(\mu) \approx 1$:

$$m_t \sim 0.1 \text{ TeV} \sim \frac{\langle \overline{T}T \rangle\big|_{M_{ETC}^{(t)}}}{\left(M_{ETC}^{(t)}\right)^2} \lesssim \frac{\langle \overline{T}T \rangle\big|_{\Lambda_{TC}}}{\Lambda_{TC} M_{ETC}^{(t)}}$$

$$\Rightarrow M_{ETC}^{(t)} \lesssim 10 \text{ TeV}$$

# Outline (reminder)

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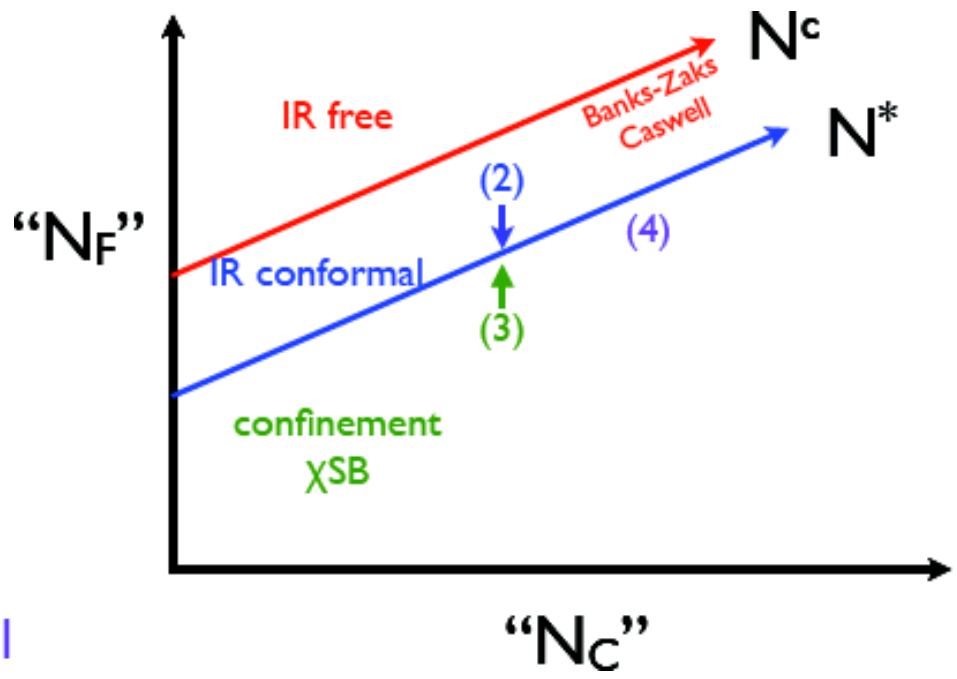
# What can the lattice contribute?

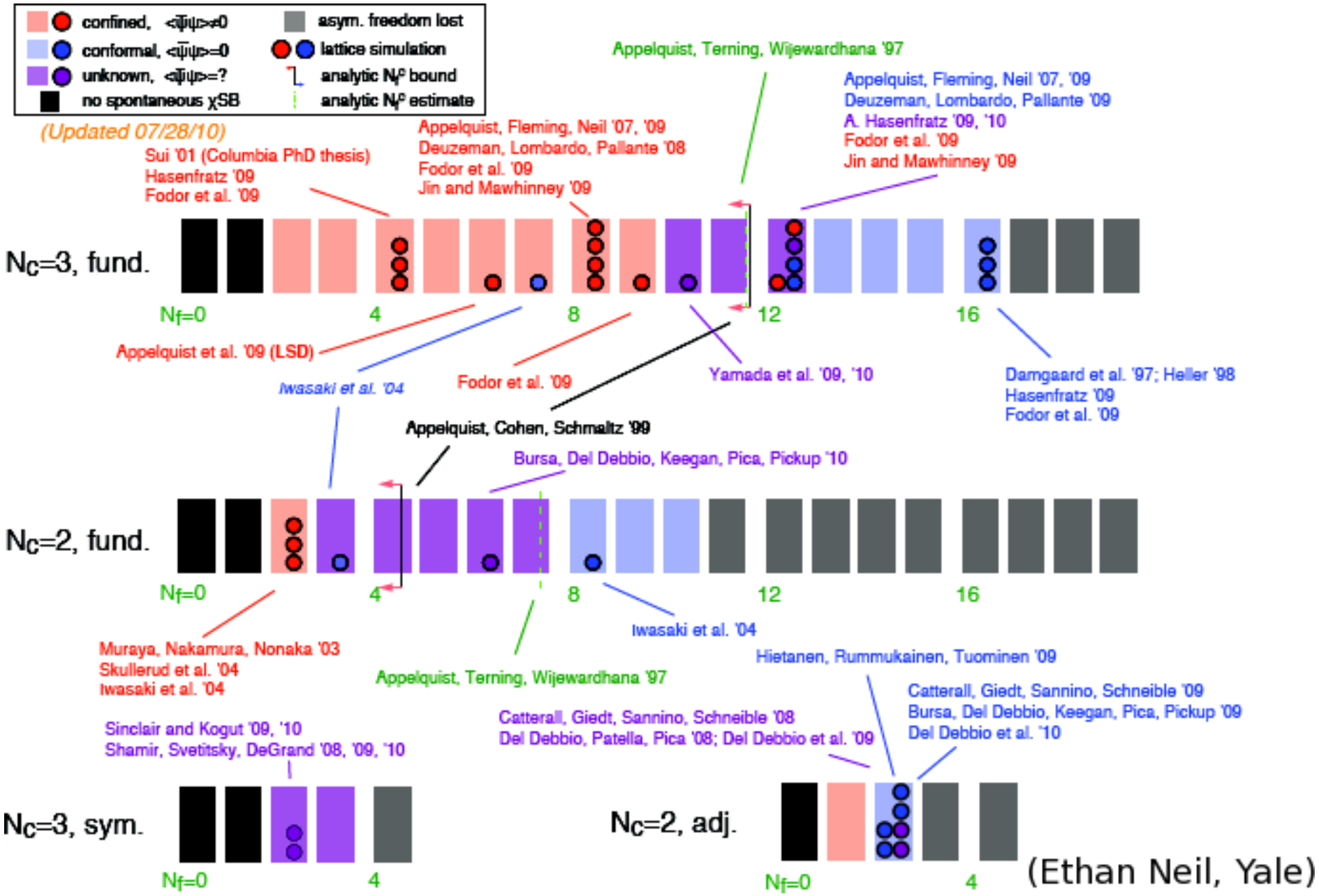
A wishlist from Lattice 2010 (S. Chivukula):

(1) Establish Phase Diagram
(2) What is $\gamma_m$? Near 1? 2?
(3) What is S? The spectrum?
  (1)Is there a $0^{++}$ (pseudo-dilaton, Higgs-like)state?
(4) Other marginal/relevant operators?
  (1)Strong-ETC and "Gauge-NJL" model




To date, most effort has focused on the phase diagram
that is, searching for conformal windows

# Searching for conformal windows




(Ethan Neil, Yale)

# How to search for conformality?

Many methods, cf. Lattice 2010 plenary by L. Del Debbio for more info

- Step scaling: search for fixed point in running coupling
  Many possible couplings: Schrödinger functional, heavy-quark potential, twisted Polyakov loop or Creutz ratio...
- Spectrum: contrast conformal vs. QCD-like,
  check scaling with quark mass $m^{1/(1+\gamma)}$ or lattice size $L$
- Monte Carlo renormalization group two-lattice matching
- Finite-temperature phase diagram
  (deconfinement and chiral transitions)
- Eigenvalue distributions

# The challenge: TeV-scale phenomenology

- Beyond classifying theories as QCD-like or (approximately) conformal, need to connect to TeV-scale phenomenology
- $S$ parameter, spectrum, $\langle \overline{\psi}\psi \rangle$
  understanding dependence on $N_c$, $N_f$ and fermion representation

(CERN)

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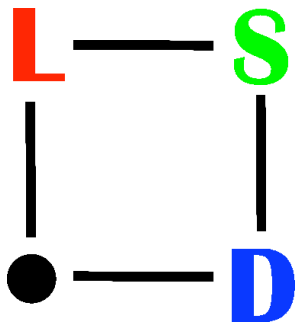
# LSD Collaboration




| | |
|---|---|
| Argonne | James Osborn |
| Boston | Ron Babich, Richard Brower, Saul Cohen, Claudio Rebbi, DS |
| Fermilab | Ethan Neil |
| Harvard | Mike Clark |
| Livermore | Mike Buchoff, Michael Cheng, Pavlos Vranas |
| UC Davis | Joseph Kiskis |
| Yale | Thomas Appelquist, George Fleming, Meifeng Lin, Gennady Voronov |

Formed in 2007 to pursue non-perturbative studies of strongly interacting theories likely to produce observable signatures at the Large Hadron Collider.

# LSD Philosophy and Simulation Details

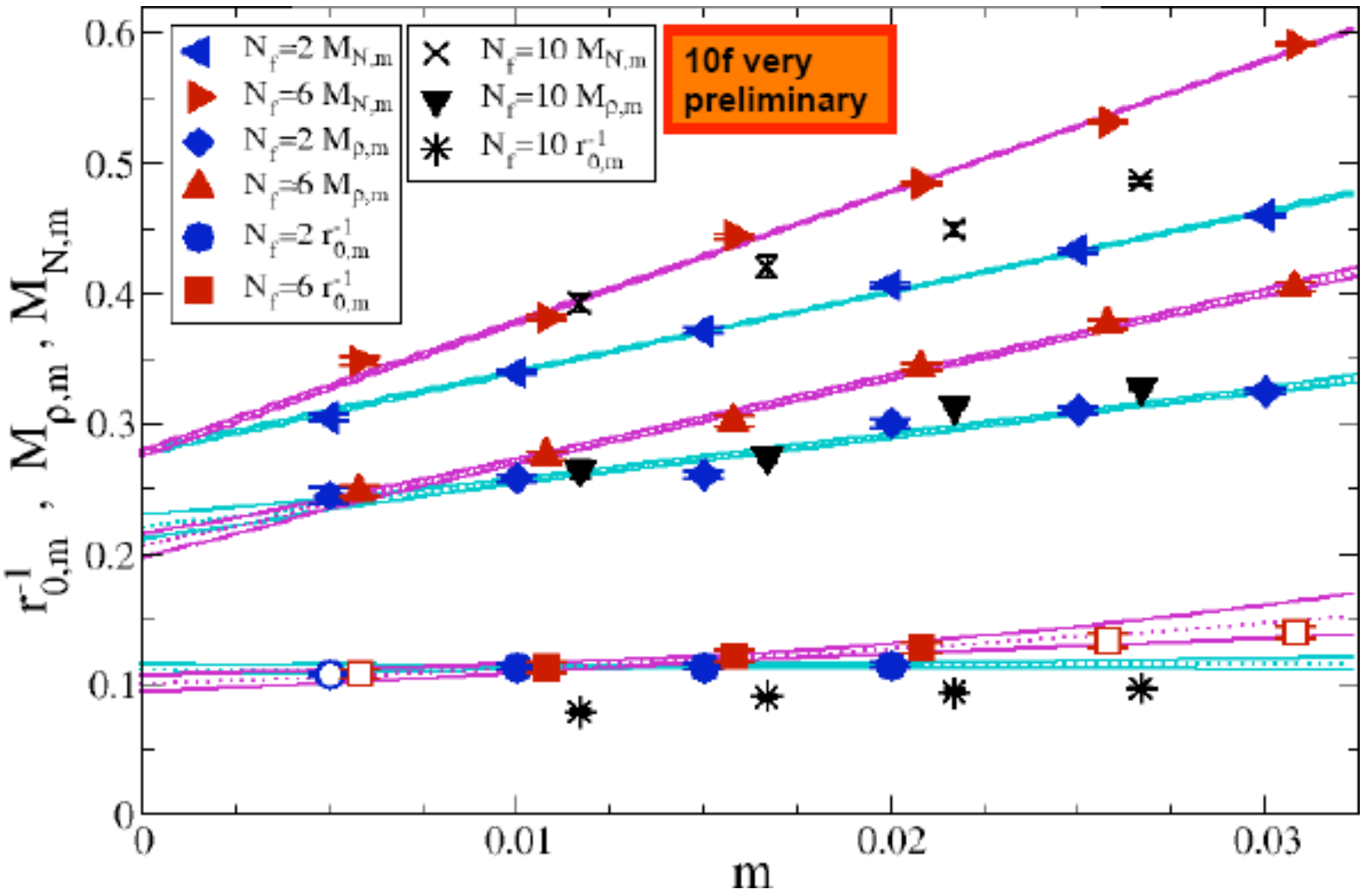
- Start from what we know (QCD) and use it as a baseline
  $\Rightarrow SU(3)$ gauge theory with $N_f =$2, 6, 10 fundamental
- Use large (matched) cutoff to observe running
  $\Rightarrow \beta =$2.7 (2f); 2.1 (6f); 1.95 (10f)
  $\Rightarrow a^{-1} \approx 3.6\ \text{GeV}^{-1} \approx 5M_\rho$;
  $a \approx 0.06$ fm; $L = 32a \approx 1.8$ fm; $M_P L \gtrsim 4$
- Exploratory calculations
  $\Rightarrow \sim$ 1000 trajectories per point
- We don't know the answer
  $\Rightarrow$ Use domain wall fermions for chiral and flavor symmetries
  $L_s = 16$: $m_{res} \approx 3\times10^{-5}$ (2f); $8\times10^{-4}$ (6f); $2\times10^{-3}$ (10f)

Anything not yet on the arXiv should be considered PRELIMINARY

# DWF are expensive, even for exploratory calculations

$\sim$ 300M core-hours on LLNL BGL, USQCD clusters, NSF Teragrid...

# Matching scales




$N_f = 2$ and $N_f = 6$ scales all matched at 10% level

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# Chiral condensate enhancement: preliminaries

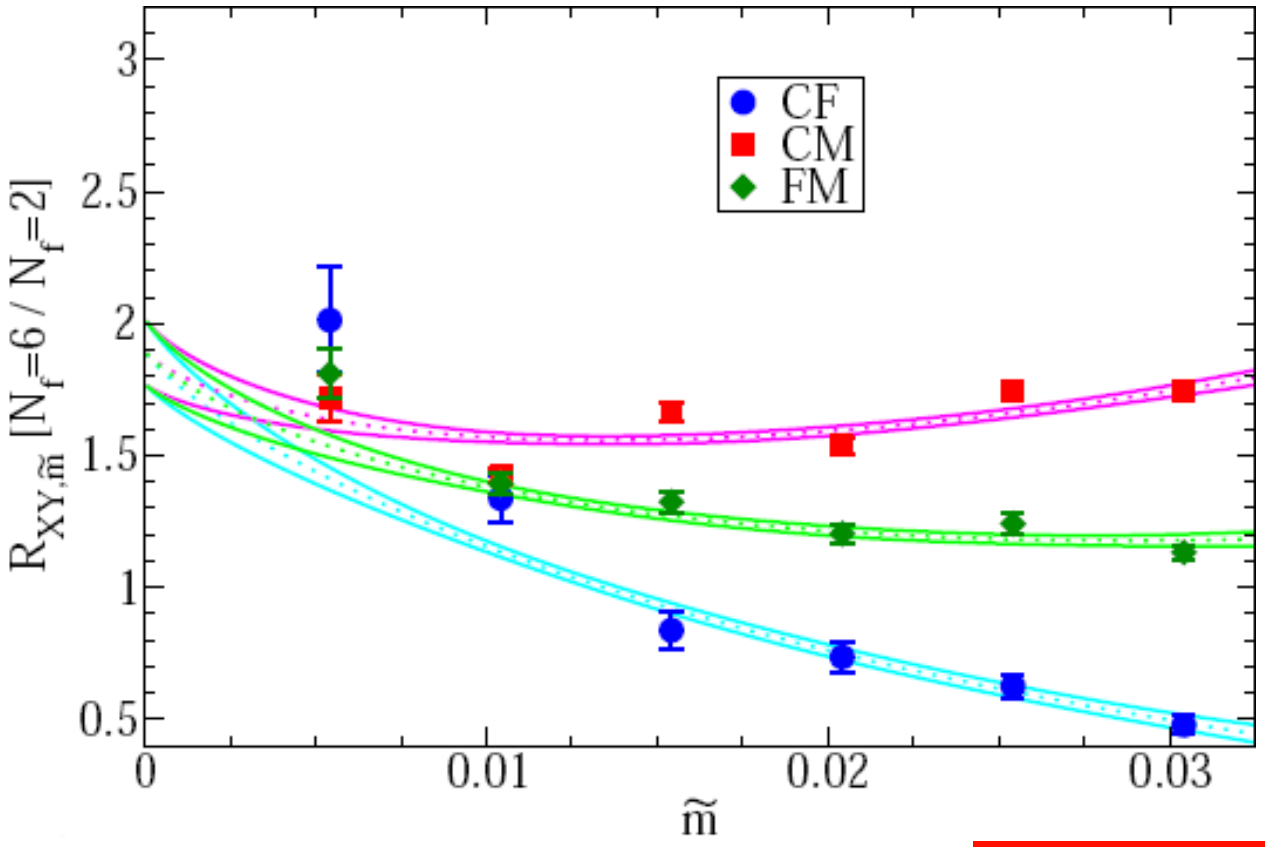
- Search for enhancement through $\langle\overline{\psi}\psi\rangle / F^3$
- Not RG invariant: keep cutoff fixed in physical units
- Focus on the ratio $R$ of $\langle\overline{\psi}\psi\rangle / F^3$ between $N_f = 6$ and $N_f = 2$

$$R = \frac{(\langle\overline{\psi}\psi\rangle / F^3)_{6f}}{(\langle\overline{\psi}\psi\rangle / F^3)_{2f}} = \frac{\exp\left(\int_{M_\rho}^{5M_\rho} \left.\frac{\gamma(\mu)}{\mu}\right|_{6f} d\mu\right)}{\exp\left(\int_{M_\rho}^{5M_\rho} \left.\frac{\gamma(\mu)}{\mu}\right|_{2f} d\mu\right)}$$

$\overline{MS}$ perturbation theory & perturbative conversion to lattice scheme predicts $R = 1.27(7)$

# Enhancement of $\langle \overline{\psi}\psi \rangle / F^3$, $N_f = 2$ to $N_f = 6$

Find significant enhancement compared with perturbative $R = 1.27(7)$

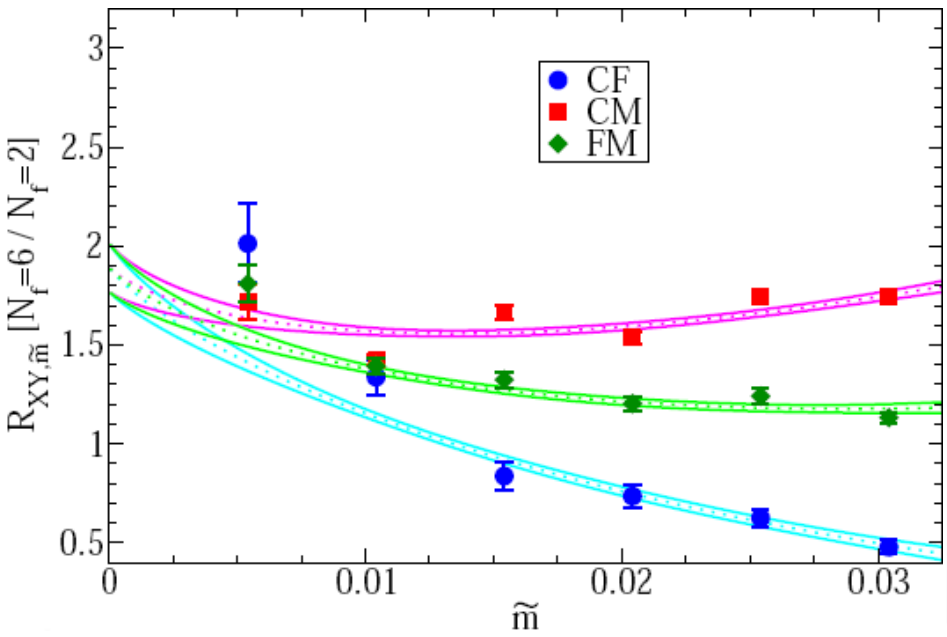


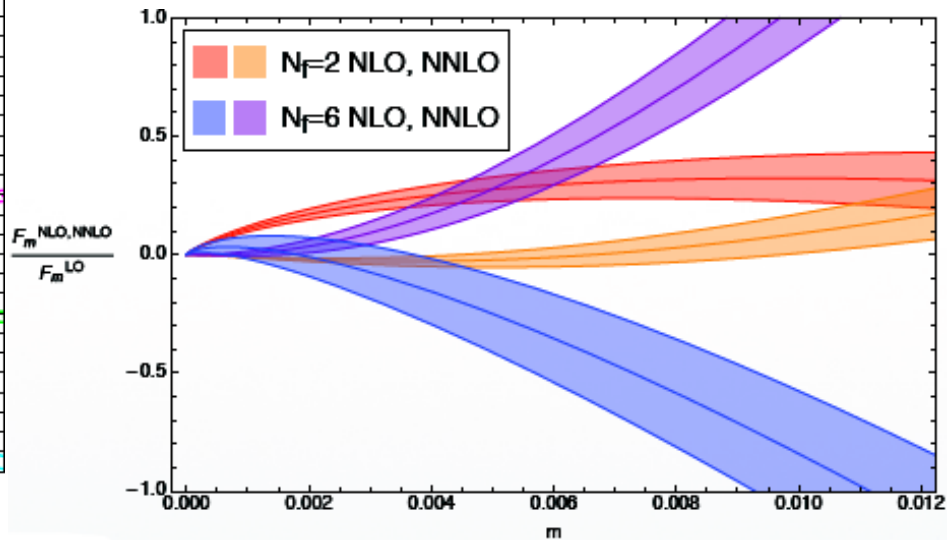

$$FM = M_\pi^2 / 2mF_\pi$$

$$CF = < \bar{\Psi}\Psi > / F_\pi^3$$

$$CM = \frac{< \bar{\Psi}\Psi >}{[\frac{\sqrt{2m\bar{\Psi}\Psi}}{m_\pi}]^3}$$

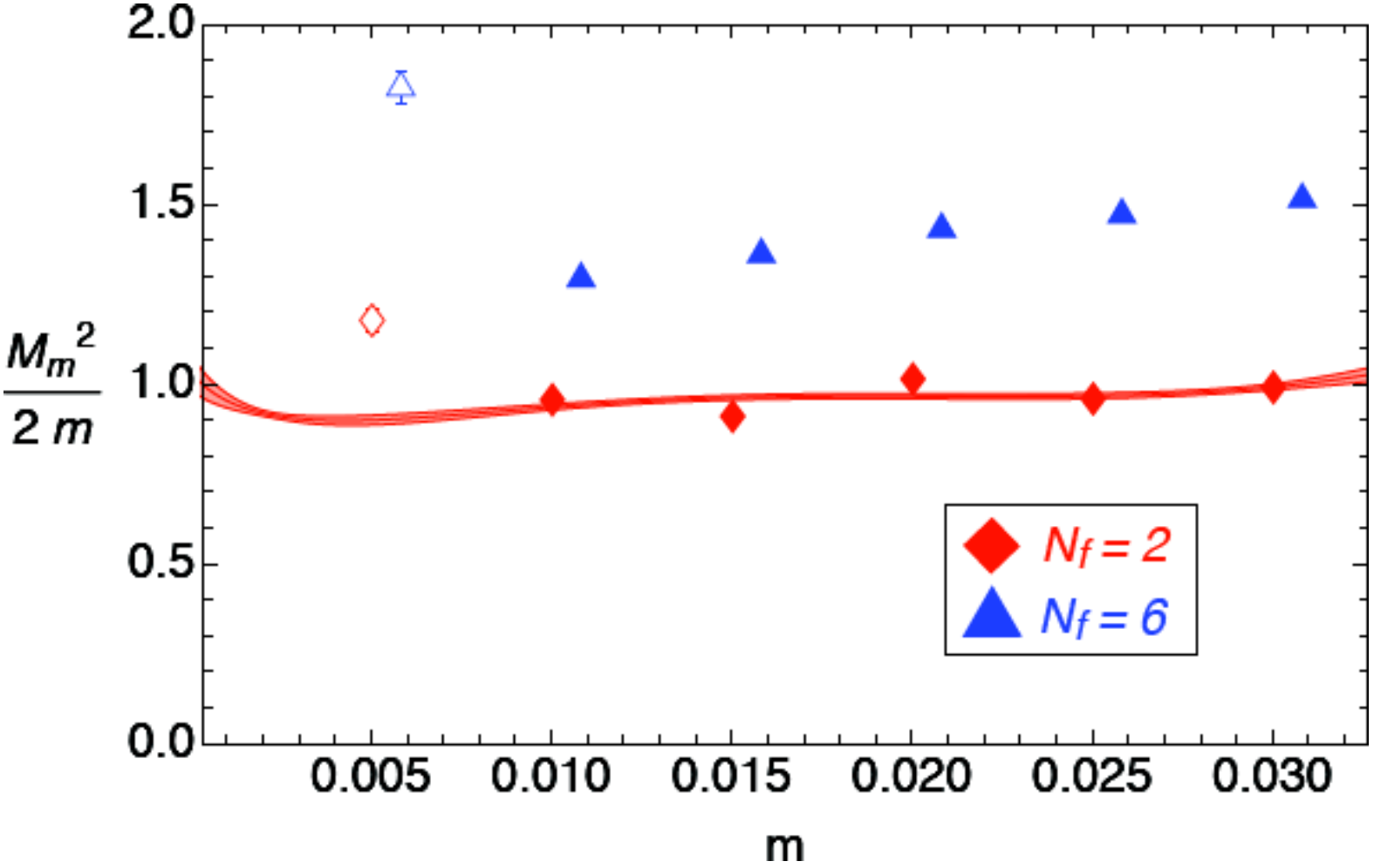
# NLO $\chi$PT fits, $N_f = 2$ and $N_f = 6$



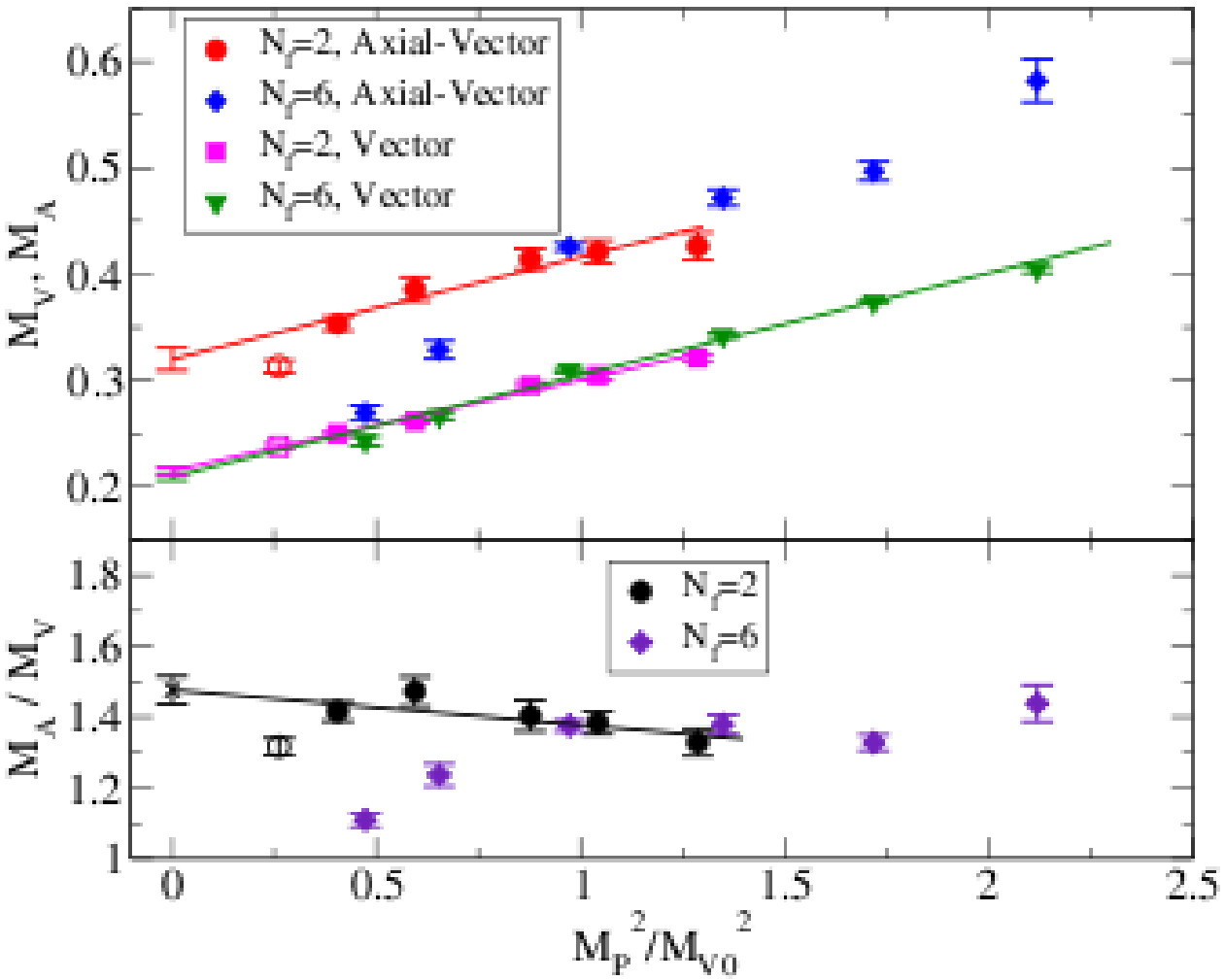

- NLO$\chi$PT fits work for $N_f = 2$ but not $N_f = 6$ (lighter $m_f$ required)
- GMOR $\Rightarrow \frac{\langle\overline{\psi}\psi\rangle}{F_\pi^3} = \frac{M_\pi^3}{\sqrt{(2m)^3\langle\overline{\psi}\psi\rangle}} = \frac{M_\pi^2}{2mF_\pi} \equiv \mathcal{R}$ as $m \to 0$
- Fit ratios to $\mathcal{R}\left[1 + \widetilde{m}(\alpha_{XY10} + \alpha_{11}\log\widetilde{m})\right]$ where $\widetilde{m} \equiv \sqrt{m_2 m_6}$

# Pseudo Nambu–Goldstone boson mass




- Slope of $M_P^2$ with $m$ significantly larger for $N_f = 6$
- Switch to plotting versus $M_P^2$, to provide more physical comparison

# Vector and axial spectrum




Signs of $N_f = 6$ parity-doubling as $M_P^2$ decreases

$\Rightarrow$ implications for $S$ parameter?

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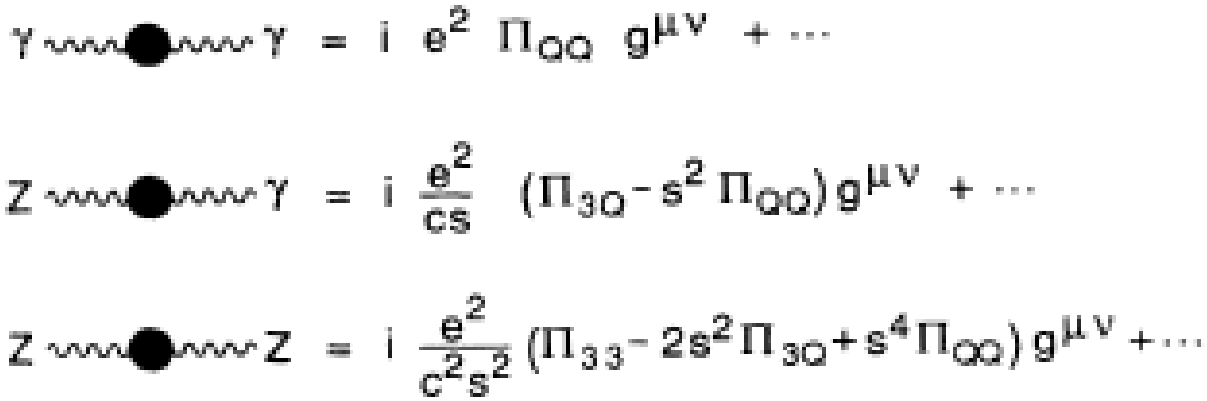
# $S$ parameter: more details




$$\Pi_{VV} = 2\Pi_{3Q}$$
$$\Pi_{AA} = 4\Pi_{33} - 2\Pi_{3Q}$$

(Peskin & Takeuchi, 1992)

With euclidean $Q^2 = -q^2 > 0$, $\widehat{Q} = 2\sin(Q/2)$

$$\Pi^{\mu\nu}_{XY}(Q) = \sum_x e^{iQ\cdot x} \left\langle J^\mu_X(x) J^\nu_Y(0) \right\rangle = g^{\mu\nu}\Pi^\perp_{XY}(Q^2) + \frac{\widehat{Q}^\mu \widehat{Q}^\nu}{\widehat{Q}^2}\Pi^{\perp+L}_{XY}(Q^2)$$

$$S \equiv 4\pi N_D \frac{d}{dQ^2}\left[\Pi^\perp_{VV}(Q^2) - \Pi^\perp_{AA}(Q^2)\right]_{Q^2=0} - \Delta S_{SM}$$

$$= \frac{1}{3\pi}\int_0^\infty \frac{ds}{s}\left\{N_D\left[R_V(s) - R_A(s)\right] - \frac{1}{4}\left[1 - (1 - M_H^2/s)^3\theta(s - M_H^2)\right]\right\}$$

$\Delta S_{SM}$ removes the Higgs boson contribution,

also cancels IR divergence from massless $\pi_T$

# Domain wall currents and correlators

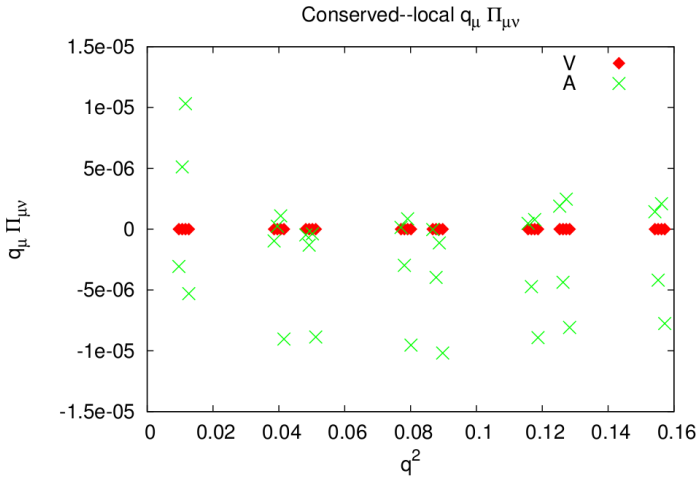
Need to use *conserved* domain wall currents $\mathcal{V}^{\mu a}$ and $\mathcal{A}^{\mu a}$
(point-split, summed over the fifth dimension)

$$\Pi^{\mu\nu}_{V-A}(Q) = Z \sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \mathrm{Tr}\left[\left\langle \mathcal{V}^{\mu a}(x) V^{\nu b}(0)\right\rangle - \left\langle \mathcal{A}^{\mu a}(x) A^{\nu b}(0)\right\rangle\right]$$

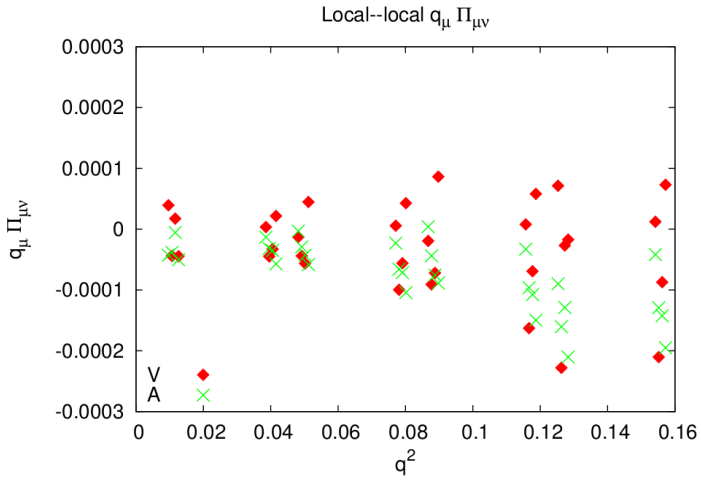
- $V^{\nu a}$ and $A^{\nu a}$ are local currents defined on the domain walls
- Conserved currents ensure that lattice artifacts cancel, needed for clean signal RBC-UKQCD
- $\langle \mathcal{V}^{\mu a}(x)\mathcal{V}^{\nu a}(0)\rangle$ and $\langle \mathcal{A}^{\mu a}(x)\mathcal{A}^{\nu a}(0)\rangle$ require $\mathcal{O}(L_s)$ inversions
- Suffices to use $\langle \mathcal{V}^{\mu a}(x) V^{\nu a}(0)\rangle$
- Renormalization constant $Z$ computed nonperturbatively
  $Z =$0.85 (2f); 0.73 (6f); 0.71 (10f)

# Ward identities and violations

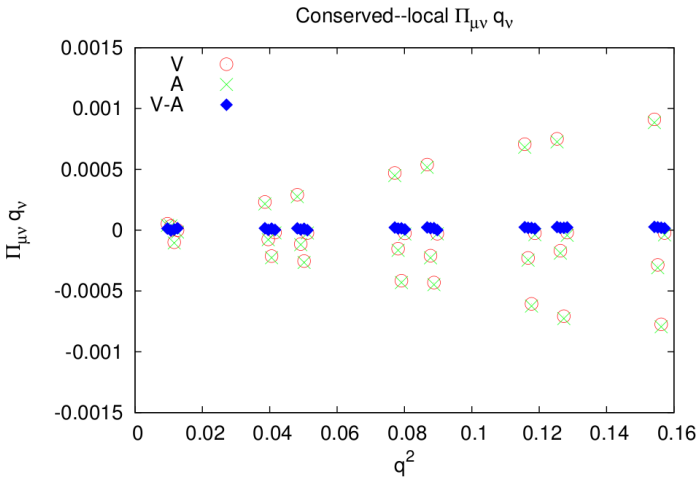
$$\widehat{Q}_\mu \left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left\langle \mathcal{V}^a_\mu(x) V^a_\nu(0)\right\rangle\right] = 0$$



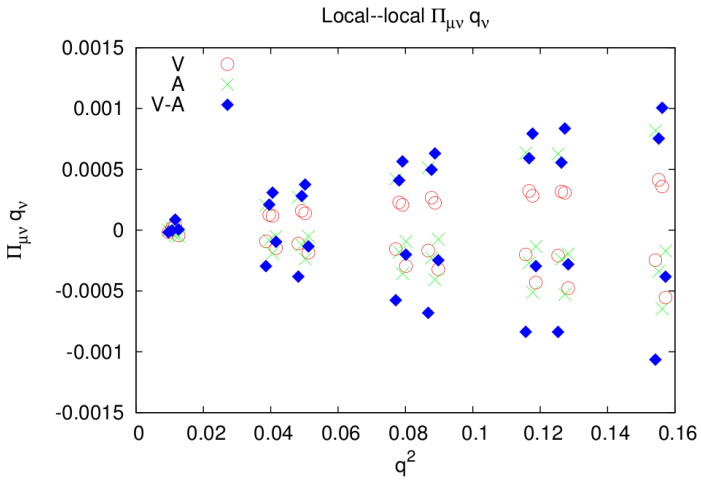

$$\widehat{Q}_\mu \left[\sum_x e^{iQ\cdot x} \left\langle V^a_\mu(x) V^a_\nu(0)\right\rangle\right] \neq 0$$



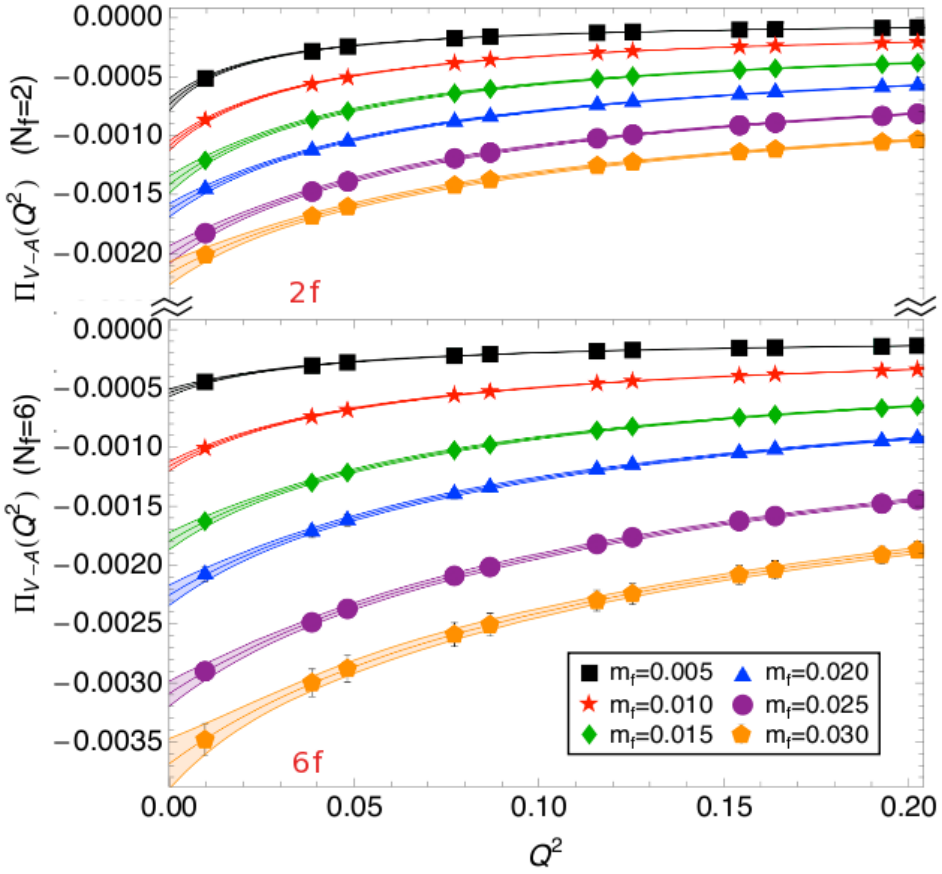

$$\left[\sum_x e^{iQ\cdot(x+\widehat{\mu}/2)} \left(\left\langle \mathcal{V}^a_\mu V^a_\nu\right\rangle - \left\langle \mathcal{A}^a_\mu A^a_\nu\right\rangle\right)\right] \widehat{Q}_\nu \approx 0$$




$$\left[\sum_x e^{iQ\cdot x} \left(\left\langle V^a_\mu V^a_\nu\right\rangle - \left\langle A^a_\mu A^a_\nu\right\rangle\right)\right] \widehat{Q}_\nu \neq 0$$



# Correlator data and fits



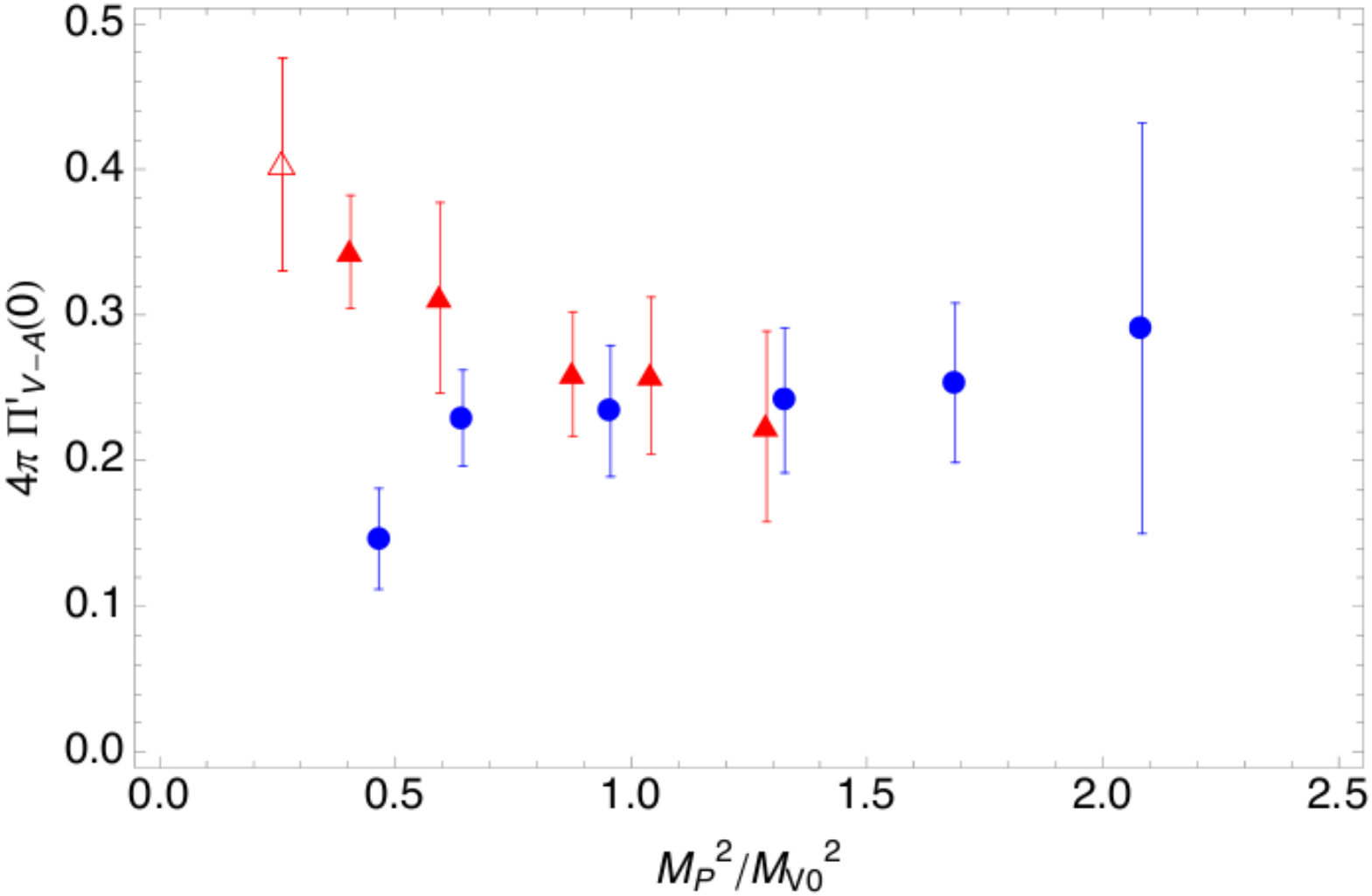

Independent fits to (1, 2) Padé, $Q^2 < 0.4$

Fits stable with $\chi^2 \ll 1$ as $Q^2$ fit range varies

$$\frac{a_0 + a_1 Q^2}{1 + b_1 Q^2 + b_2 Q^4} = \left[ -F_0^2 + \frac{Q^2 F_1^2}{M_1^2 + Q^2} - \frac{Q^2 F_2^2}{M_2^2 + Q^2} \right]_{F_0^2 = F_1^2 - F_2^2}$$

# Fit results for $\Pi'_{V-A}(0)$, $N_f = 2$ and $N_f = 6$

2f in red 6f in blue



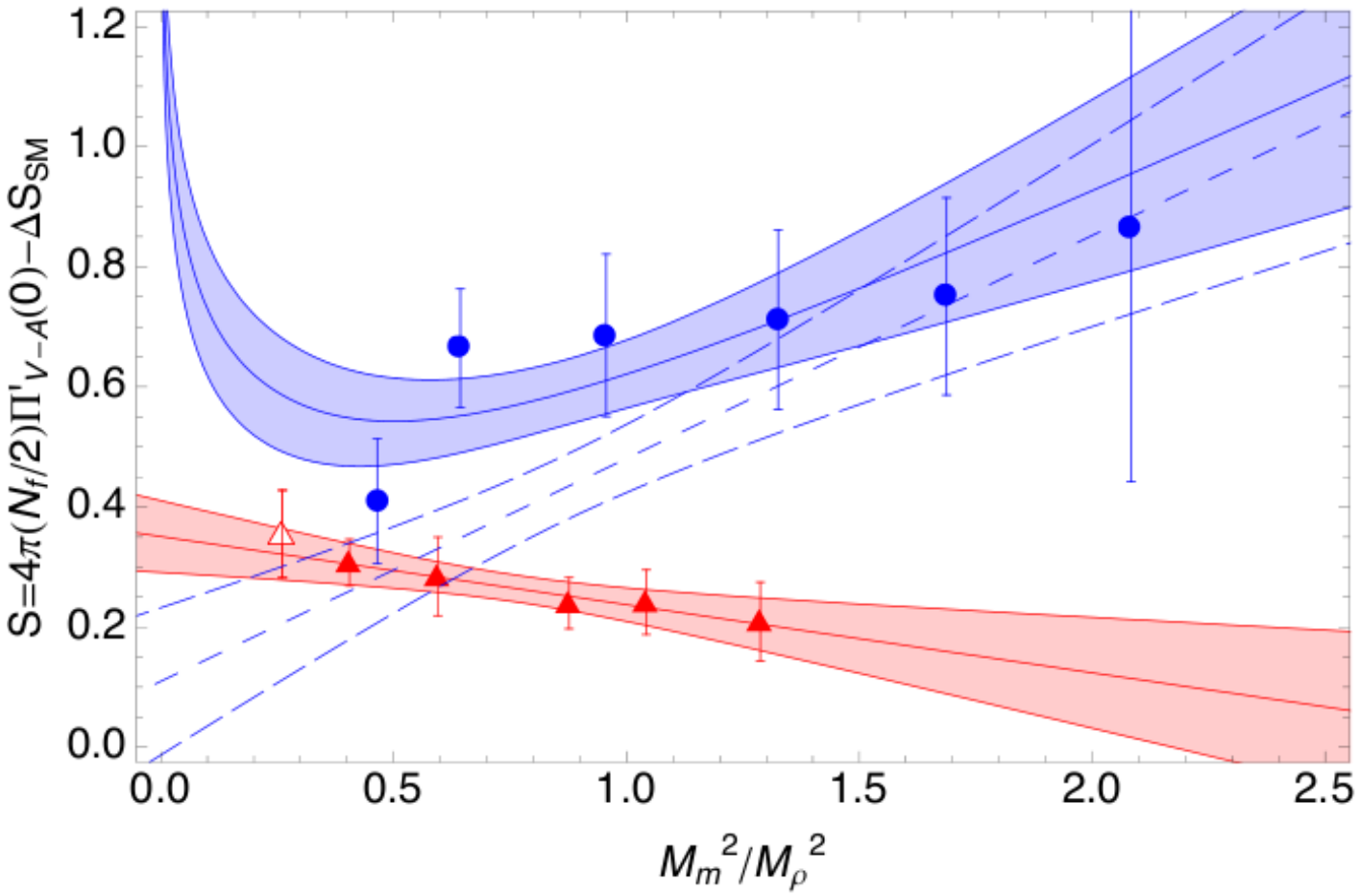
# $\Delta S_{SM}$ with $m_f > 0$

- $\Delta S_{SM}$ cancels IR divergence from massless $\pi_T$
- With $m_f > 0$, need IR cutoff $4M_P^2 > 0$ on $\Delta S_{SM}$ spectral integral
- For $N_f = 2$, $\Delta S_{SM}$ and $\pi_T$ continue to cancel as $m_f \to 0$
- For $N_f > 2$, extra $N_f^2 - 4$ pseudo Nambu–Goldston bosons receive masses from other interactions
- Set reference Higgs mass $M_H^{ref} = \lim_{m\to 0} M_V \equiv M_{V0} \sim 1$ TeV

$$\Delta S_{SM} = \frac{1}{4}\int_{4M_P^2}^{\infty} \frac{ds}{s}\left[1 - \left(1 - \frac{M_{V0}^2}{s}\right)^3 \Theta(s - M_{V0}^2)\right]$$

Numerically, $\Delta S_{SM} \lesssim 0.04$, at most 10% reduction

# $S$ parameter, $N_f = 2$ and $N_f = 6$




$$\text{Fit to } S = A + BM_P^2 + \frac{1}{12\pi}\left[\frac{N_f^2}{4} - 1\right]\log\left(\frac{M_{V0}^2}{M_P^2}\right)$$

# Recap

- Physics beyond the standard model may be strongly coupled
- Strongly-coupled gauge theories need not resemble QCD
- Lattice gauge theory can provide non-perturbative information

For $SU(3)$ gauge theory with $N_f = 6$ compared to $N_f = 2$
The LSD Collaboration has found:

- Significant enhancement of the condensate $\langle \overline{\psi}\psi \rangle / F^3$
- $S$ parameter smaller than naïve scaling

Further studies underway:

- $N_f = 10$
- Dirac operator eigenvalue spectrum
- Effects of finite volume, topology
- $SU(2)$
- OPE for $\Pi_{V-A}$
- ...

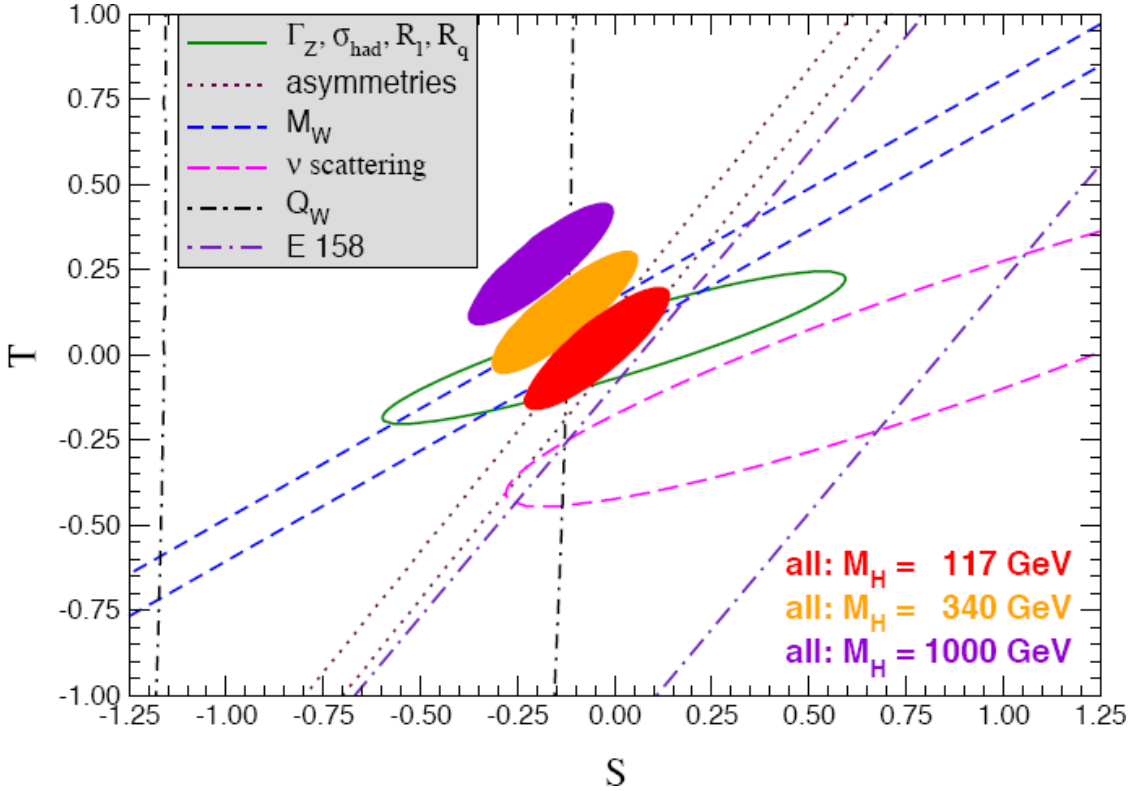
Bonus slides!

# Experiments find $S \lesssim 0$

Extract $S$ from global fit to experimental data for

- $Z$ decay partial widths and asymmetries
- Deep inelastic neutrino scattering
- $M_W/M_Z$
- Atomic parity violation

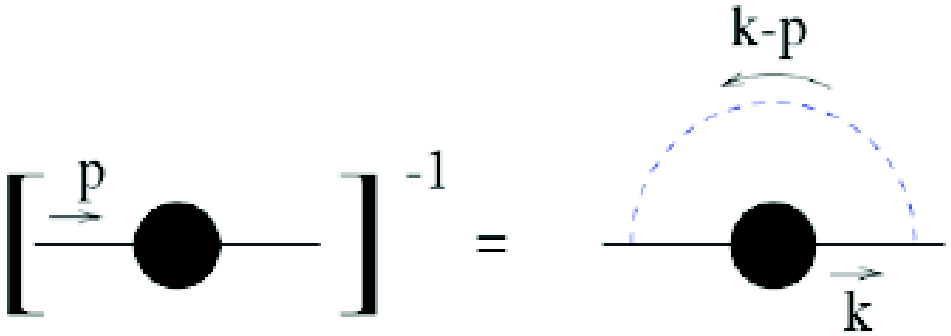
Result: $S \lesssim 0$




(PDG)

# Anomalous dimension

From “rainbow approximation” to “gap” (Schwinger–Dyson) equation




$$\gamma(\mu) = 1 - \sqrt{1 - 3C_2(r)\alpha(\mu)/\pi} \leq 1$$

Assume spontaneous chiral symmetry breaking when

$$\alpha(\mu) \geq \frac{\pi}{3C_2(r)} \equiv \alpha_{\chi SB}$$

When $\alpha(\mu) = \alpha_{\chi SB}$, this gives $\gamma(\mu) = 1$

# Perturbative Yang–Mills $\beta$ function

For $SU(N_c)$ Yang–Mills theory with $N_f$ fermions in representation $r$

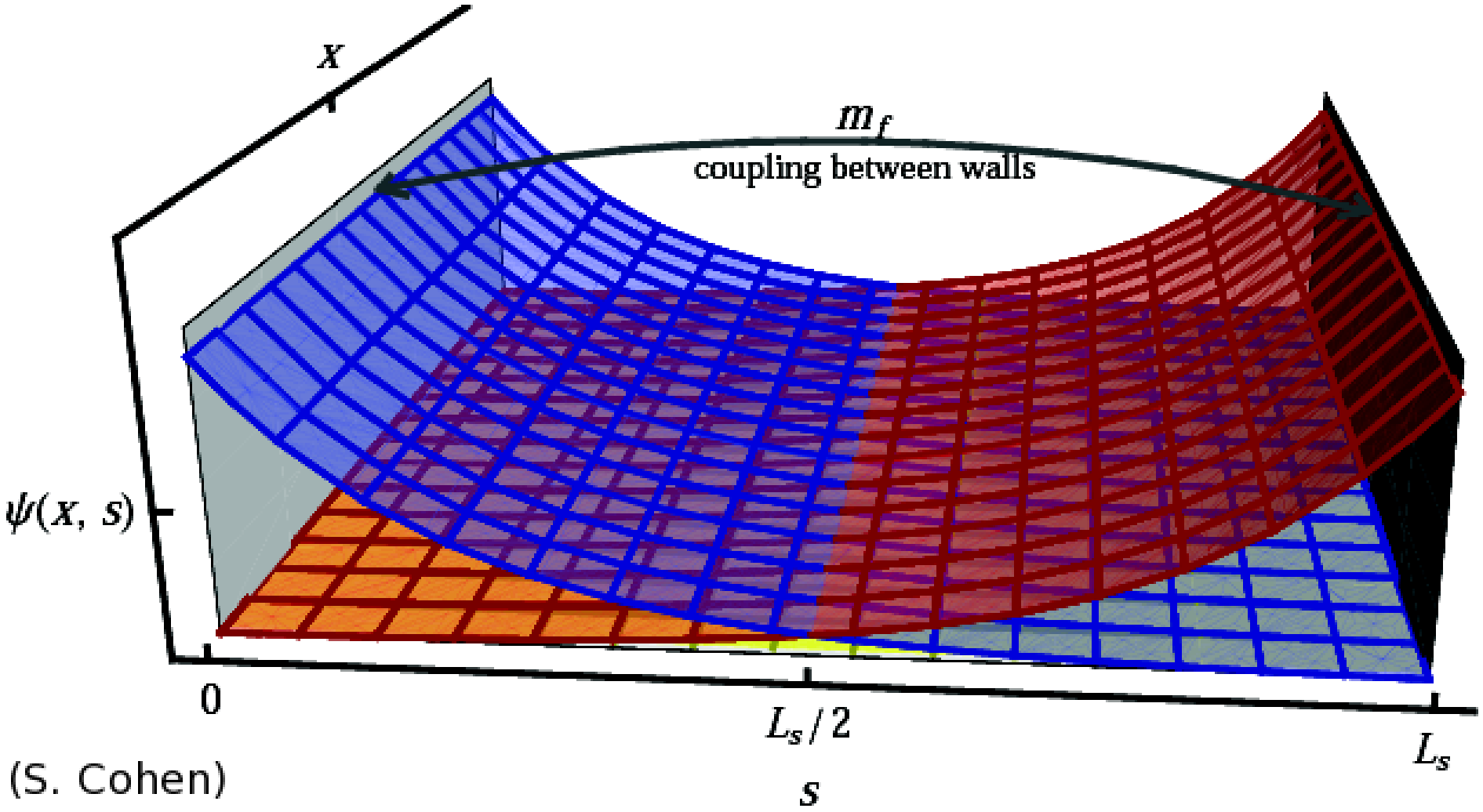
$$\beta(g) = \beta_0 g^3 + \beta_1 g^5 + \beta_2 g^7 + \cdots$$

$$\beta_0 = -\frac{1}{(4\pi)^2}\left(\frac{11}{3}N_c - \frac{4}{3}N_f C(r)\right)$$

$$\beta_1 = -\frac{1}{(4\pi)^4}\left[\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{N_c}\right)N_f C(r)\right]$$

$$C(N) = \frac{1}{2} \qquad C(Adj) = N_c \qquad C_2(N) = \frac{d(Adj)}{d(N)}C(N) = \frac{N_c^2 - 1}{2N_c}$$

# Domain wall fermions




(S. Cohen)

# NLO$\chi$PT for general $N_f$

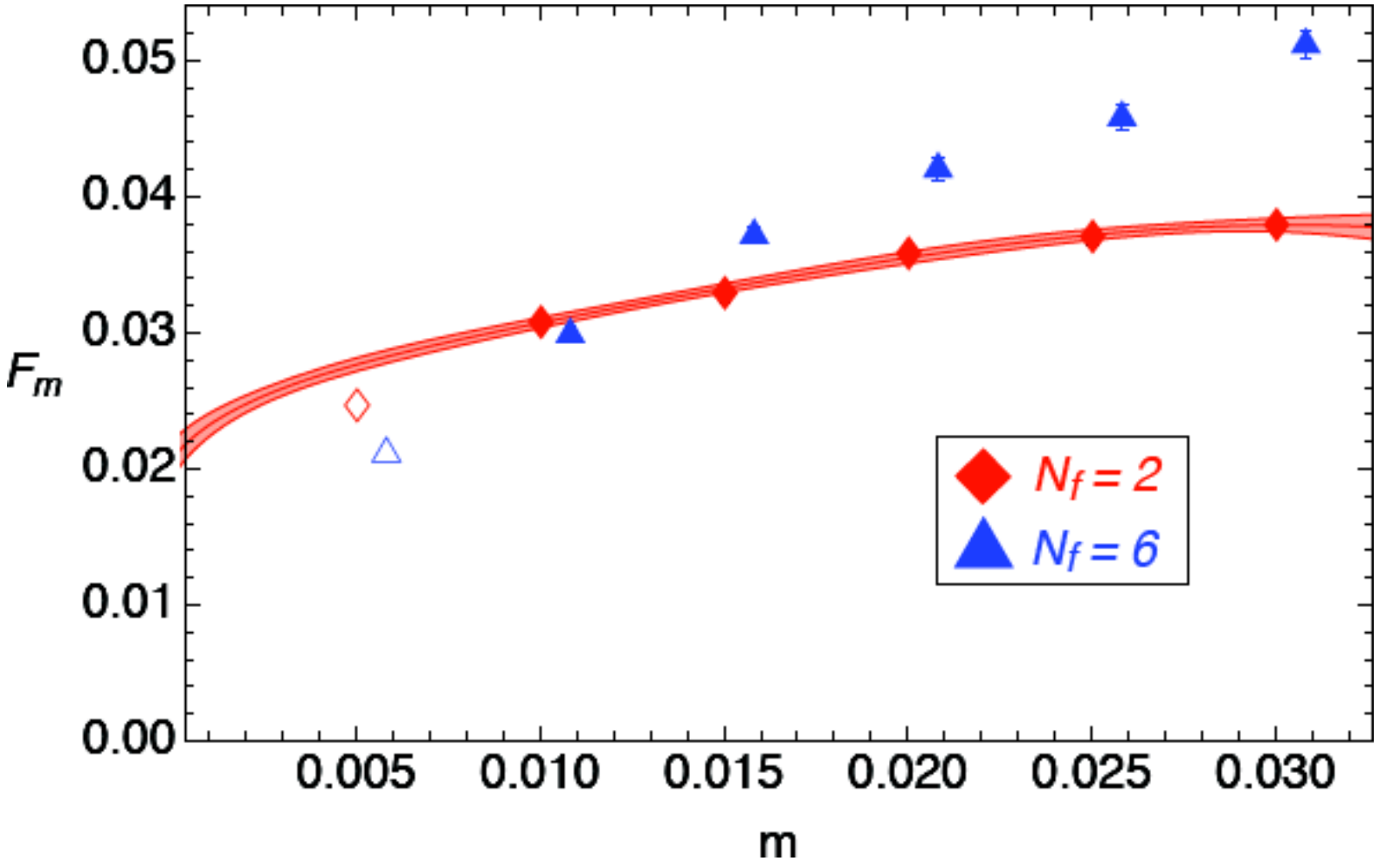
$$\frac{M_P^2}{2m} = B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_m + \frac{1}{N_f}\log\left(\frac{2mB}{(4\pi F)^2}\right)\right]\right\}$$

$$F_P = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_F - \frac{N_f}{2}\log\left(\frac{2mB}{(4\pi F)^2}\right)\right]\right\}$$

$$\langle\overline{\psi}\psi\rangle = F^2 B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\alpha_C - \frac{N_f^2 - 1}{N_f}\log\left(\frac{2mB}{(4\pi F)^2}\right)\right]\right\}$$

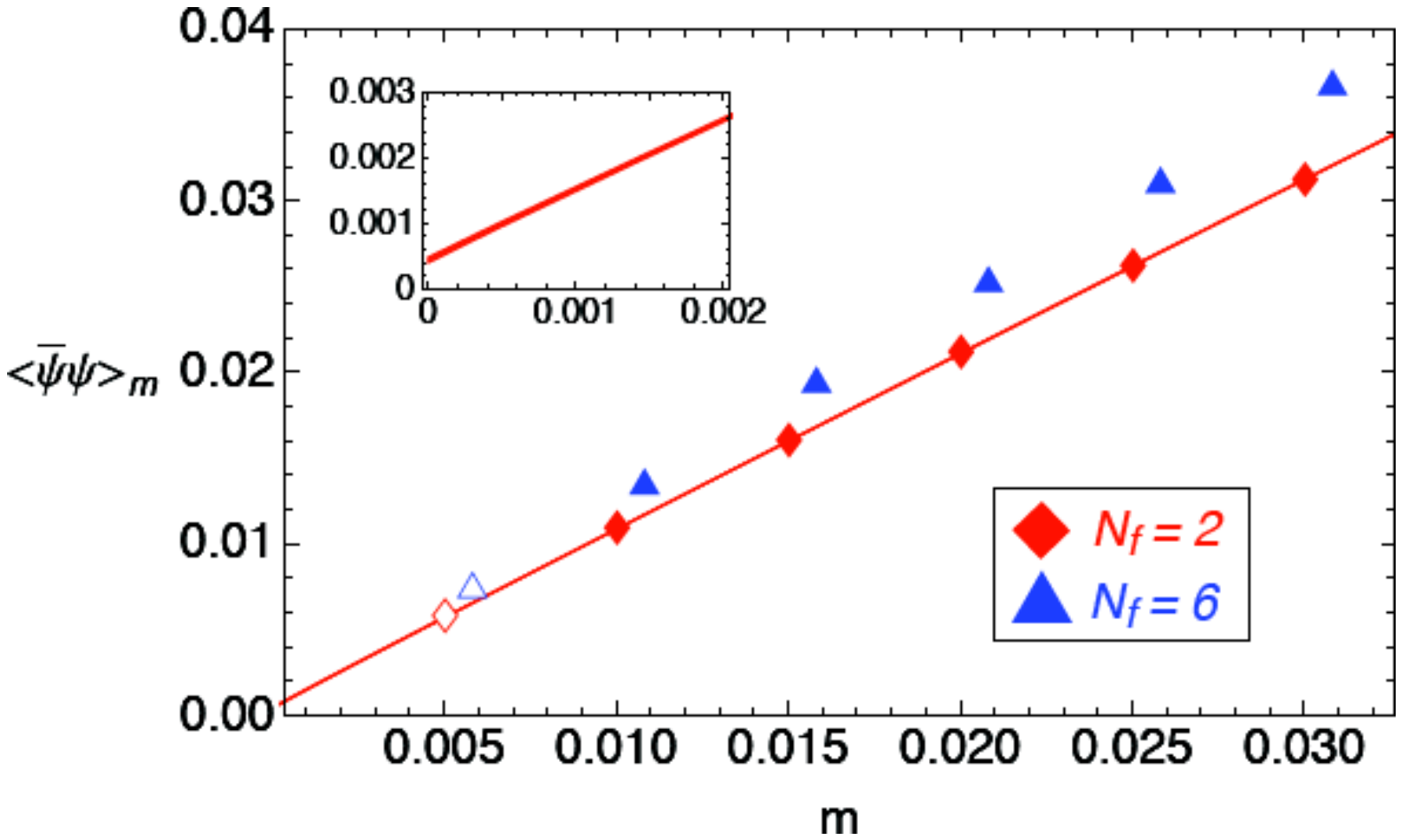
- $\alpha_C$ includes "contact term" $m\Lambda^2 \sim ma^{-2}$
- NNLO $M_P^2$ coefficients enhanced by $N_f^2$ (Bijnens & Lu, 2009)

# Goldstone decay constant



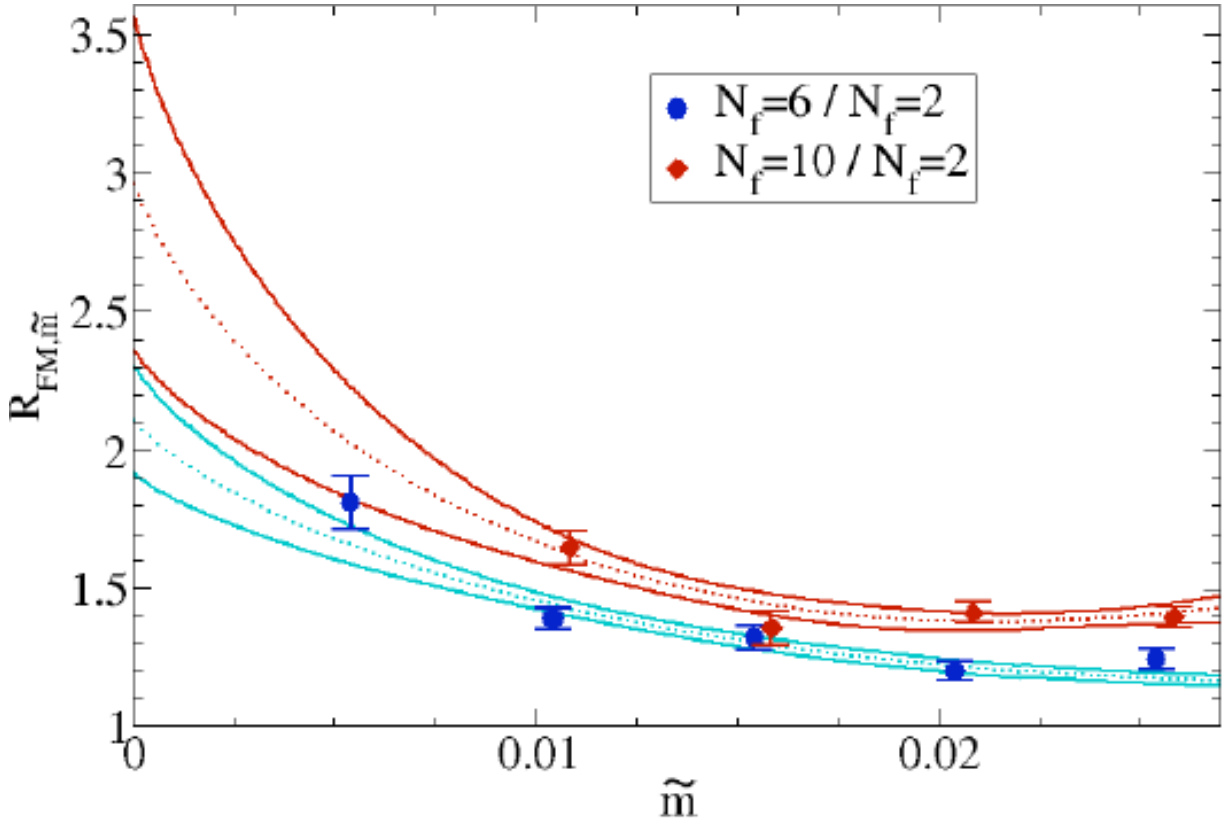

Joint NNLO$\chi$PT fit to $N_f = 2$ $F_P$, $M_P^2$, $\langle \overline{\psi}\psi \rangle$

# Chiral condensate




Joint NNLO$\chi$PT fit to $N_f = 2$ $F_P$, $M_P^2$, $\langle \overline{\psi}\psi \rangle$
Linear term clearly dominant

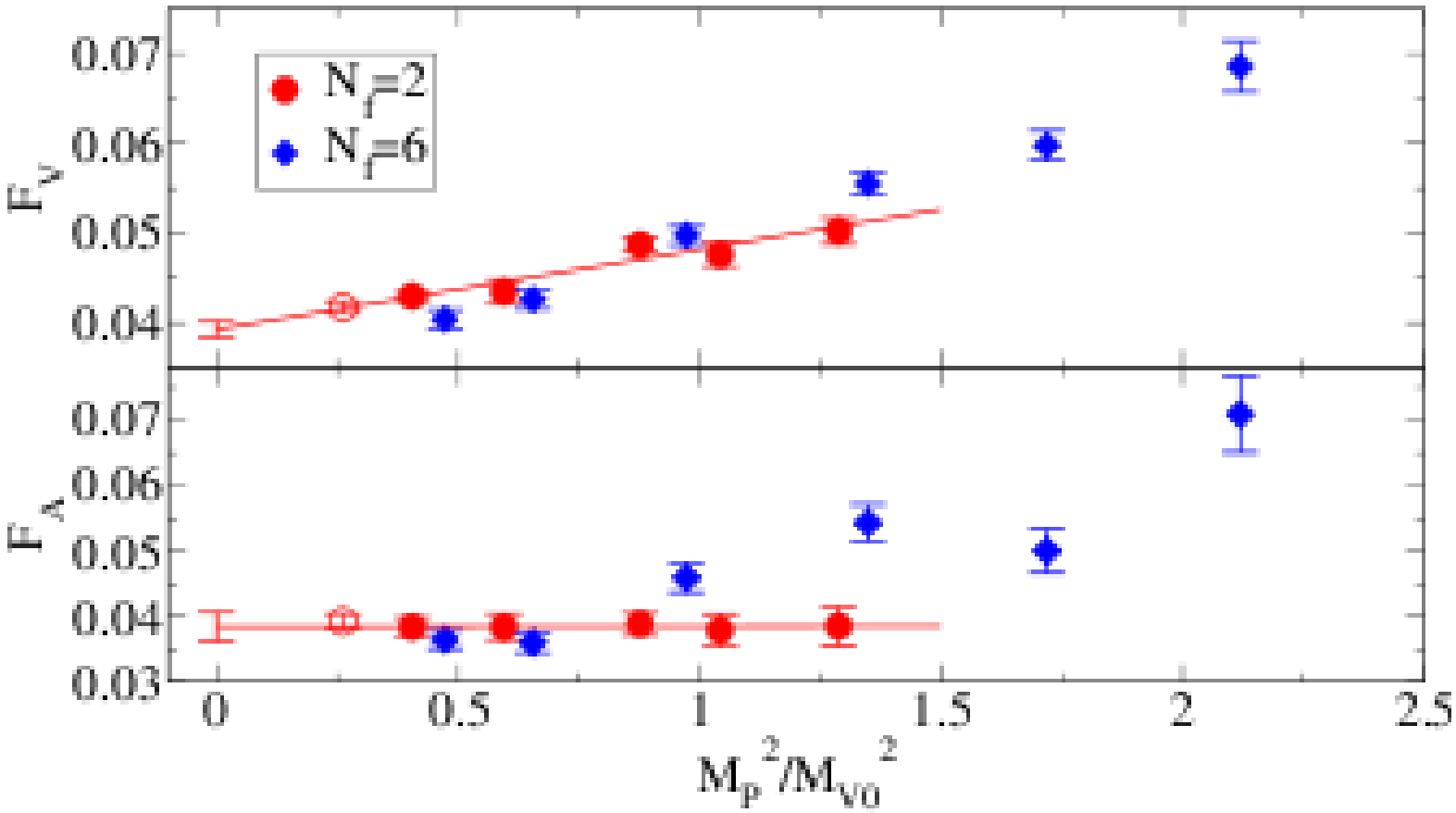
# Very preliminary $N_f = 10$ condensate enhancement




✦**Very Preliminary:** Observing more enhancement at 10f

*P. Vranas, LLNL*

# Vector and axial decay constants



# Conserved and local domain wall currents

Conserved currents:

$$\mathcal{V}^{\mu a}(x) = \sum_{s=0}^{L_s-1} j^{\mu a}(x,s) \qquad \mathcal{A}^{\mu a}(x) = \sum_{s=0}^{L_s-1} \text{sign}\left(s - \frac{L_s-1}{2}\right) j^{\mu a}(x,s)$$

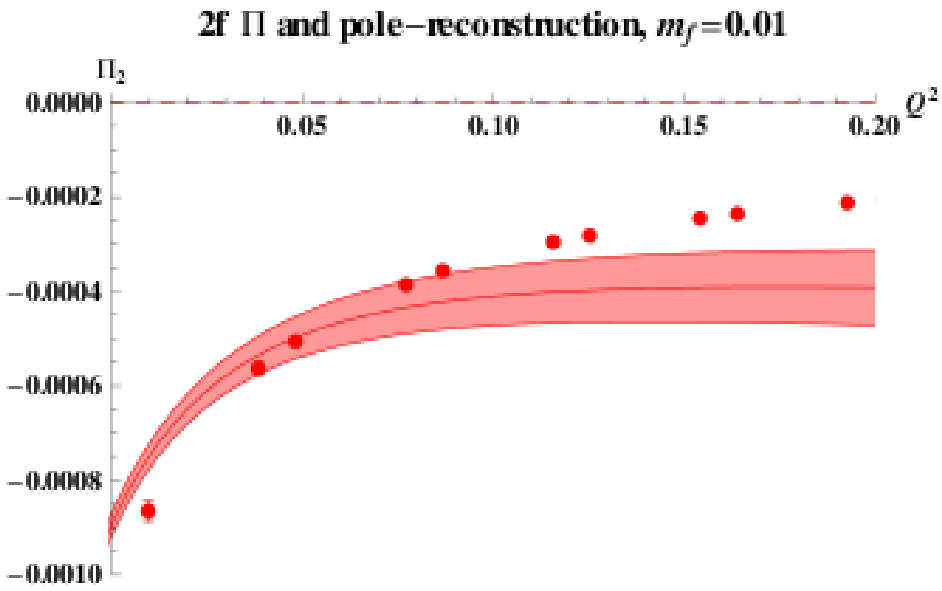
$$j^{\mu a}(x,s) = \overline{\Psi}(x+\widehat{\mu},s)\frac{1+\gamma^\mu}{2}\tau^a U^\dagger_{x,\mu}\Psi(x,s) - \overline{\Psi}(x,s)\frac{1-\gamma^\mu}{2}\tau^a U_{x,\mu}\Psi(x+\widehat{\mu},s)$$

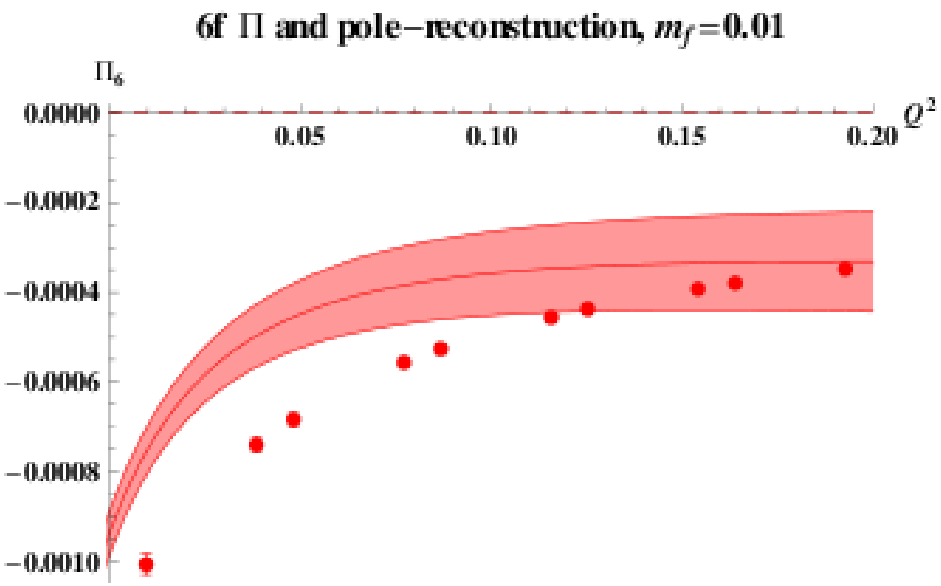
Local currents:

$$V^\mu(x) = \overline{q}(x)\gamma^\mu\tau^a q(x) \qquad A^\mu(x) = \overline{q}(x)\gamma^\mu\gamma^5\tau^a q(x)$$

$$q(x) = P_L\Psi(x,0) + P_R\Psi(x,L_s-1)$$

# Single-pole approximations to $\Pi_{V-A}$



# $S$ in $\chi$PT, for $N_f = 2$

$$S = \frac{1}{12\pi}\left(\bar{\ell}_5 + \log\left[\frac{m_\pi^2 \frac{v^2}{f_\pi^2}}{M_H^2}\right] - \frac{1}{6}\right)$$

$\bar{\ell}_5$ is extracted from

Gasser & Leutwyler, 1984

$$\Pi^{\perp}_{V-A}(q^2) = -F_\pi^2 + q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5 - \frac{1}{3}\right) + \frac{2}{3}(1+x)\bar{J}(x)\right]$$

$$\bar{J}(x) = \frac{1}{16\pi^2}\left(\sqrt{1+x}\log\left[\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1}\right] + 2\right), \quad x \equiv 4M_\pi^2/q^2$$

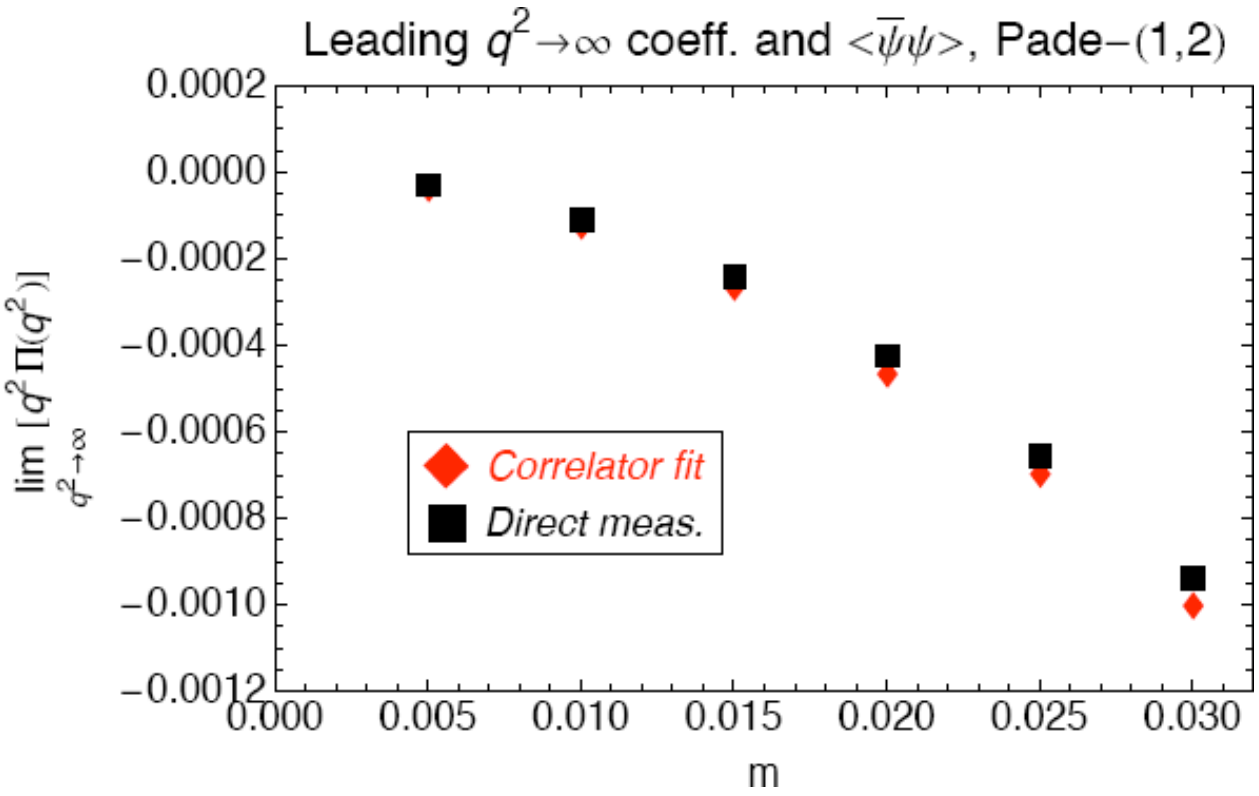
- As discussed above, $\chi$PT inapplicable for $N_f = 6$
- General-$N_f$ corrections for $\bar{\ell}_5$ not yet known
- Must take only two flavors to the chiral limit,

  any others remain massive

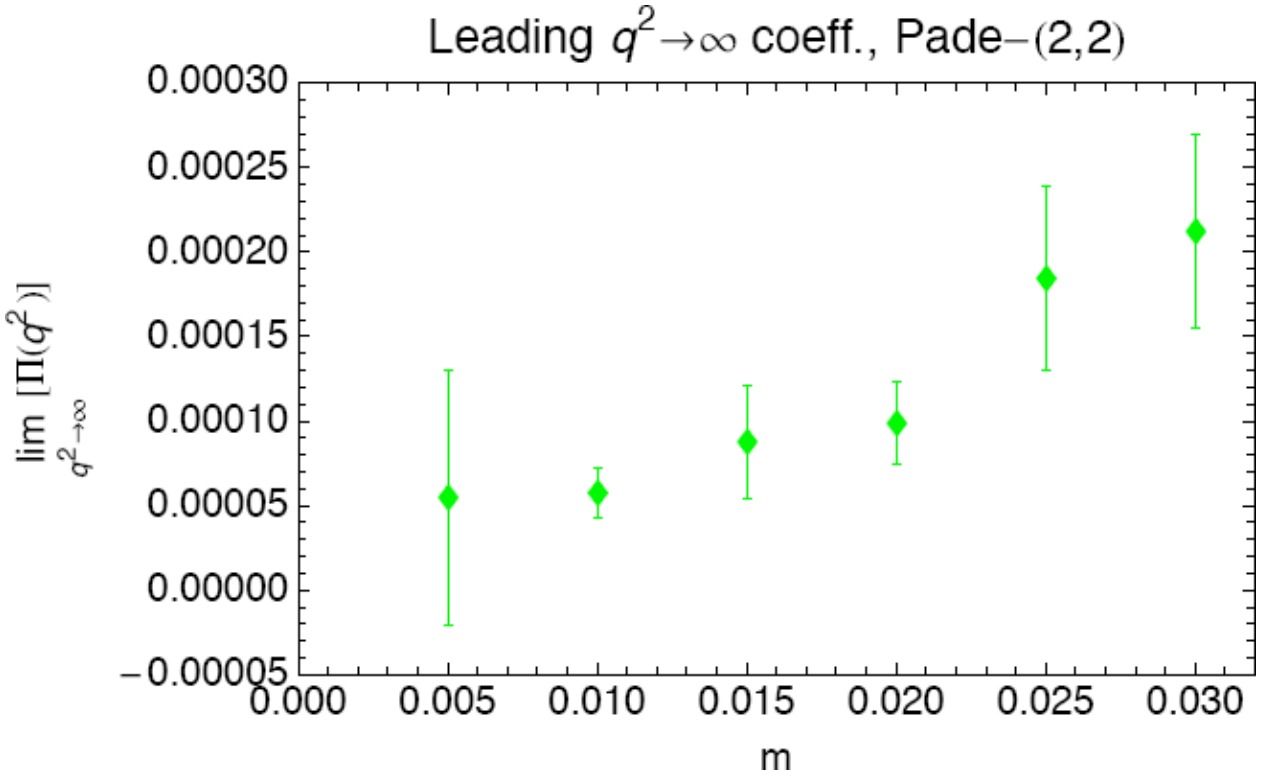
# Comparing Padé and OPE, $N_f = 2$

As $Q^2 \to \infty$,

$$\Pi_{V-A}(Q^2) \to \frac{N_c}{8\pi^2} m^2 + \frac{m \langle \overline{\psi}\psi \rangle}{Q^2} + \mathcal{O}(\alpha) + \mathcal{O}(Q^{-4})$$



# Corrections to the first Weinberg sum rule, $N_f = 2$




$Q^4$ term in numerator of (2, 2) Padé is small

$$\frac{a_0 + a_1 Q^2 + a_2 Q^4}{1 + b_1 Q^2 + b_2 Q^4} = \left[ -F_0^2 + \frac{Q^2 F_1^2}{M_1^2 + Q^2} - \frac{Q^2 F_2^2}{M_2^2 + Q^2} \right]$$